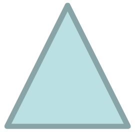
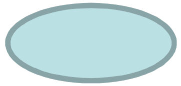
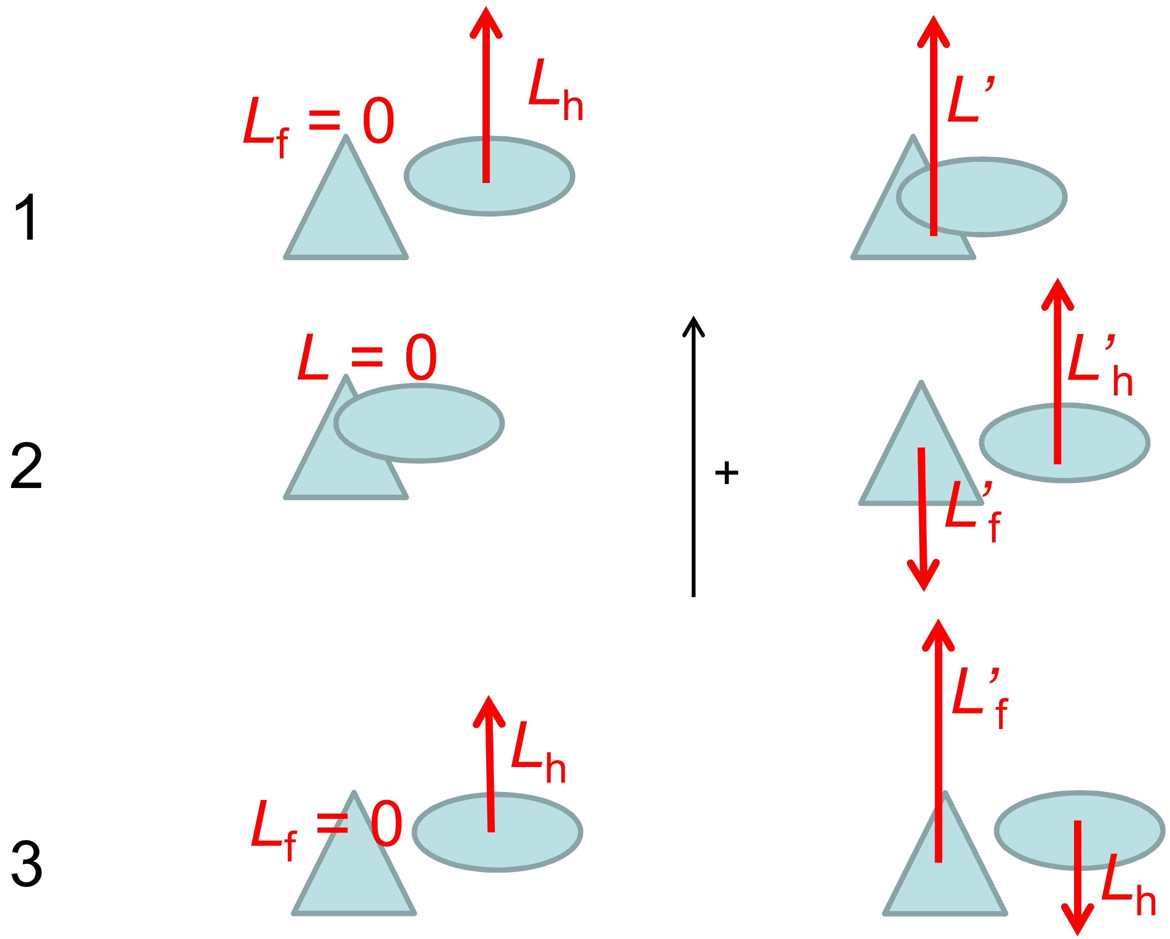
Kap 9+10 Rotasjon, spinn.

Eksempler og demonstrasjoner i forelesning

(N1-rot):
Bevaring av spinn i kontorstol

 = stol+ foreleser, spinn L_f  = hjul, spinn L_h

$$L = L_f + L_h$$



$L' = L_h$

$0 = L'_h - L'_f$

$\Rightarrow \quad L'_f = L'_h$

$L_h = L'_f - L_h$

$\Rightarrow \quad L'_f = 2L_h$

Spinn: $L = I\,\omega$ = **konstant !**

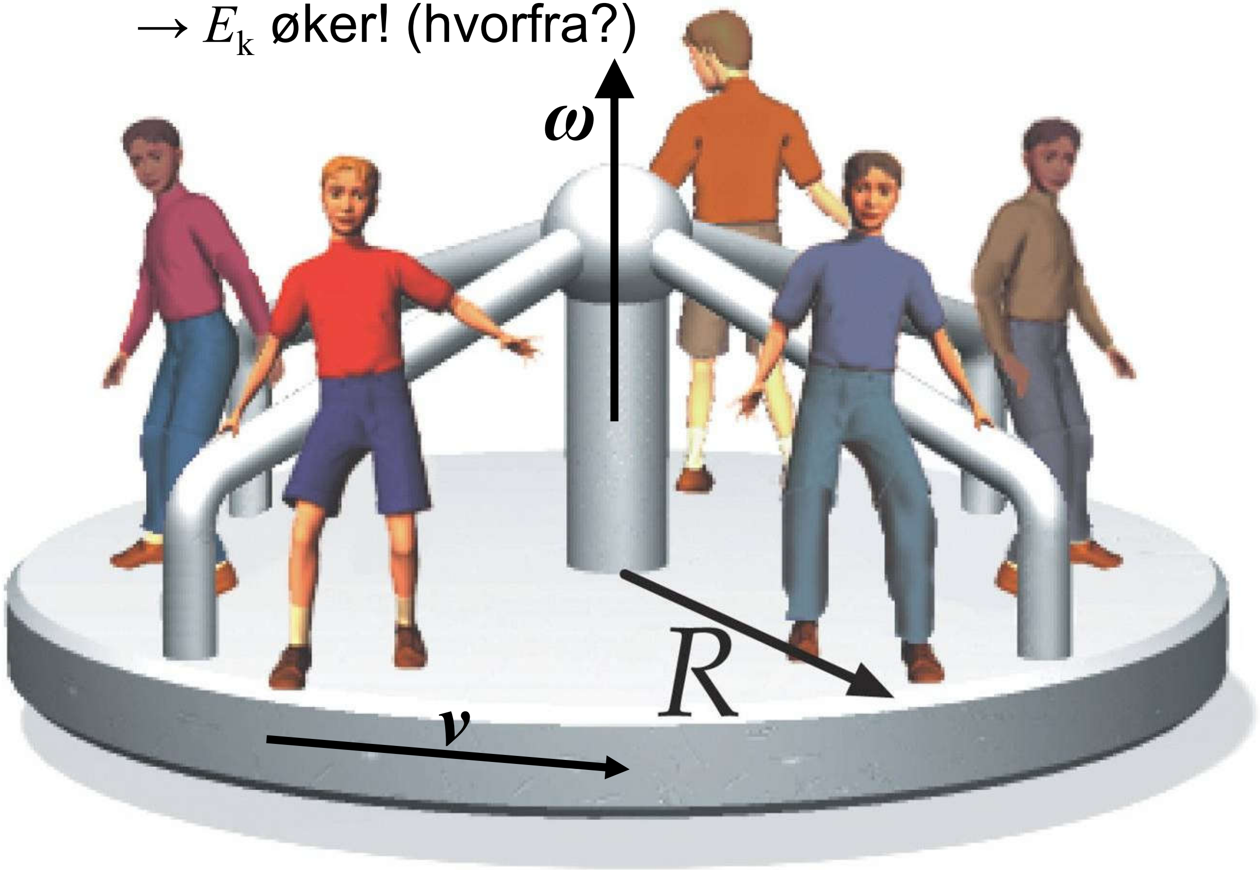
Personer inn mot sentrum: $I = \Sigma\, m_i r_i^2$ avtar — ω må øke!

Ikke stivt legeme!

Kinetisk energi: $E_k = \frac{1}{2} I \omega^2 = \frac{1}{2} L\, \omega$

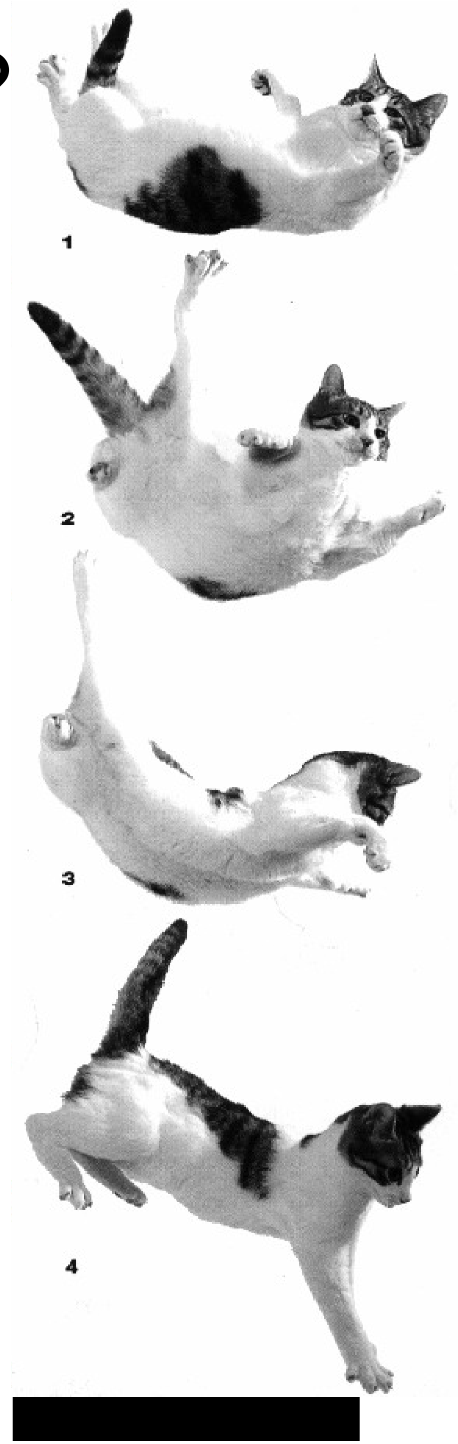
→ L konstant, ω øker

→ E_k øker! (hvorfra?)



Spinn for fallende katt bevart?

Katter lander
- alltid på føttene!



L = **0** ved start og ved slutt
L = **0** underveis !?

Gyroskop

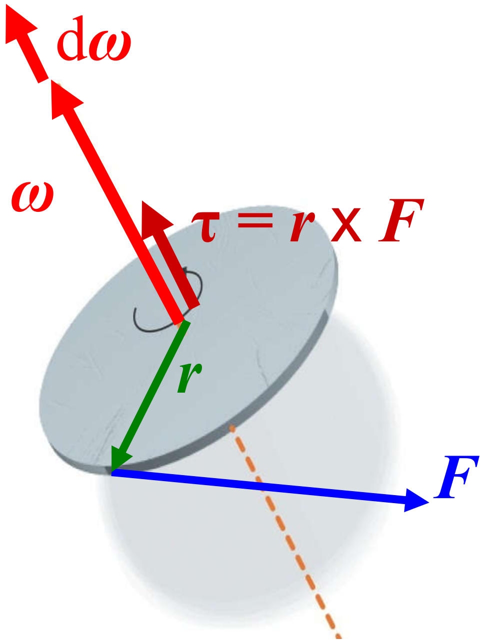
1. Lodd holder hjulet i balanse

2. $\boldsymbol{L} = I\boldsymbol{\omega}$ konstant (uten τ_{ytre})
   → gyrokompass

3. Stor motstand mot endring

4. Endring av akseretning ved kraft normalt på endringen

(N2-rot)



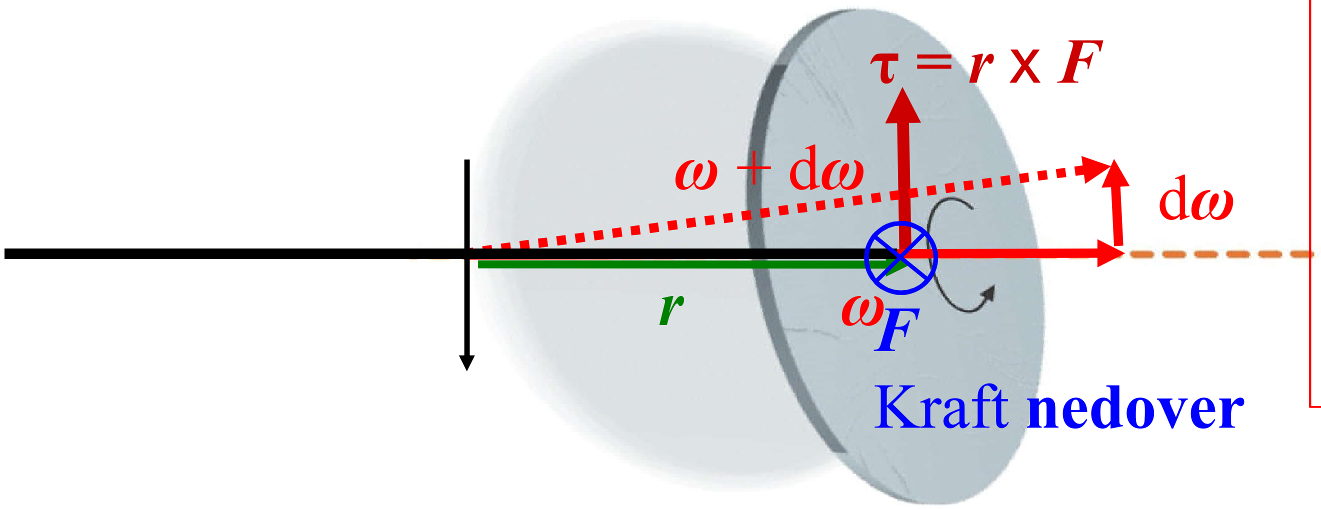
Alle legemer:
$\boldsymbol{\tau} = \mathrm{d}\boldsymbol{L} / \mathrm{d}t$ (10.29)

Stivt legeme
om symmetriakse:
$\boldsymbol{L} = I \ \boldsymbol{\omega}$
=>
$\boldsymbol{\tau} = I \ \mathrm{d}\boldsymbol{\omega} / \mathrm{d}t$ (10.7)

Raskere rotasjon om samme akse:
$\boldsymbol{\omega} \rightarrow \boldsymbol{\omega} + \mathrm{d}\boldsymbol{\omega}$ alle i samme retning
(N2-rot): $\boldsymbol{\tau} \, \mathrm{d}t = I \ \mathrm{d}\boldsymbol{\omega}$
=> $\boldsymbol{\tau}$ i samme retning som $\mathrm{d}\boldsymbol{\omega}$
=> $\boldsymbol{F}$ som i figuren

Hva hvis akseretningen skal endres?

Endring av akseretning
Sett ovenfra:



Alle legemer:

$\boldsymbol{\tau} = \mathrm{d}\boldsymbol{L} / \mathrm{d}t$ (10.29)

Stivt legeme om symmetriakse:

$\boldsymbol{L} = I\ \boldsymbol{\omega}$

=>

$\boldsymbol{\tau} = I\ \mathrm{d}\boldsymbol{\omega} / \mathrm{d}t$ (10.7)

Endring akseretning:

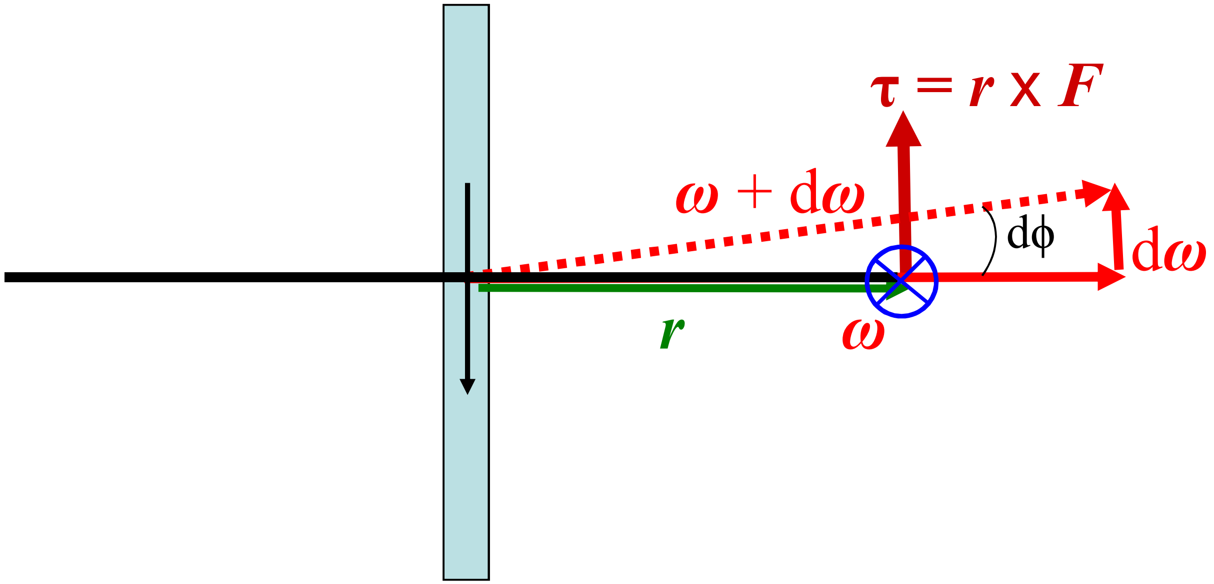
$\boldsymbol{\omega} \rightarrow \boldsymbol{\omega} + \mathrm{d}\boldsymbol{\omega}$

(N2-rot): $\boldsymbol{\tau}\,\mathrm{d}t = I\ \mathrm{d}\boldsymbol{\omega}$

=> $\boldsymbol{\tau}$ i samme retning som $\mathrm{d}\boldsymbol{\omega}$

=> $\boldsymbol{F}$ nedover

Med vedvarende ***F*** får vi **presesjon**



Alle legemer:

$\boldsymbol{\tau} = \mathrm{d}\boldsymbol{L} / \mathrm{d}t$ (10.29)

Stivt legeme om symmetriakse:

$\boldsymbol{L} = I\,\boldsymbol{\omega}$

=>

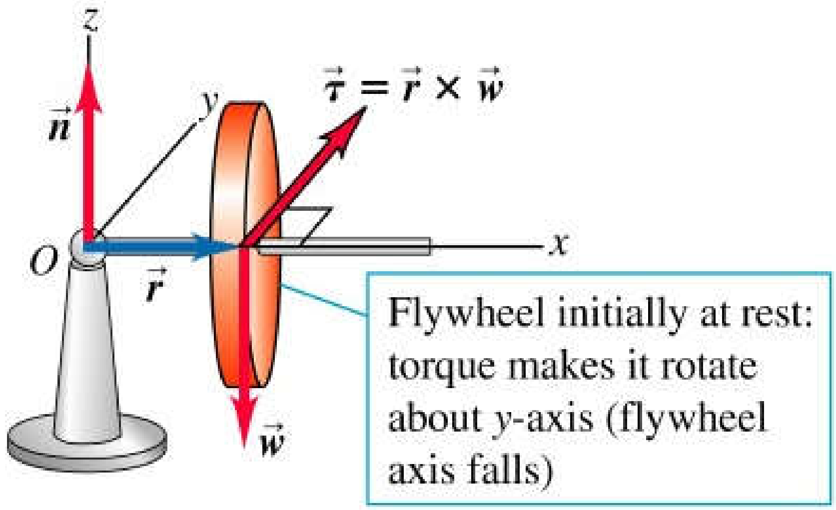
$\boldsymbol{\tau} = I\,\mathrm{d}\boldsymbol{\omega} / \mathrm{d}t$ (10.7)

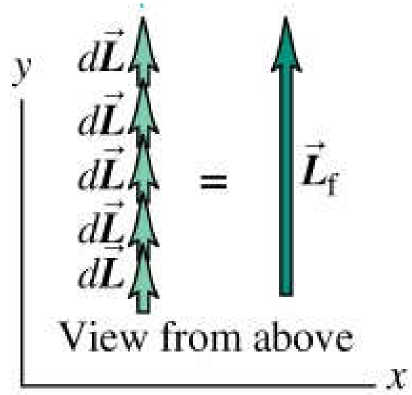
$$\mathrm{d}\phi = \frac{\mathrm{d}\omega}{\omega}$$

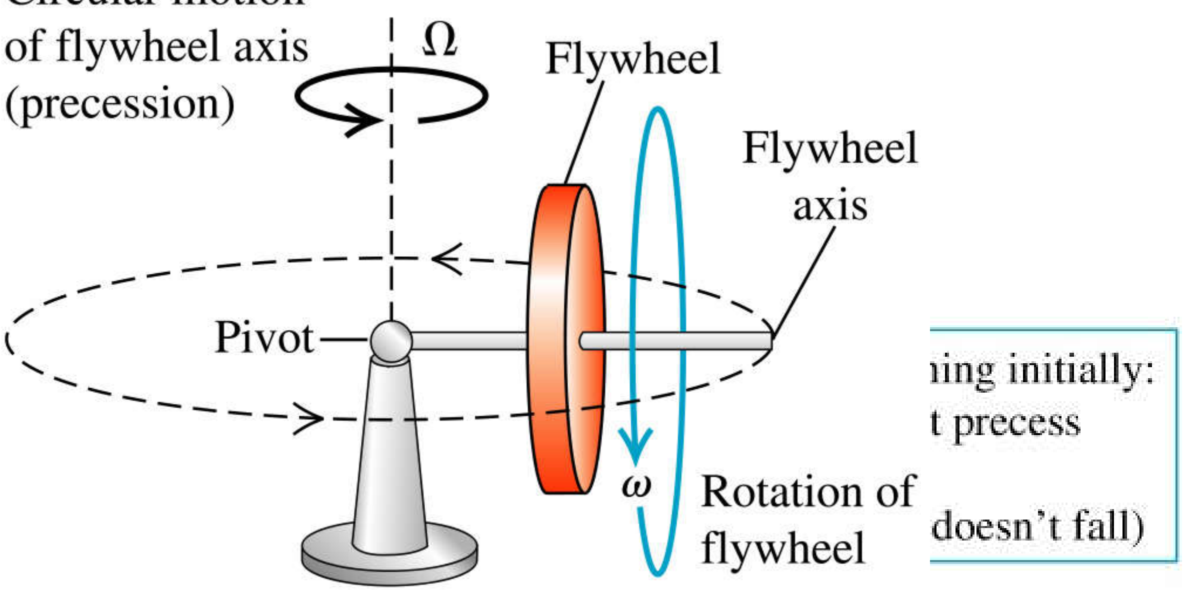
Presesjonsfrekvens: $\Omega_p = \frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{\mathrm{d}\omega}{\mathrm{d}t}\frac{1}{\omega} \overset{\text{(N2-rot)}}{=} \frac{\tau}{I}\frac{1}{\omega} = \frac{Fr}{I\omega}$

Sykkelhjul

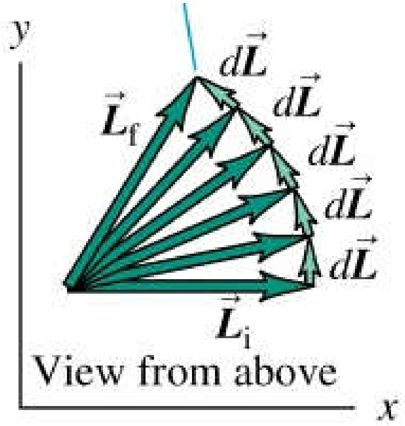
Ikke-roterende hjul:







Sett ovenfra:



Snurrebasser

Alle legemer:

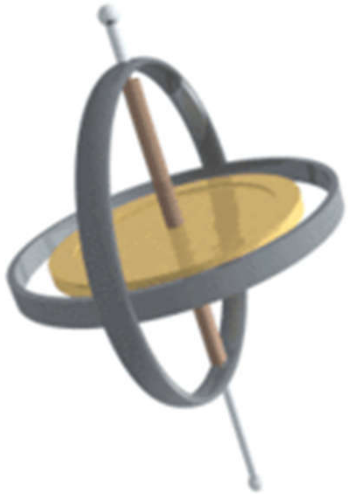
$$\boldsymbol{\tau} = \mathrm{d}\boldsymbol{L} / \mathrm{d}t \qquad (10.29)$$

Stivt legeme om symmetriakse:

$$\boldsymbol{L} = I\ \boldsymbol{\omega}$$

=>

$$\boldsymbol{\tau} = I\ \mathrm{d}\boldsymbol{\omega} / \mathrm{d}t \qquad (10.7)$$



Rotasjon av stive legemer

- Treghetsmoment $I = \Sigma\, r_i^2\, m_i$ (om en gitt akse)
- Rotasjonsenergi $E_k = ½\, \Sigma\, m_i\, v_i^2 = ½\, I\, \omega^2$
- Kraftmoment: $\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}$
- Spinn (dreieimpuls) $\boldsymbol{L} = \boldsymbol{r} \times m\, \boldsymbol{v}$
- Spinnsatsen (N2-rot): $\boldsymbol{\tau} = \mathrm{d}\boldsymbol{L}/\mathrm{d}t$
- Ingen ytre moment (N1-rot): $\boldsymbol{L}$ = konst.

stivt legeme om sym.akse:

$= I\, \boldsymbol{\omega}$

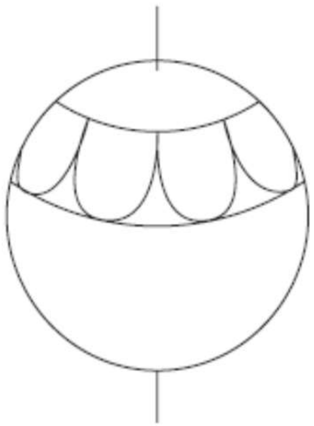
$= I\, \mathrm{d}\boldsymbol{\omega}/\mathrm{d}t$

Matematisk forklaring av fysikken ofte eneste mulige

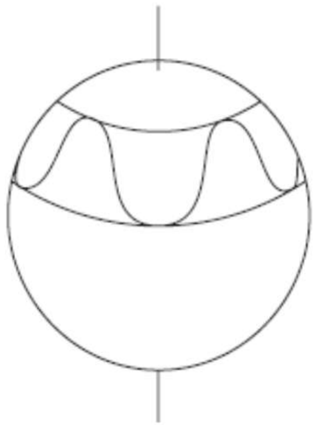
Richard Feynman (am. fysiker/pedagog, 1918-1988):

"…many simple things can be deduced mathematically more rapidly than they can really be understood in a fundamental or simple sense. This is a strange characteristic, and as we get into more and more advanced work there are circumstances in which mathematics will produce results which *no one* has really been able to understand in any direct fashon."

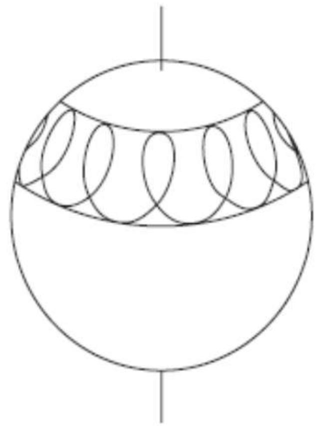
Nutasjon



(A) Released from rest



(B) Released with forward speed



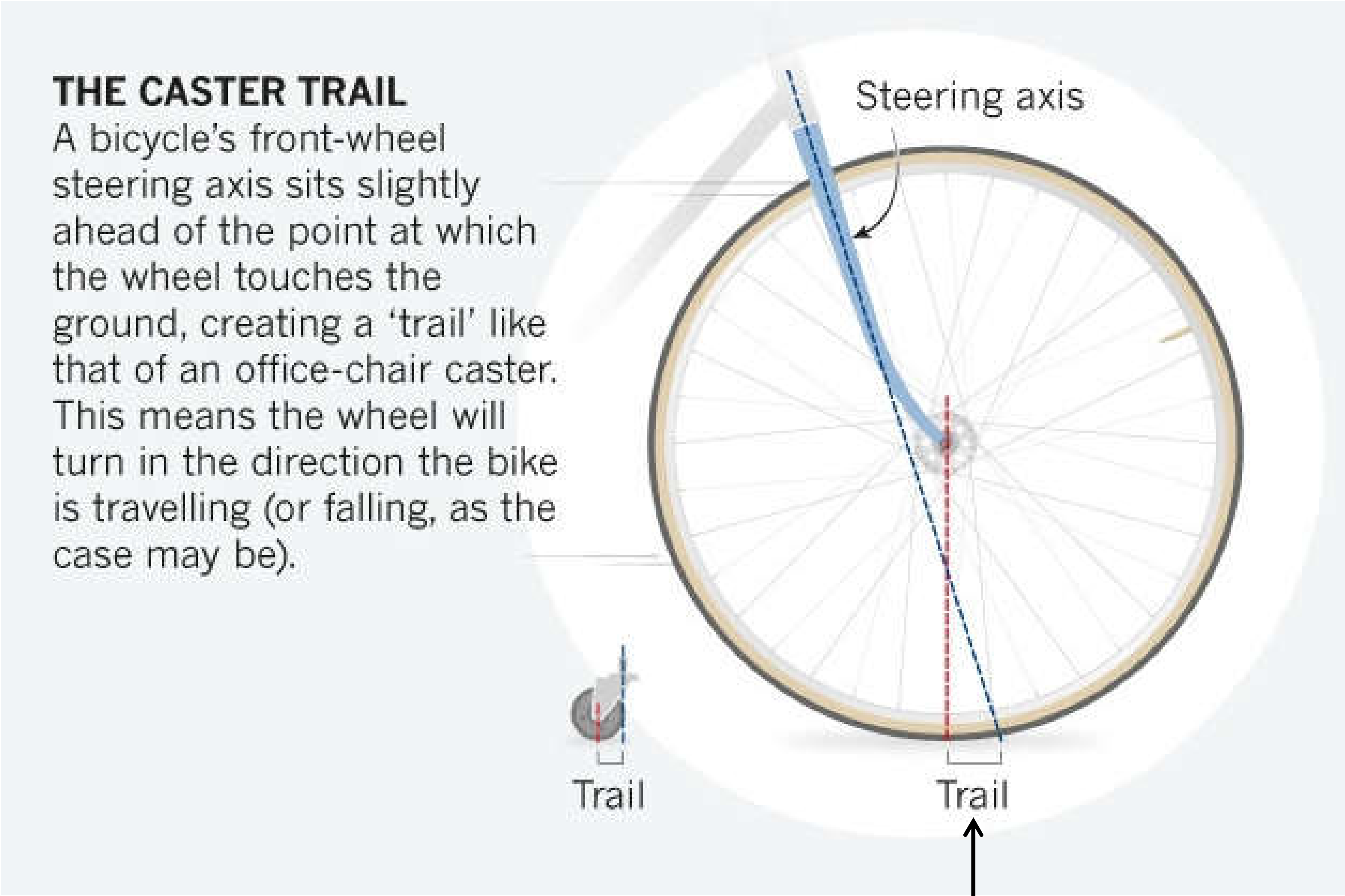
(C) Released with backward speed

Hva betyr gyroeffekten for å holde sykkel oppe?

Mr. Jones testet dette med hjul som roterte motsatt retning, dvs. motsatt gyroeffekt.

=> En URB (UnRidableBicycle)?

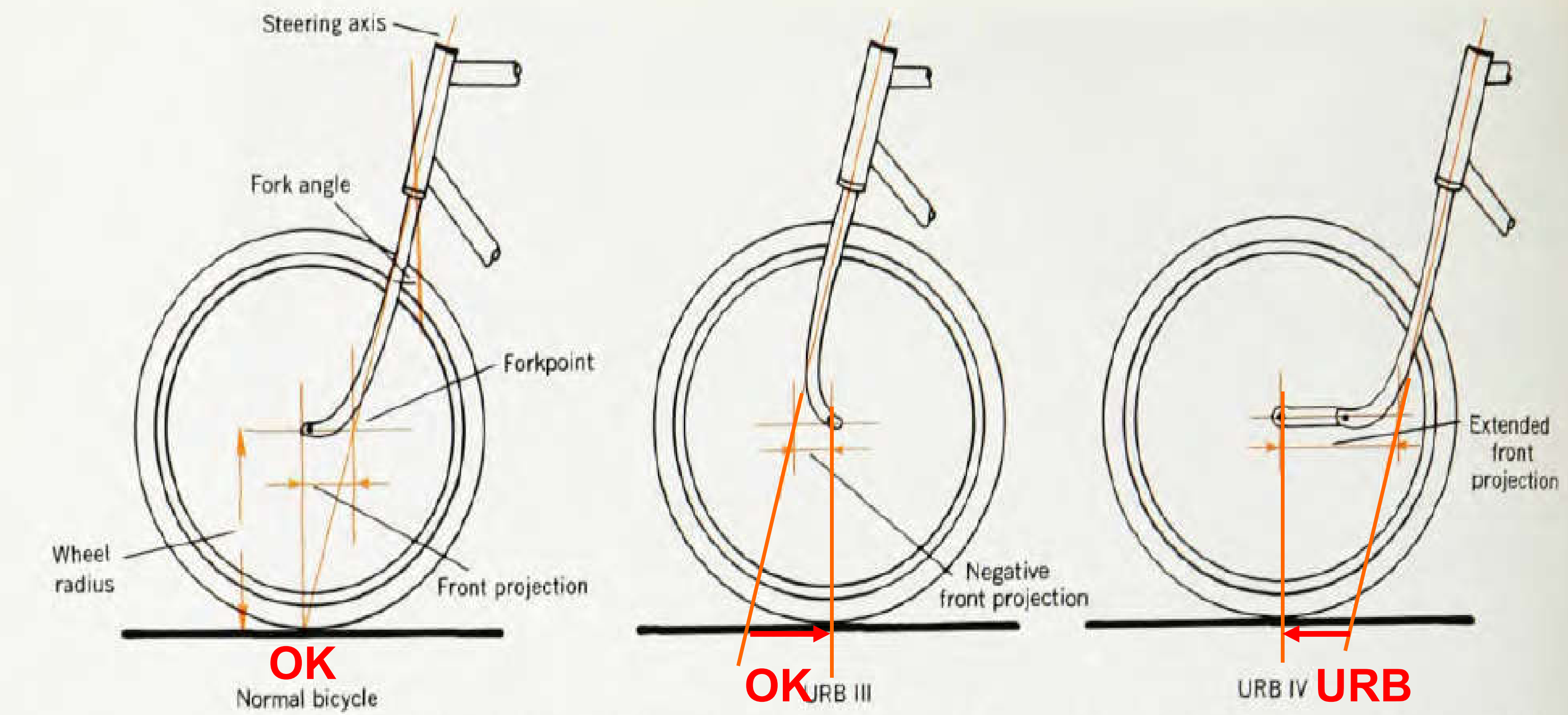
Gyroeffekten ingen praktisk betydning!



Viktig at bakkekontakt er bak styreaksen

Fra: Nature 535, 21.July 2016, pp. 338-342

Mr. Jones' URB-sykler:



FRONT-FORK GEOMETRY. On left is a normal bicycle. Center shows URB III with reversed forks giving a negative front projection, and on right is URB IV with extended front projection. —FIG. 2

Snøsykler (snowbikes): Null gyroeffekt

Fra wikipedia

For spesielt interesserte:

Sykkelens stabilitet, referanser.

Noen av lenkene krever IP-adresse fra NTNU for å få tilgang.

Kooijman et al: A Bicycle Can Be Self-Stable Without Gyroscopic or Caster Effects
Science 332 (2011), pp. 339-342 (med mange flere referanser)
http://science.sciencemag.org/content/332/6027/339.full

B. Borrell: The bicycle problem that nearly broke mathematics,
Nature 535, 21.July 2016, pp. 338-342.
http://www.nature.com/news/the-bicycle-problem-that-nearly-broke-mathematics-1.20281

J. Matson: A Bicycle Built for None: What Makes a Riderless Bike Stable?
Scientific American April 2011
http://www.scientificamerican.com/article/self-stable-bike/

D.E.H. Jones: The stability of the bicycle,
Physics Today, April 1970, pp. 34-40
http://www.phys.lsu.edu/faculty/gonzalez/Teaching/Phys7221/vol59no9p51_56.pdf
http://home.phys.ntnu.no/brukdef/undervisning/tfy4145/arkiv/2010/diverse/UnridableBicycle.pdf

J. Lowell and H.D. McKell, The Stability of Bicycles, (teknisk avansert)
Am. J. Phys. 50 (1982), pp. 1106-1112.
https://www.me.utexas.edu/~longoria/VSDC/09_vehicle_roll_dynamics_and_control/09_3b_Lowell_1982_The%20Stability%20of%20Bicycles.pdf

Bicycle wheel / Turning a bicycle:
http://hyperphysics.phy-astr.gsu.edu/hbase/mechanics/bicycle.html#c2