

# Magnetisme

- **Magnetostatikk** (ingen tidsvariasjon):
- Kap 27. Magnetiske krefter
- **Kap 28: Magnetiske kilder**
- **Elektrodynamikk:**
- Kap 29-32:  
Tidsvariasjon: Induksjon mm.

# Kap 28: Magnetiske kilder

- **Elektrostatikk:**

Ladning  $q$  påvirkes av kraft  $q\mathbf{E}$  (Coulombs lov)

→ Definisjon  $E$ -felt

$E$ -feltet skapes fra ladninger (Coulombs lov)

- **Magnetostatikk:**

Ladning  $q$  i **bevegelse** påvirkes av kraft  $q\mathbf{v} \times \mathbf{B}$

→ Definisjon  $B$ -felt (Lorentzkrafta)

$B$ -feltet skapes fra ladninger i **bevegelse**

(Biot-Savarts lov)

- **Hjelpelover:**

Elektrostatikk: Gauss' lov

Magnetostatikk: Amperes lov

- **Magnetiske materialer**

Ferromagnetisk materiale. Magnetisering.  $\mathbf{M}$ -vektor og  $\mathbf{H}$ -vektor.

# Kap 28: Magnetiske kilder

28.1  $B$ -felt fra **enkeltiladninger** i bevægelse

28.2  $B$ -felt fra **strøm** i ledning

## 28.1+28.2 Bevegelse av ladninger gir magnetfelt $B$

- Enkeltladning  
i bevegelse:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2} \quad \text{Enhetsvektor} \quad (28.2)$$
$$= \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^3}$$

- Strøm i leder:  
(Biot-Savarts lov)

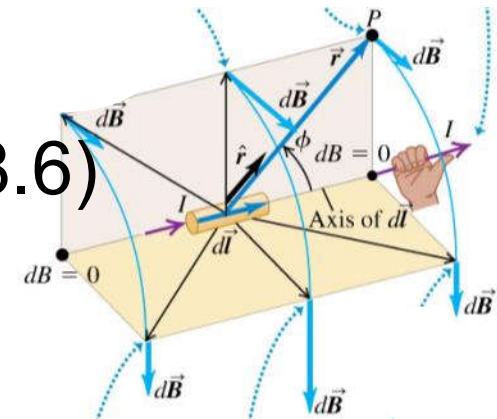
1819-25: Vitenskapelig arbeid:  
Hans Christian Ørsted, André Ampere,  
Jean-Baptist Biot, Felix Savart,  
Michael Faraday, Joseph Henry

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{s} \times \hat{r}}{r^2} \quad \text{Enhetsvektor}$$
$$= \frac{\mu_0}{4\pi} \frac{Id\vec{s} \times \vec{r}}{r^3} \quad (28.6)$$

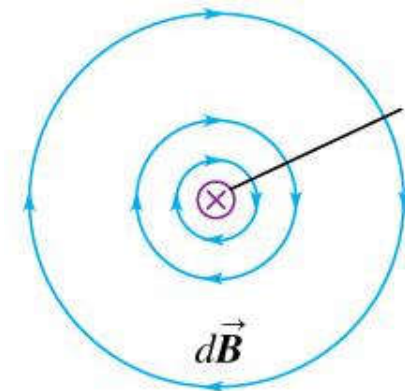
## 28.1+28.2 Bevegelse av ladninger gir magnetfelt $B$

- Enkeltladning: 
$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2} \quad (28.2)$$

- Strømelement: 
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2} \quad (28.6)$$



- Strøm i leder:  
(Biot-Savarts lov) 
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \int_{\text{ledning}} \frac{d\vec{s} \times \hat{r}}{r^2} \quad (28.7)$$



# Elmag og relativitetsteori i [Notat 3](#)

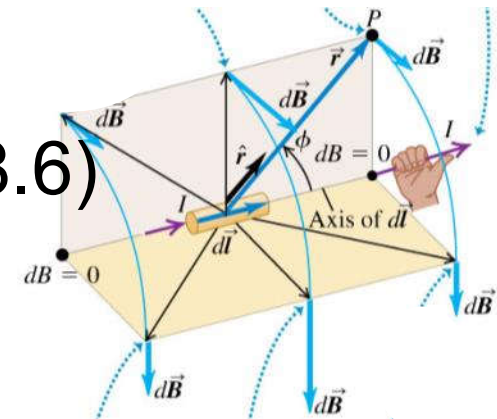
Einsteins utledning av spesiell relativitetsteori var drevet av dette problemet i elektromagnetismen:

Elektriske og magnetiske krefter er to sider av samme sak, avhengig av referansesystemet det hele observeres i.

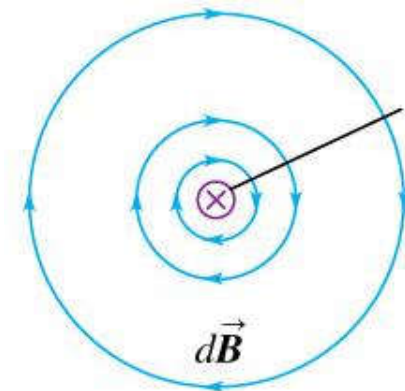
## 28.1+28.2 Bevegelse av ladninger gir magnetfelt $B$

- Enkeltladning: 
$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2} \quad (28.2)$$

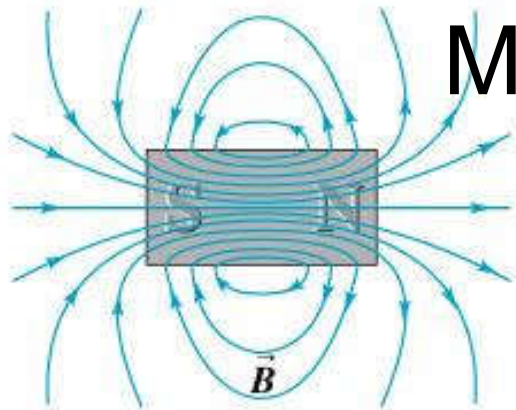
- Strømelement: 
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2} \quad (28.6)$$



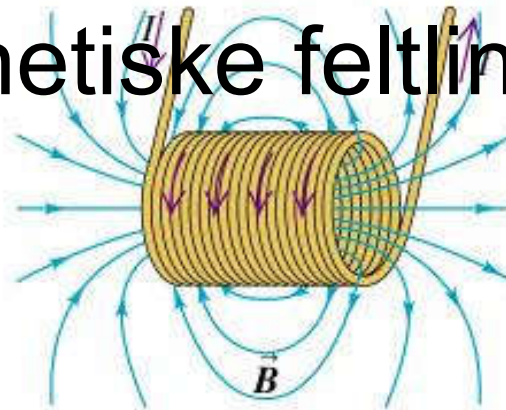
- Strøm i leder:  
(Biot-Savarts lov) 
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \int_{\text{ledning}} \frac{d\vec{s} \times \hat{r}}{r^2} \quad (28.7)$$



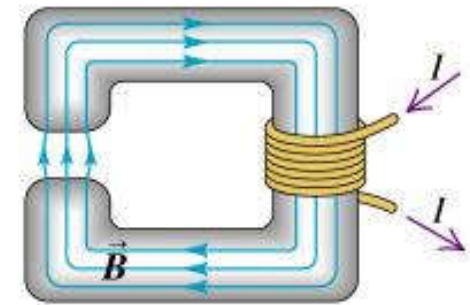
# Magnetiske feltlinjer



(a) Magnetic field lines through the center of a permanent magnet

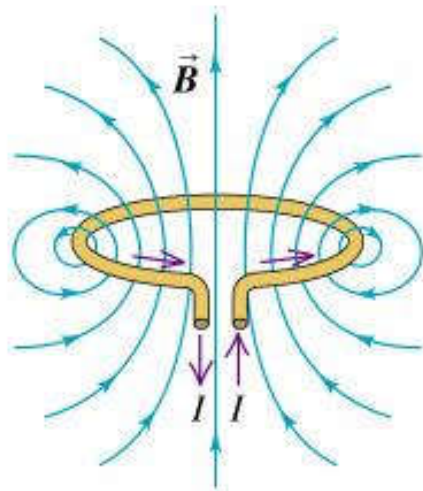


(b) Magnetic field lines through the center of a cylindrical current-carrying coil



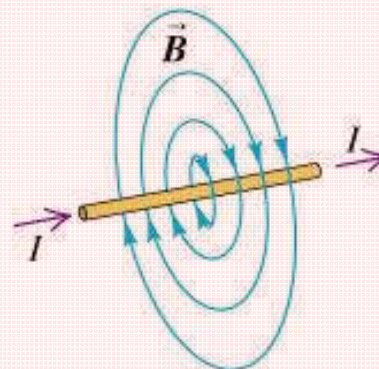
(c) Magnetic field lines through the center of an iron-core electromagnet

(Fig 27.11)

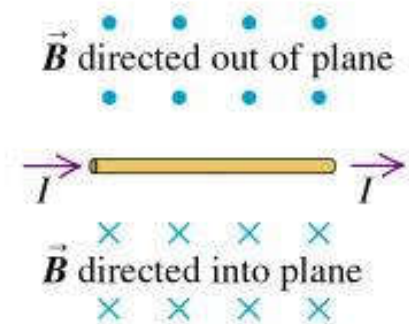


(d) Magnetic field lines in a plane containing the axis of a circular current-carrying loop

Eks. 1



(e) Magnetic field lines in a plane perpendicular to a long, straight, current-carrying wire

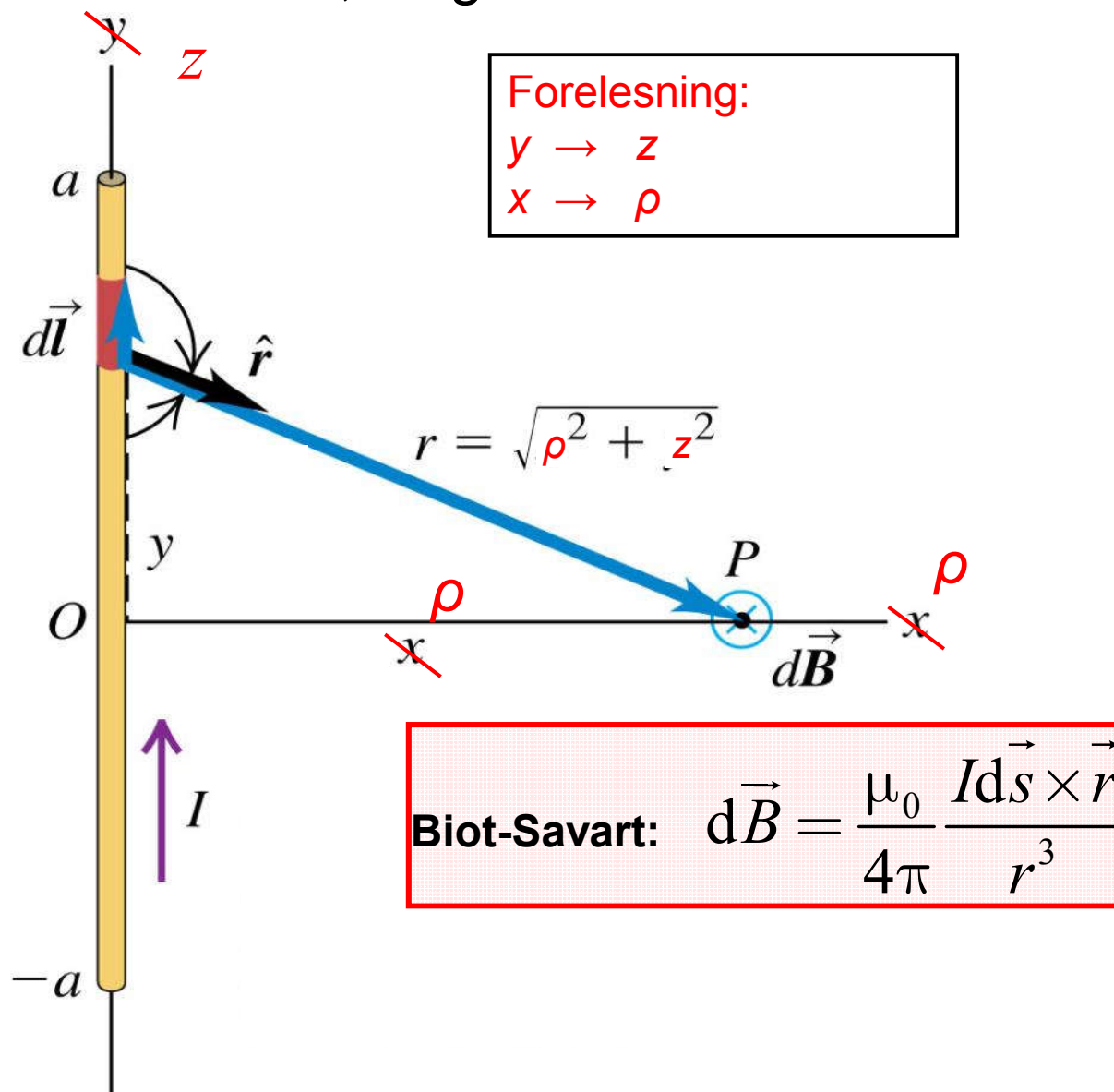


(f) Magnetic field lines in a plane containing a long, straight, current-carrying wire



Eks. 1 (Y&F Kap. 28.3):

$B$ -felt på midtnormal til rett leder, lengde  $2a$



# Rottmann integraltabell (s. 137)

$$47) \int \frac{x}{X^{3/2}} dx = \frac{-1}{ac - b^2} \frac{bx + c}{\sqrt{X}} + C$$

$$48) \int \frac{dx}{X^{3/2}} = \frac{1}{ac - b^2} \frac{ax + b}{\sqrt{X}} + C$$

$$49) \int x(ax^2 + c)^{k+1/2} dx = \frac{1}{(2k+3)a} (ax^2 + c)^{k+3/2} + C, \quad k \neq -\frac{3}{2}$$

$$50) \int \frac{dx}{x\sqrt{ax^2 + c}} = \begin{cases} \frac{-1}{\sqrt{c}} \ln C_1 \frac{\sqrt{c} + \sqrt{ax^2 + c}}{x}, & \text{for } c > 0; \\ \frac{-1}{\sqrt{-c}} \arcsin \sqrt{\frac{-c}{a}} \frac{1}{|x|} + C_2, & \\ \text{for } c < 0 \text{ og } |x| \geq \sqrt{\frac{-c}{a}} \end{cases}$$

$$*) \quad X = ax^2 + 2bx + c$$

$$X = z^2 + \rho^2, \quad \text{dvs:}$$

$$x = z$$

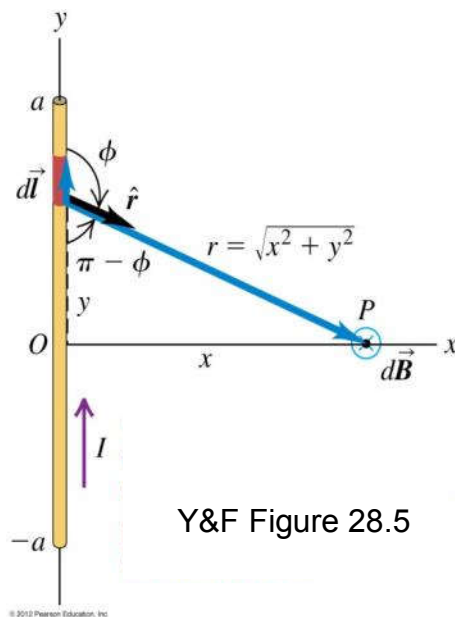
$$a = 1$$

$$b = 0$$

$$c = \rho^2$$

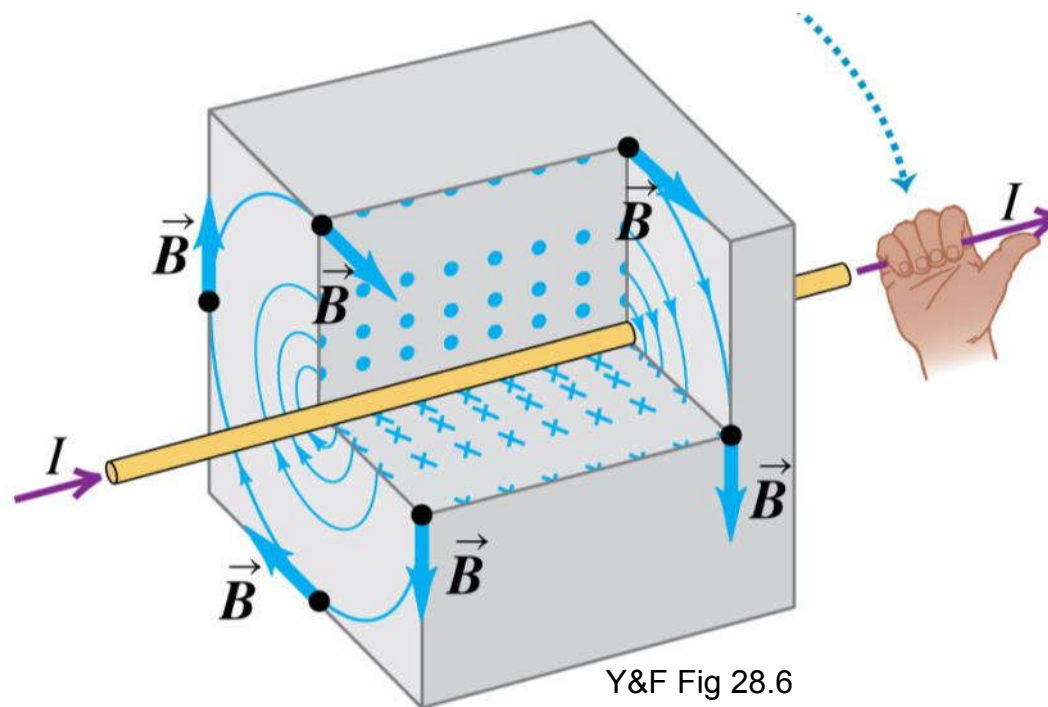
Rett leder lengde  $2a$ :

$$\vec{B} = \frac{\mu_0}{4\pi} I \frac{2a}{\rho} \frac{1}{\sqrt{a^2 + \rho^2}} \hat{\phi} \quad (28.8)$$



Nærme rett leder ( $a \gg \rho$ ):

$$B = \frac{\mu_0}{2\pi} \frac{I}{\rho} \quad (28.9)$$



Felt rundt uendelig lang, rett leder:

$$B = \frac{\mu_0 I}{2\pi \rho}, \quad \frac{\mu_0}{2\pi} = 2 \cdot 10^{-7} \text{ Tm/A}$$

**Eksempler:**

1) Under én kraftledning:

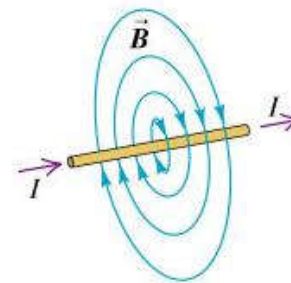
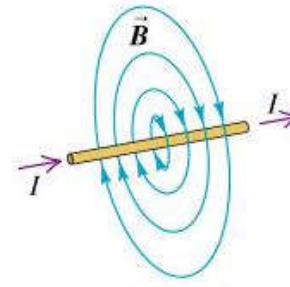
$$I = 1 \text{ kA}, \quad \rho = 100 \text{ m} \quad \Rightarrow \quad B = 2 \mu\text{T}$$

2) Nær f. eks. hårføner:

$$I = 3 \text{ A}, \quad \rho = 5 \text{ cm} \quad \Rightarrow \quad B = 12 \mu\text{T}$$

Jordmagnetismen:  $B = 0,5 \text{ G} = 50 \mu\text{T}$   
(statisk felt)

1) og 2) gjelder for enkeltledere:



For to ledere med motsatt strøm blir  $B$ -feltet betydelig lavere.

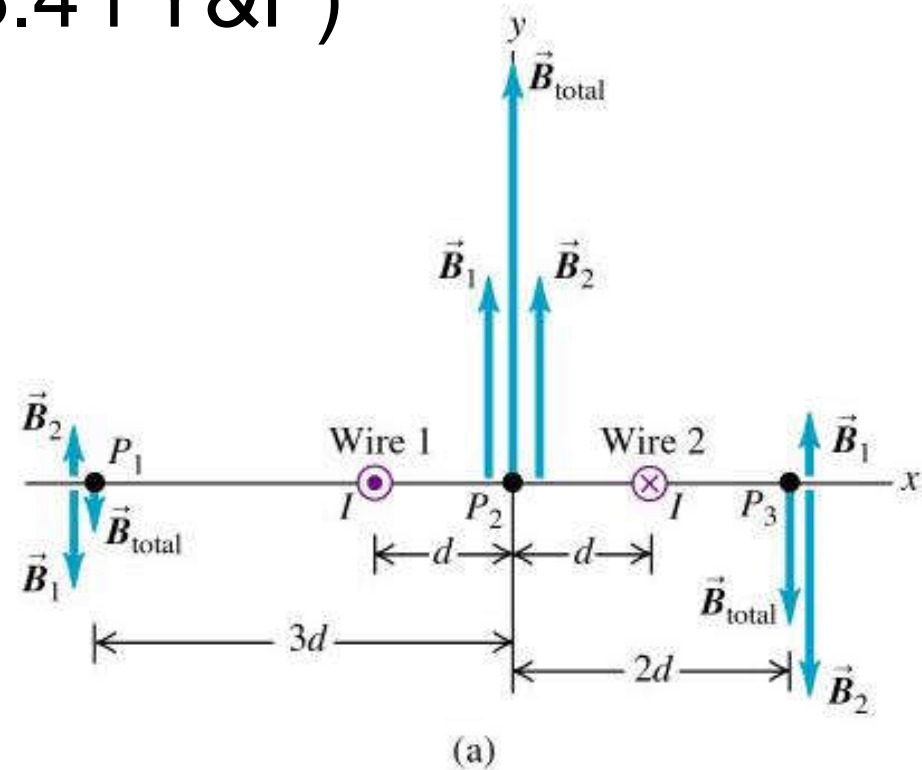
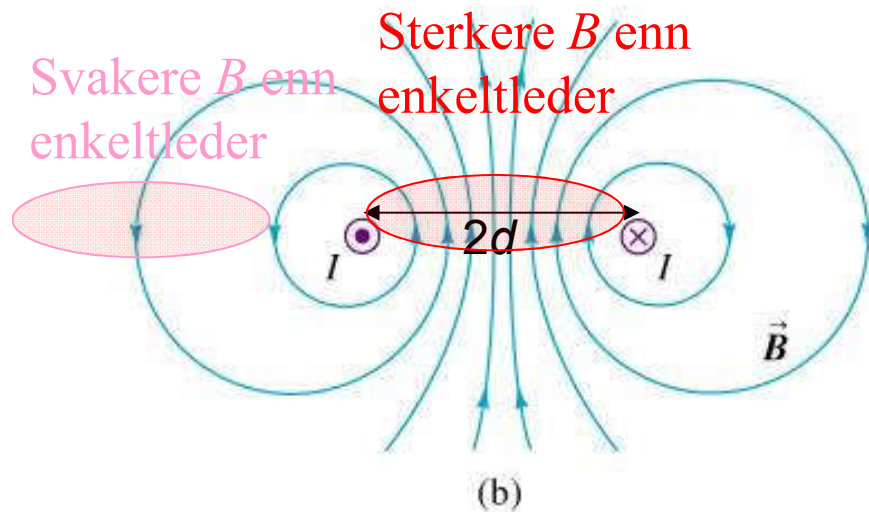
Eksempler på magnetfeltnivå ved høyspentledninger:

- Gjelder vekselfelt 50 Hz:
- Grenseverdien er 200  $\mu\text{T}$  for befolkningen
- Ved nybygg eller nye anlegg hvor årsgjennomsnittet overskrider 0,4  $\mu\text{T}$ , skal tiltak vurderes.

Spenningsnivå (kilovolt)	Strømstyrke (Ampere)	Avstand i meter som gir 0,4 $\mu\text{T}$
22	150	15
22	200	18
66	200	20
66	300	25
132	300	35
132	400	40
300	450	60
300	650	70
420	800	85
420	1100	100

Tabellen viser eksempler på hvor langt fra nærmeste ledning magnetfeltet vil være nede i utredningsnivået 0,4  $\mu\text{T}$ . Eksemplene gjelder vanlig planoppheng, og er satt opp ut fra typiske gjennomsnittsverdier på strømstyrke i ledninger med ulike spenningsnivå.

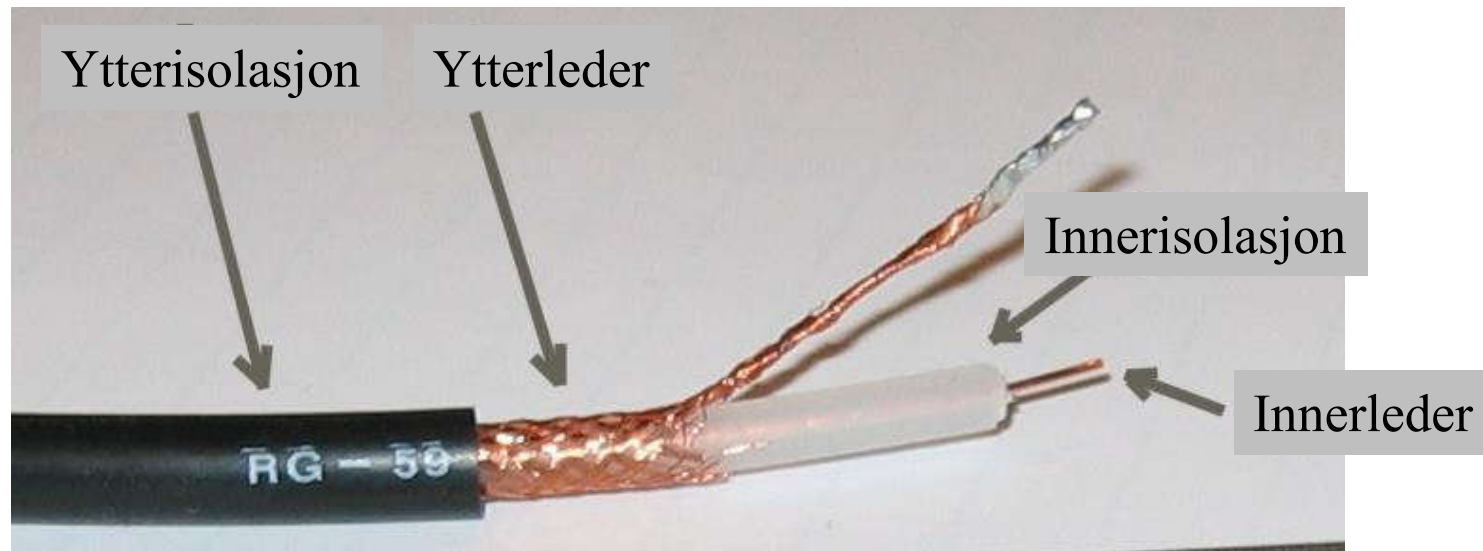
# Magnetfelt fra to parallelle ledere (Ex. 28.4 i Y&F)



For  $r \gg d$ :  
 $B$ -felt avtar med  $1/r^2$ ,  
 dvs. betydelig raskere  
 enn for enkeltleder.

(Fig 28.7)

# Utafor koaksialkabel er $B$ -feltet null!



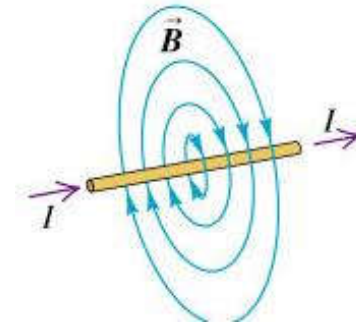
Mer seinere, bl.a. oppgave i regneøving.





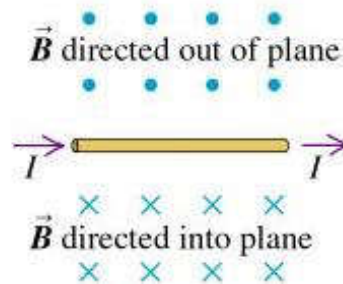
**B-felt** rundt uendelig lang, rett leder:

$$B = \frac{1}{2\pi} \mu_0 \frac{I}{r}$$



Retning: asimutalt ( $\varphi$ -retning)

$r$  = avstand fra lederen



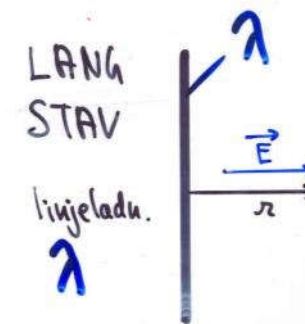
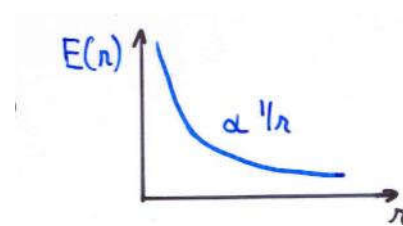
### Sammenlikn med:

**E-felt** rundt uendelig lang, ladd rett leder:

$$E = \frac{1}{2\pi} \frac{1}{\epsilon_0} \frac{\lambda}{r}$$

Retning: radielt ( $r$ -retning)

$r$  = avstand fra lederen





# Kap 28: Magnetiske kilder

- **Elektrostatikk:**

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q\hat{r}}{r^2} \quad (\text{Coulombs lov})$$

- **Magnetostatikk:**

Enkeltladning:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2} \quad (28.2)$$

Strøm i leder:

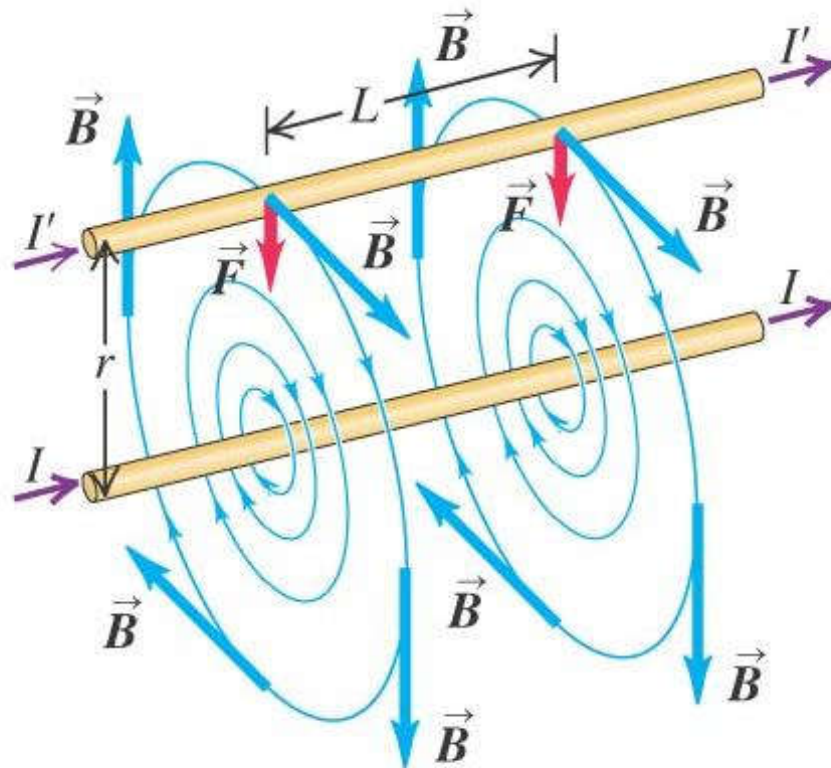
$$\vec{B} = \int_{\text{ledn.}} d\vec{B} = \frac{\mu_0 I}{4\pi} \int_{\text{ledn.}} \frac{d\vec{s} \times \hat{r}}{r^2} \quad (28.7) \text{ (Biot-Savart)}$$

1819-25: Vitenskapelig arbeid:

Hans Christian Ørsted, André Ampere, Jean-Baptist Biot,  
Felix Savart, Michael Faraday, Joseph Henry

- **Eks. 1: Rett leder**
- **28.4: Definisjon 1 ampere**
- **Eks. 2: Sirkulær sløyfe**
  
- **Amperes lov**

## 28.4 Kraft mellom to parallelle ledere



$$F' = I_1 I_2 \mu_0 / (2\pi r)$$

**Definisjon 1 A:**

$$2 \cdot 10^{-7} \text{ N/m} = 1 \text{ A} \cdot 1 \text{ A} \cdot \mu_0 / (2\pi \cdot 1 \text{ m})$$

.. er i praksis definisjon av  $\mu_0$ :

$$\mu_0 = 2\pi \cdot 2 \cdot 10^{-7} \text{ N/A}^2 = 4\pi \cdot 10^{-7} \text{ Tm/A}$$

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Figure 28.9

# Definisjon av 1 ampere (grunnenhet i SI-systemet)

- En ampere er den konstante elektriske strømmen som frambringer en gjensidig lineær kraft på  $2 \cdot 10^{-7}$  newton per meter leder når strømmen går gjennom hver av to rettlinjete, parallelle, uendelige lange ledere med sirkulært og neglisjerbart lite tverrsnitt, og lederne er anbrakt i én meters innbyrdes avstand i tomt rom.

- ampere er en av sju SI-grunnenheter:

meter	- lengde
kilogram	- masse
sekund	- tid
ampere	- strømstyrke
kelvin	- temperatur
mol	- stoffmengde
candela	- lysstyrke

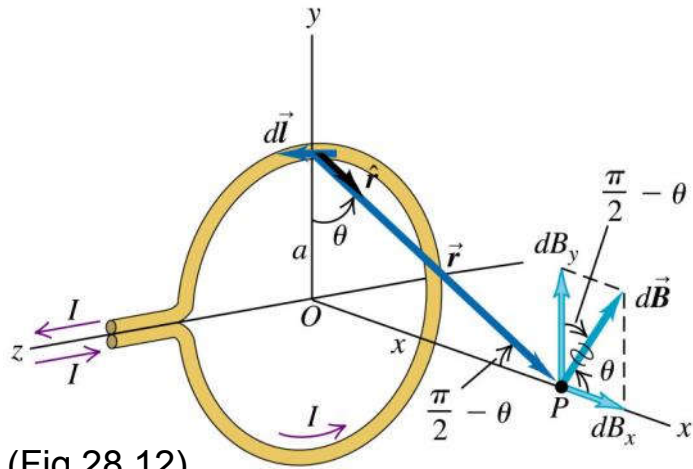
Alle andre enheter er avledet fra disse, for eksempel

$$N = \text{kg m s}^{-2}$$

$$V = \text{J/C} = \text{kg m}^2 \text{s}^{-3} \text{A}^{-1}$$

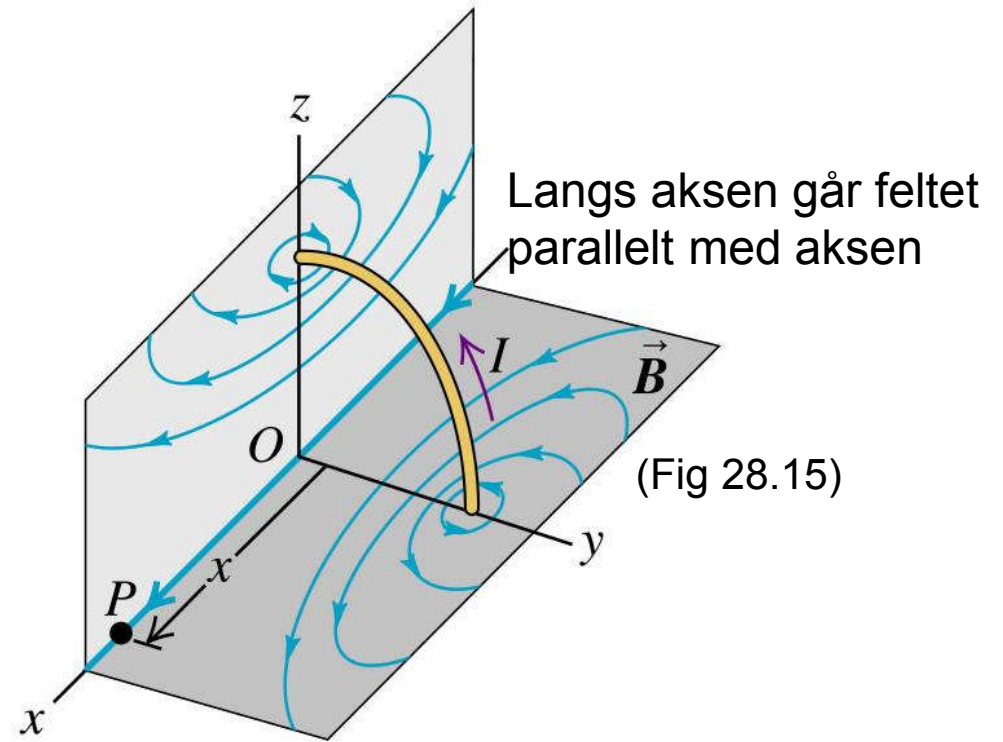
(se formelarket)

# Eks. 2: $B$ -feltet på aksene i en sirkulær strømsløyfe: (kap 28.5)



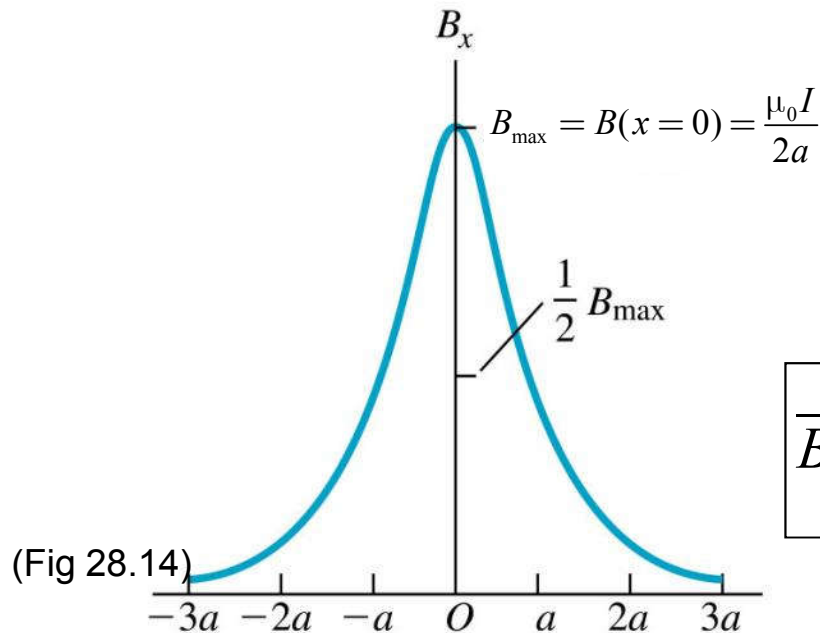
(Fig 28.12)

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(Fig 28.15)

Langs aksene går feltet parallelt med aksene



(Fig 28.14)

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$$\vec{B}(x) = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} \hat{i} \quad (28.15)$$

## Kap. 28:

Eks. 1:  $B$ -feltet på midtnormal til rett leder, lengde  $2a$

$$\vec{B} = \frac{\mu_0}{4\pi} I \frac{2a}{\rho} \frac{1}{\sqrt{a^2 + \rho^2}} \hat{\varphi} \quad \vec{B}(\rho \ll a) = \frac{\mu_0}{2\pi} \frac{I}{\rho} \hat{\varphi}$$

Eks. 2:  $B$ -feltet på aksene i en sirkulær strømsløyfe, radius  $a$

$$\vec{B}(x) = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} \hat{i} \quad \vec{B}(x = 0) = \frac{\mu_0 I}{2a} \hat{i} \quad \vec{B}(x \gg a) = \frac{\mu_0 I a^2}{2x^3} \hat{i}$$

Eks. 3:  $B$ -feltet i sentrum av kvadratisk strømsløyfe  
(Øving 10, opg. 4:  $B$ -feltet på hele midtnormalen)

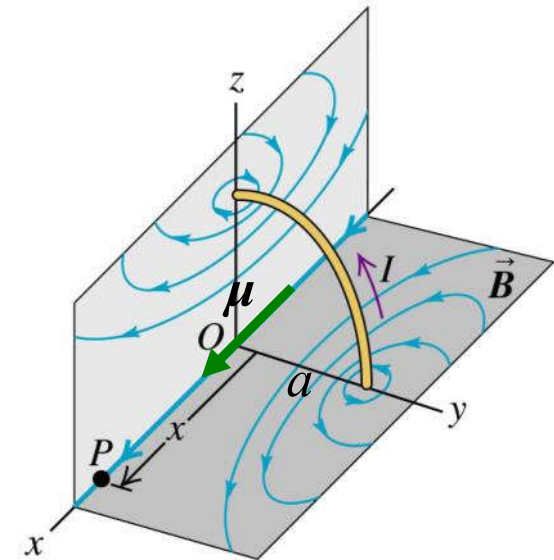
$$\begin{aligned} \vec{B}(x = 0) &= \frac{\mu_0 I}{2a} \frac{4}{\sqrt{2\pi}} \hat{i} \\ &= \vec{B}_{\text{sirkulær}}(x = 0) \cdot \frac{4}{\sqrt{2\pi}} \\ &= \vec{B}_{\text{sirkulær}}(x = 0) \cdot 0,900 \end{aligned}$$

## Eks. 2: Feltet på akse i en sirkulær strømsløyfe

$$\vec{B}(x) = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} \hat{i} \quad (28.15)$$

Langt unna  $x \gg a$ :  $\vec{B}(x) = \frac{\mu_0 I a^2}{2x^3} \hat{i} = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{x^3}$

sløyfas dipolmoment  $\mu = I\pi a^2$



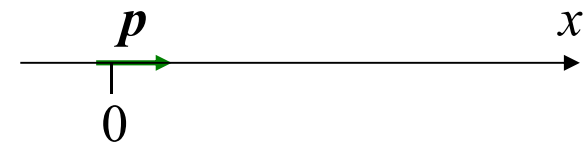
(Fig 28.15)

Analogi:

Langt unna elektrisk dipol:

(Øv. 6, opg. 2d)

$$\vec{E}(x) = \frac{1}{2\pi\epsilon_0} \frac{\vec{p}}{x^3}$$



# Kap 28: Magnetiske kilder

- **Elektrostatikk:**

Coulombs lov

→ hjelpelov: Gauss' lov (når symmetri)

- **Magnetostatikk:**

Biot-Savarts lov

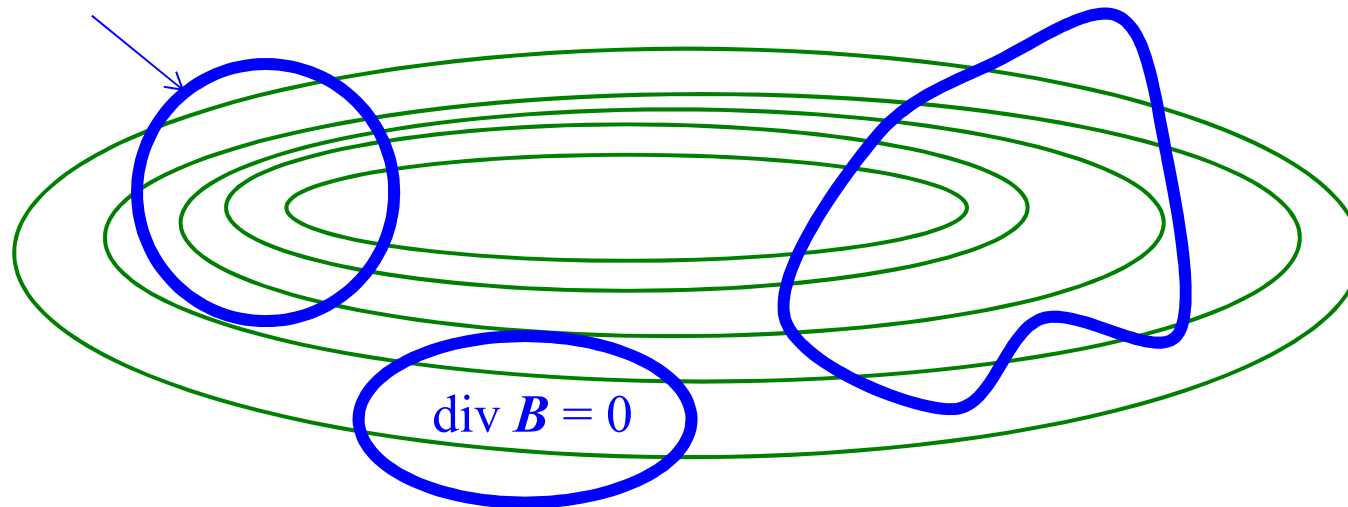
→ hjelpelov: Amperes lov (når symmetri)

# Gauss' lov for $B$ -felt:

Feltlinjer er lukka kurver

=> Nettofluks = 0 ut fra **lukka flate**

Lukka flate



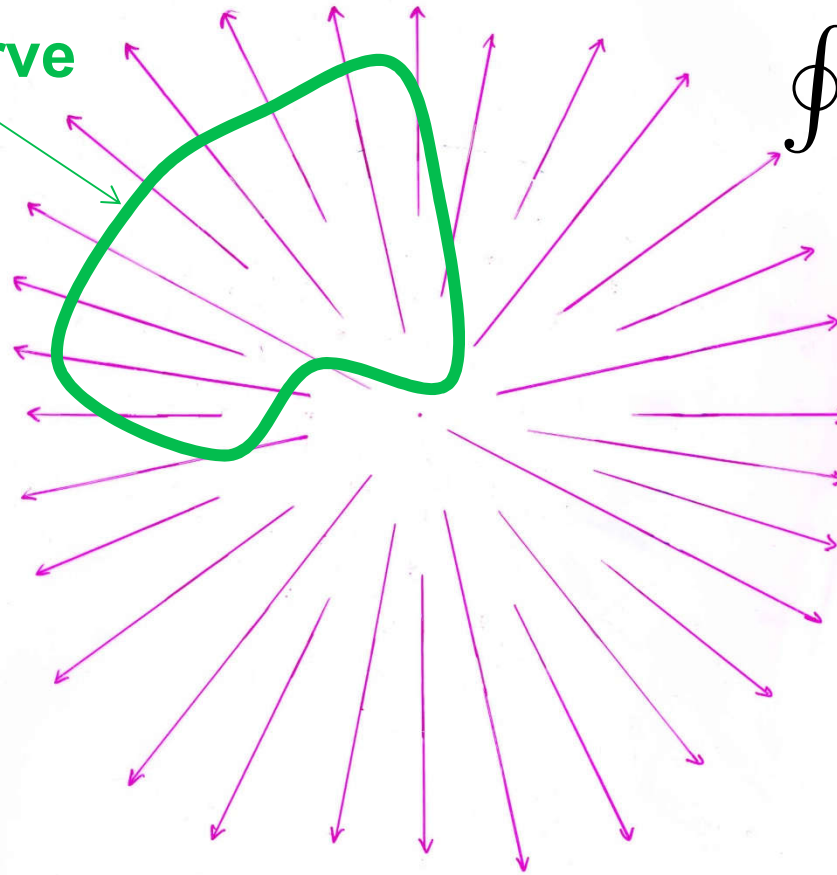


# Kap. 23: Sirkulasjonsloven for $E$ -felt

(konservativt felt):

Integrasjon over **lukka kurve** = 0

Lukka kurve

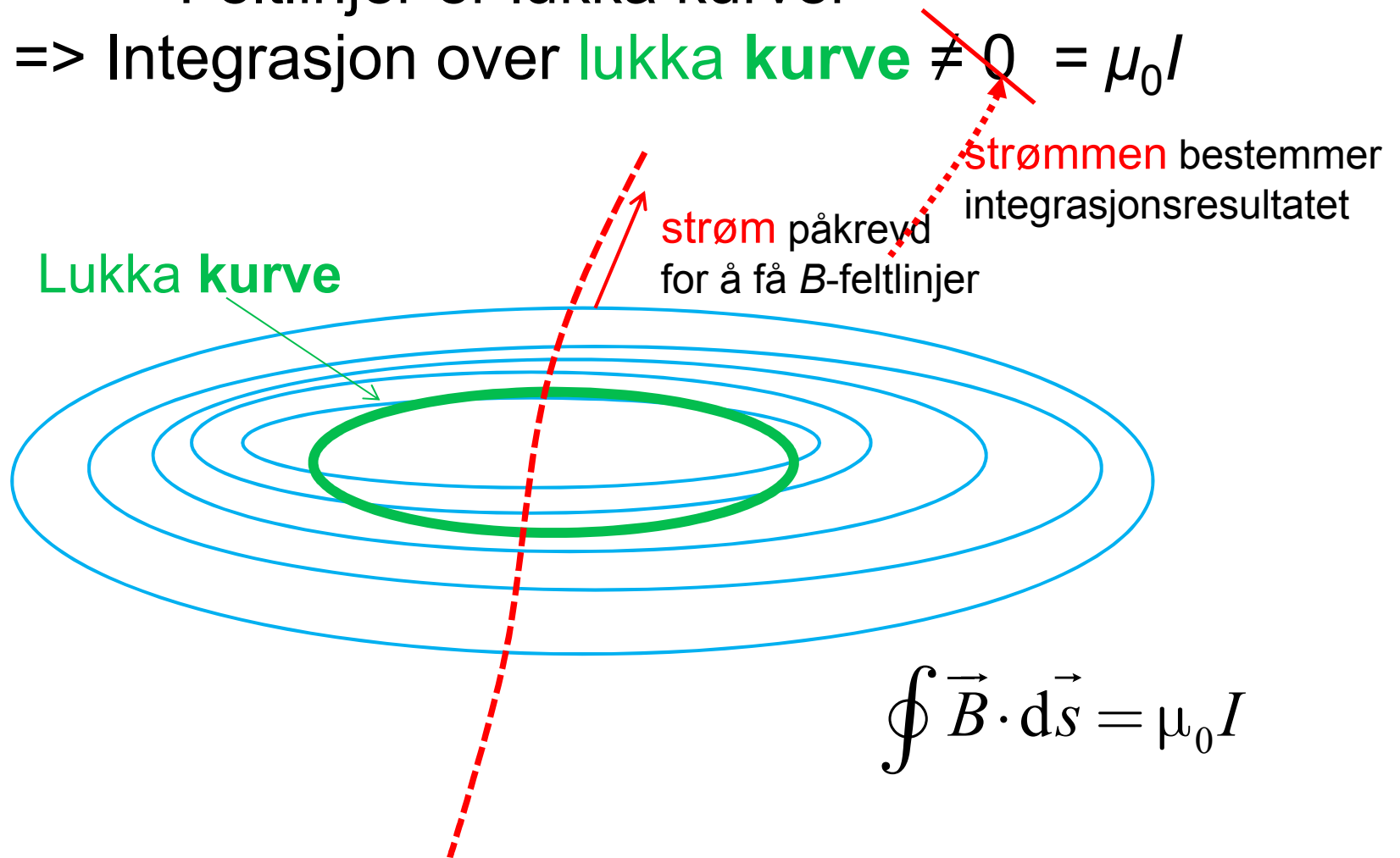


$$\oint \vec{E} \cdot d\vec{s} = 0$$

# Sirkulasjonsloven for $B$ -felt:

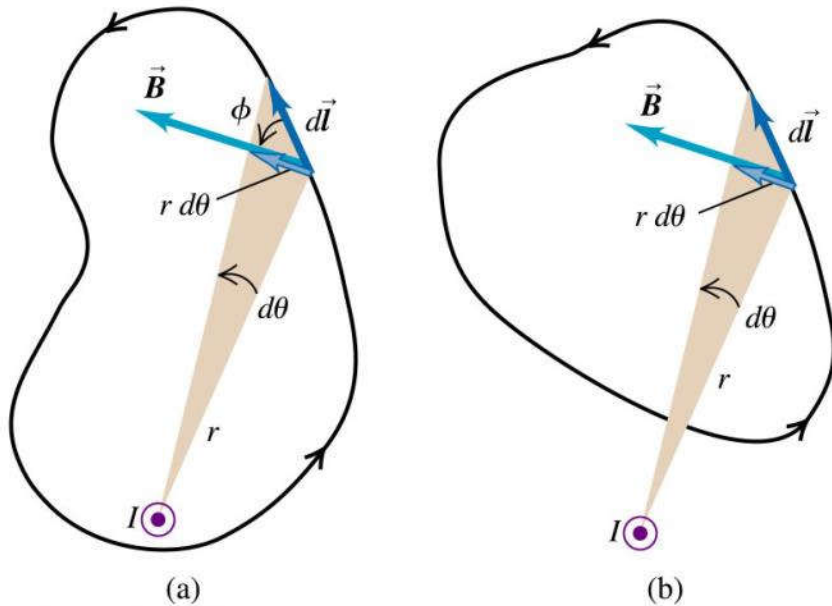
Feltlinjer er lukka kurver

=> Integrasjon over **lukka kurve**  $\neq 0 = \mu_0 I$

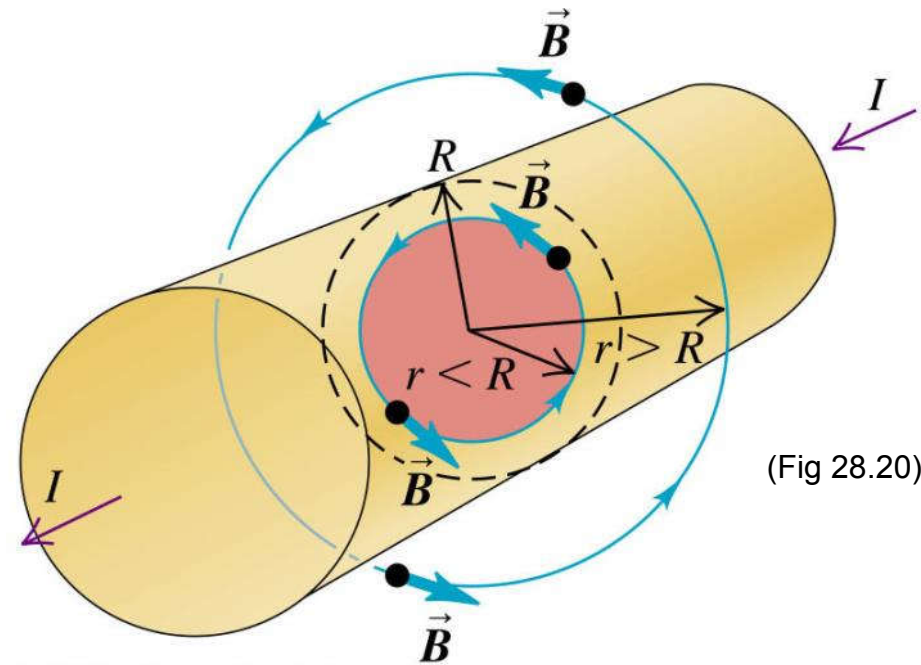


# Amperes lov

$\int \vec{B} \cdot d\vec{s} = \mu_0 I$  over lukka kurve, der  $I$  er totalstrøm innenfor kurva



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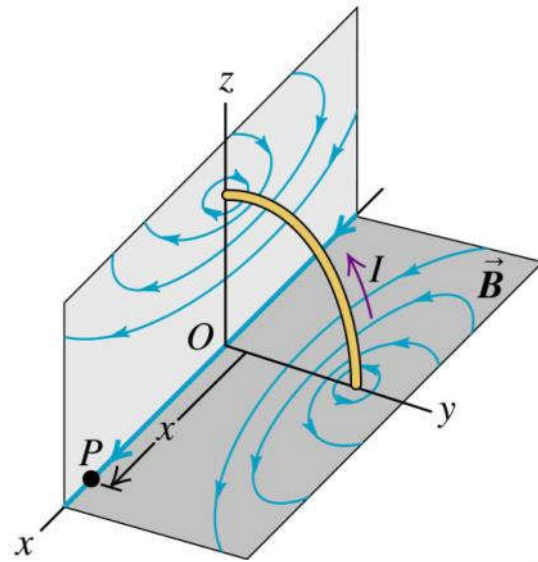


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Gjelder alle integrasjonsveger,

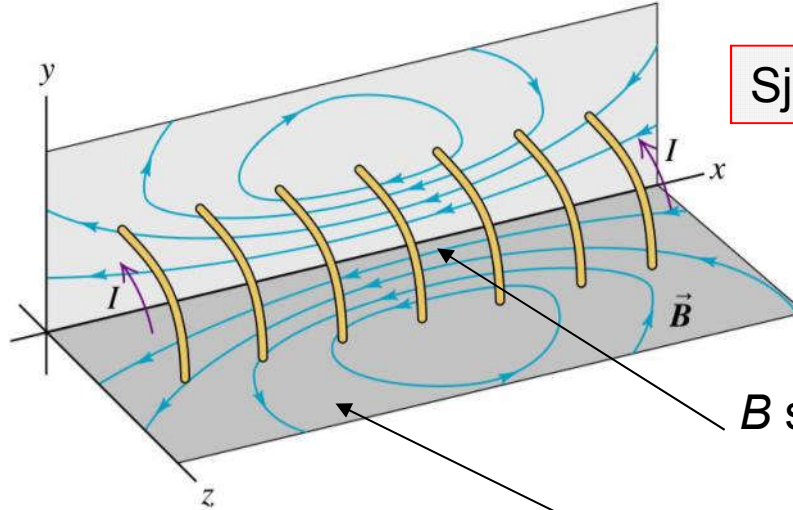
men er nyttig kun i (sylinder)symmetriske konfigurasjoner. F. eks. rundt leder:  $B = \mu_0 I / 2\pi r$

# Eks. 4. Solenoide (Ex. 28.10)



Én sløyfe

(Fig 28.15)



Sju sløyfer

(Fig 28.22)

$B$  sterkt inni

$B$  svakt utafor

Mange sløyfer: Antar  $B$  konst. inni,  
 $B = 0$  utafor

# Eks. 4: Solenoide (mange sirkulære strømsløyfer)

Eks:

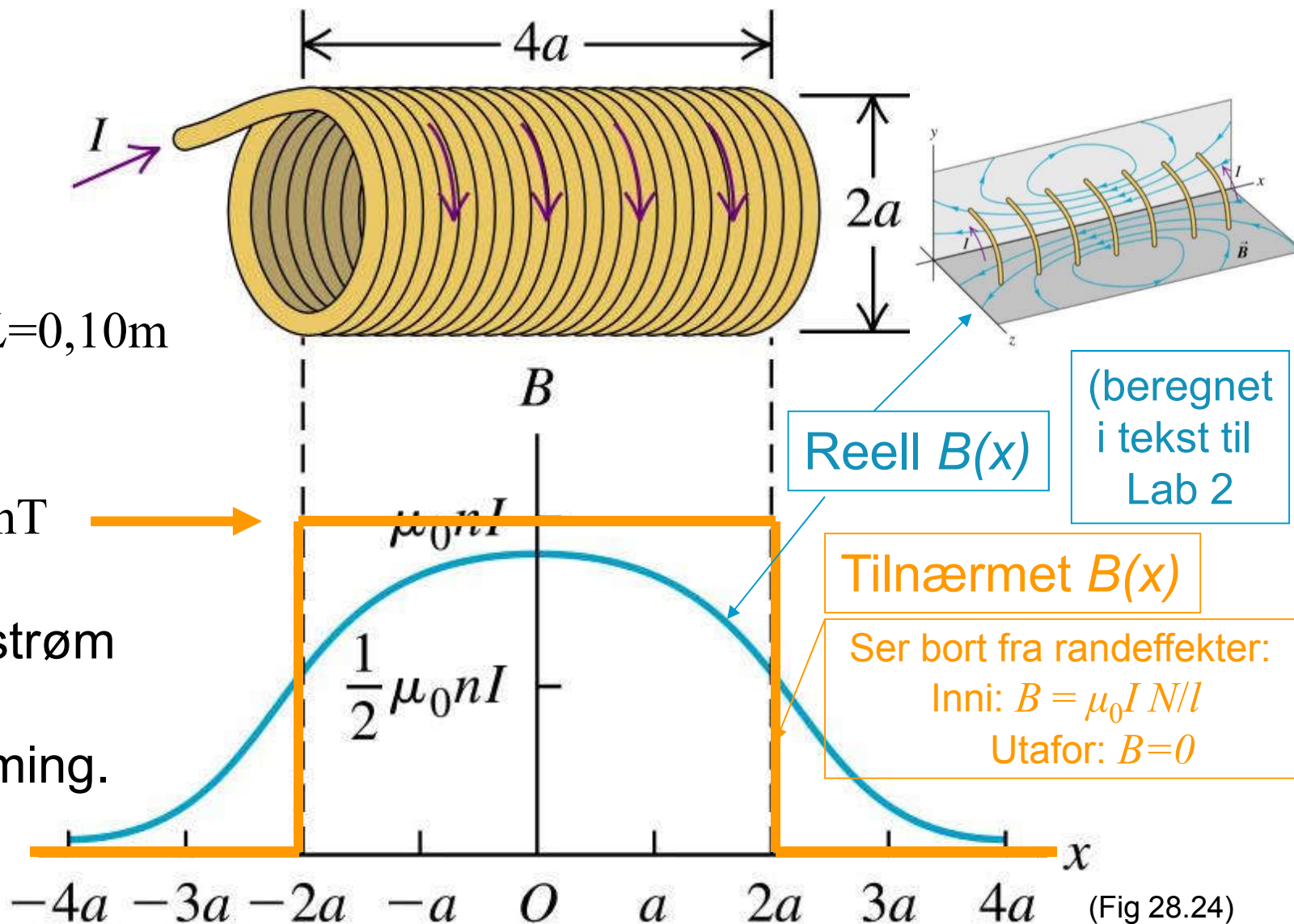
$N=400$   $L=0,10\text{m}$

$I=5,0\text{A}$

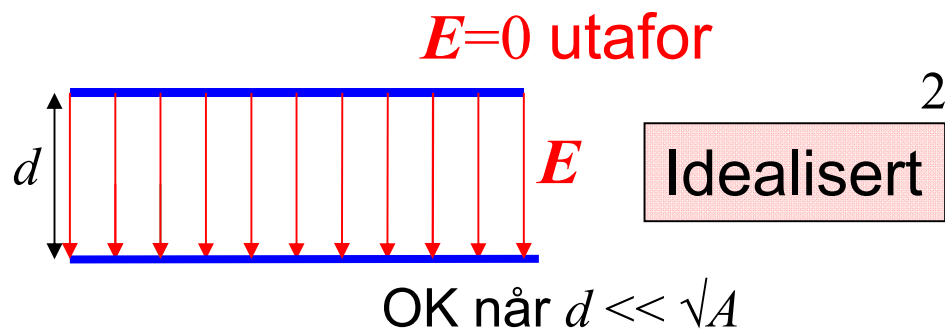
$\Rightarrow$

$B = 25 \text{ mT}$

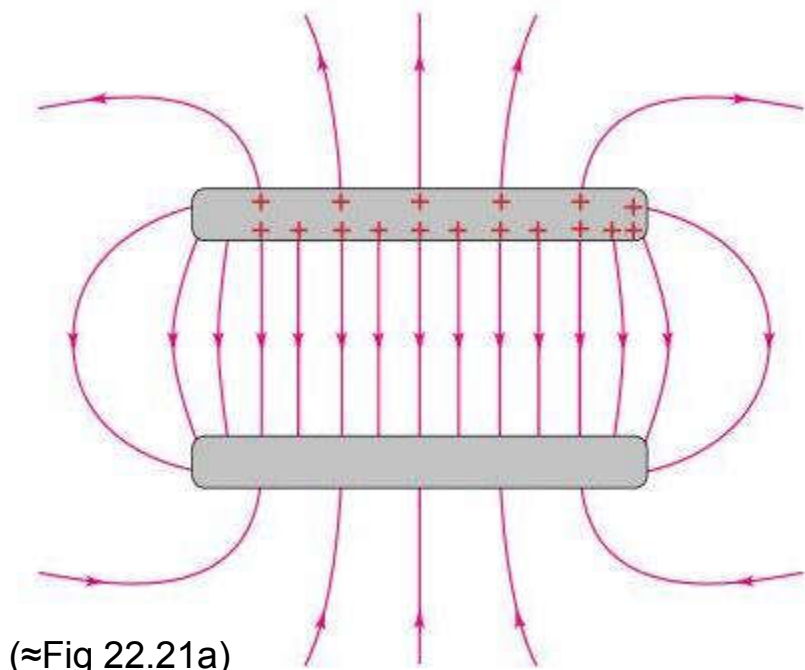
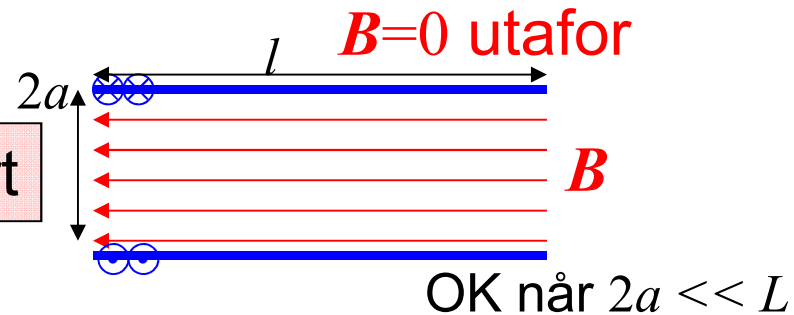
Større strøm  
gir  
oppvarming.



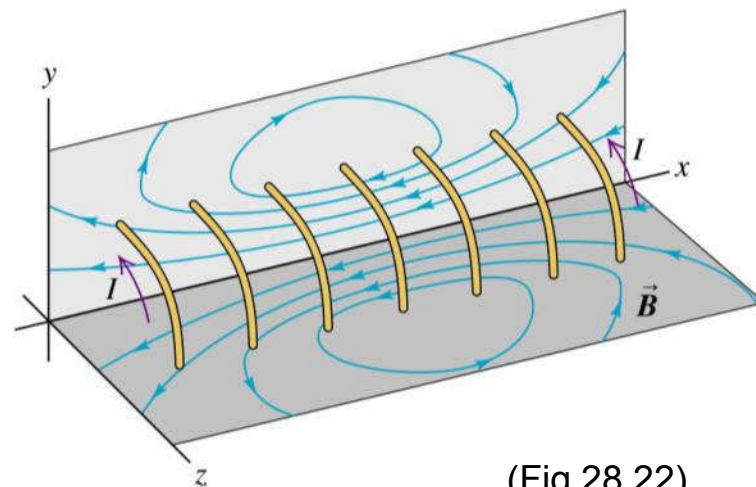
# $E$ i parallellplatekondensator



# $B$ i solenoide

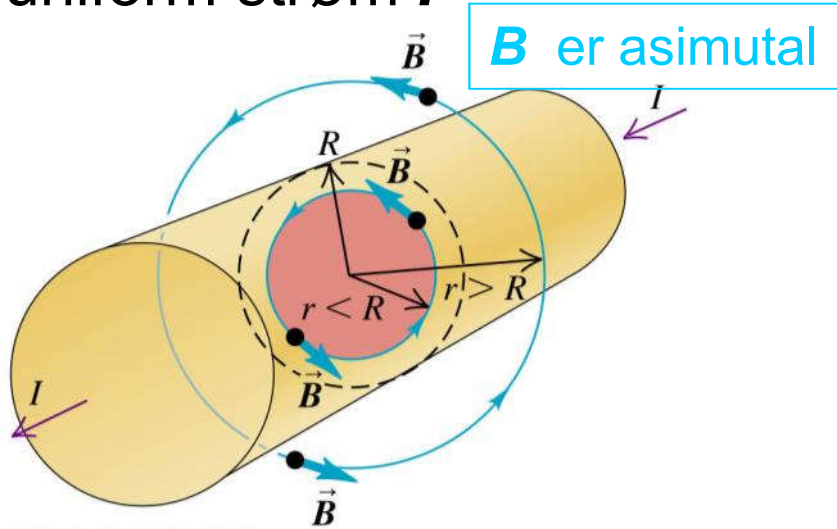


Reelt

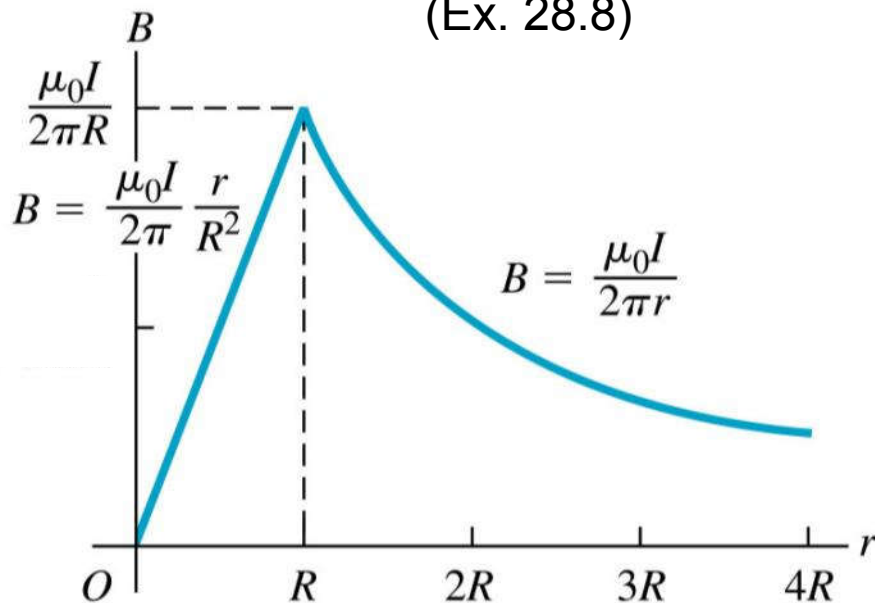




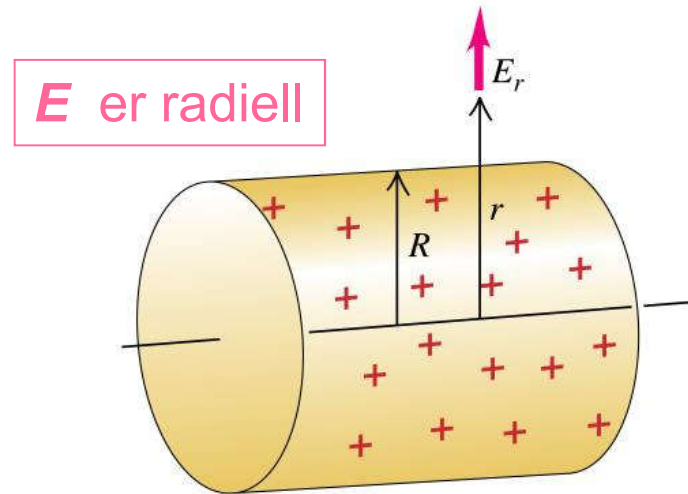
**$B$**  inni og utafor en  
sylinderleder med analog til  
uniform strøm  $I$



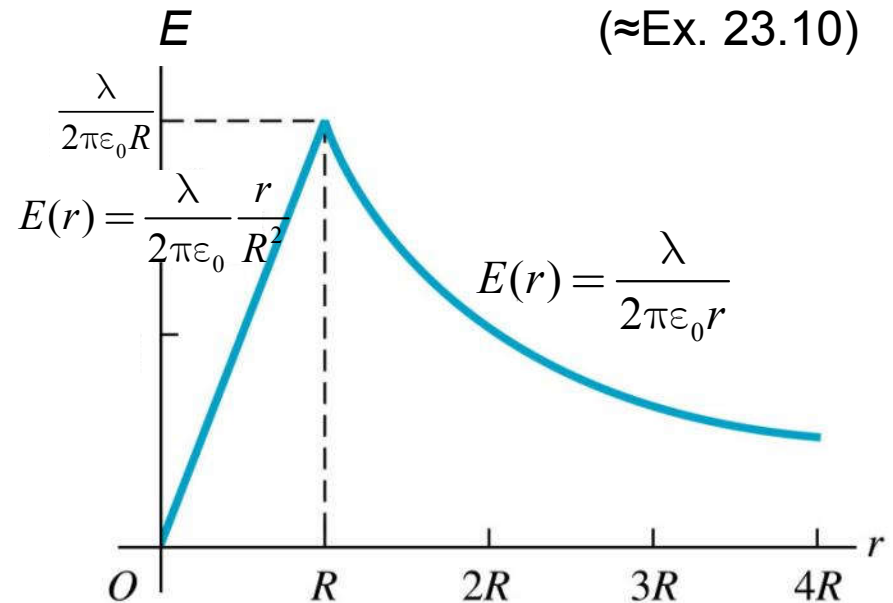
(Ex. 28.8)



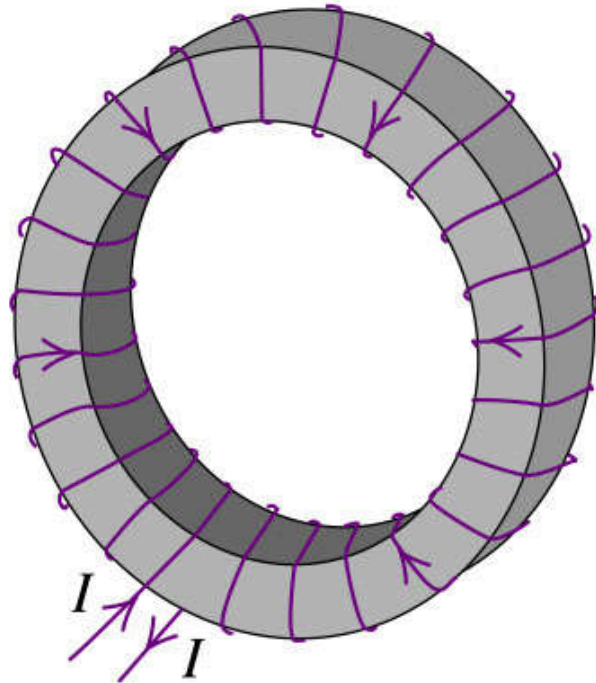
**$E$**  inni og utafor en  
sylinder med  
uniform ladning  $\lambda$



(≈Ex. 23.10)



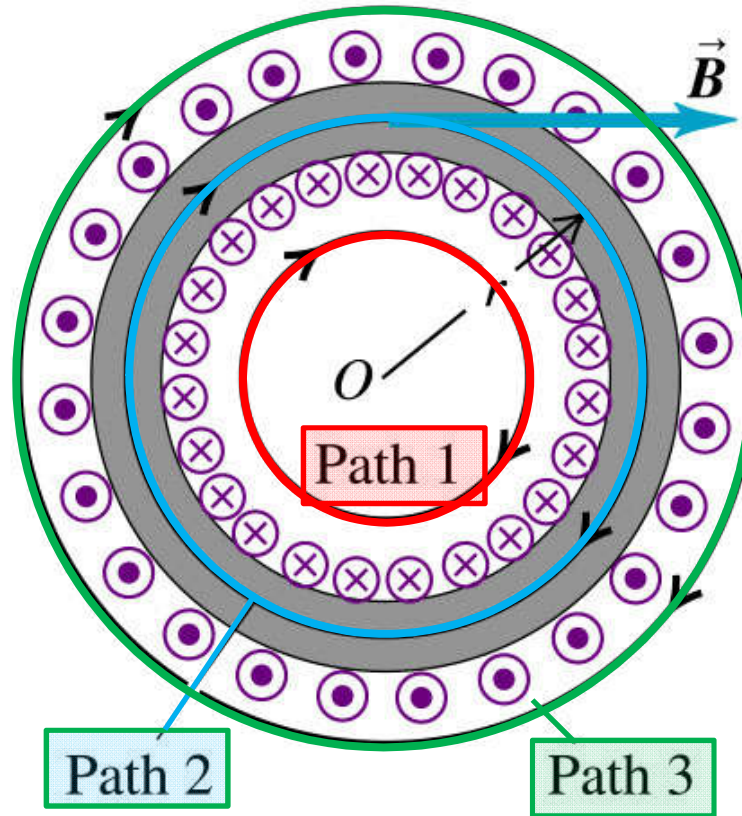
# Feltet i toroid solenoide: (Ex. 28.10)



(a)

(Fig 28.25)

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(b)

Mer  
Amperes lov  
i Øv. 11 og 12

Path 1:  $I_{\text{incl}} = 0 \Rightarrow B = 0$

Path 2:  $I_{\text{incl}} = NI \Rightarrow B \approx \mu_0 NI / 2\pi r$

Path 3:  $I_{\text{incl}} = NI + N(-I) = 0 \Rightarrow B = 0$



## Øving 9, flervalg:

a) En punktladning  $q$  er plassert i det ene hjørnet av en kube. Hva blir elektrisk fluks gjennom den skraverte (høyre) sideflata i figuren?

A)  $q$

D)  $q/8$

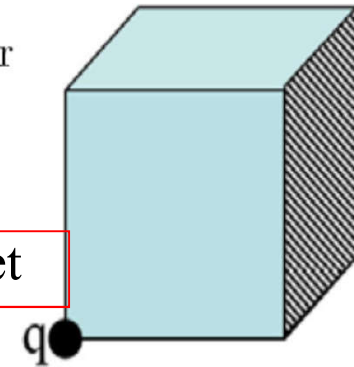
B)  $q/3$

E)  $q/24$

C)  $q/4$

Elek. fluks = fluks til  $D$ -feltet

$$\Phi = D A$$



Young & Freedman, kap. 22.2:

bruker

Elek. fluks

= fluks til  $E$ -feltet

$$\Phi_E = E A$$

If the area  $A$  is flat but not perpendicular to the field  $\vec{E}$ , then fewer field lines pass through it. In this case the area that counts is the silhouette area that we see when looking in the direction of  $\vec{E}$ . This is the area  $A_{\perp}$  in Fig. 22.6b and is equal to  $A \cos \phi$  (compare to Fig. 22.5b). We generalize our definition of electric flux for a uniform electric field to

$$\Phi_E = EA \cos \phi \quad (\text{electric flux for uniform } \vec{E}, \text{ flat surface}) \quad (22.1)$$

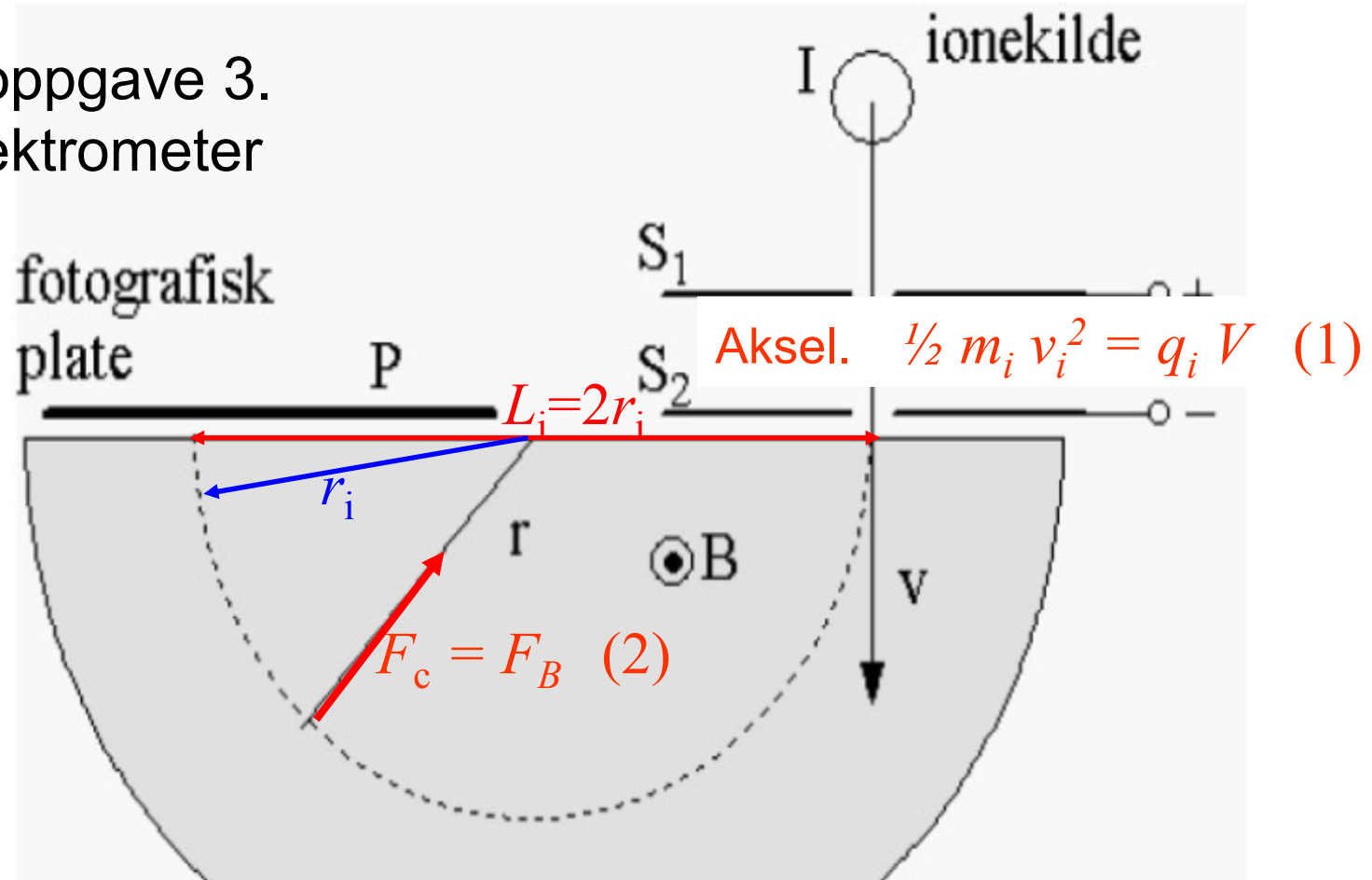
Since  $E \cos \phi$  is the component of  $\vec{E}$  perpendicular to the area, we can rewrite Eq. (22.1) as

$$\Phi_E = E_{\perp} A \quad (\text{electric flux for uniform } \vec{E}, \text{ flat surface}) \quad (22.2)$$

In terms of the vector area  $\vec{A}$  perpendicular to the area, we can write the electric flux as the scalar product of  $\vec{E}$  and  $\vec{A}$ :

$$\Phi_E = \vec{E} \cdot \vec{A} \quad (\text{electric flux for uniform } \vec{E}, \text{ flat surface}) \quad (22.3)$$

Øving 9, oppgave 3.  
Massespektrometer



a) Likn. (1) for protonet

b) Likn (2) for protonet  $m_i v_i^2 / r_i = q_i v_i B$  (2)

c) Søk etter masseforholdet  $m_1/m_p$   
med likn (1) og (2) for  $i=1$  (masse 1) og  $i=p$  (protonet).  
Tilsvarende for  $m_2/m_p$ .

# Amperes lov, rekap.

$$\int \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \quad (\text{Amp})$$

over lukka kurve, der  $I$  er totalstrøm innenfor kurva

$$\text{curl } \mathbf{B} = \mu_0 \mathbf{J} \quad (\text{Amp-diff})$$

# Maxwells likninger (så langt, statikk)

Integralform

Differensialform

$$\oiint \vec{E} \cdot d\vec{A} = Q / \epsilon \quad \boxed{\text{Gauss' lov } \mathbf{E}}$$

$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon$$

$$\oiint \vec{B} \cdot d\vec{A} = 0 \quad \boxed{\text{Gauss' lov } \mathbf{B}}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I \quad \boxed{\text{Amperes lov}}$$

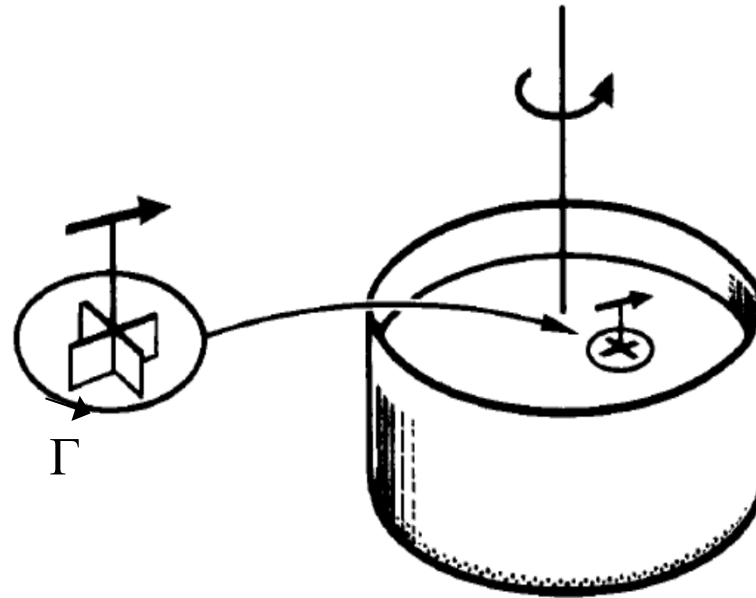
$$\text{curl} \vec{B} = \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\oint \vec{E} \cdot d\vec{s} = 0$$

$$\text{curl} \vec{E} = \vec{\nabla} \times \vec{E} = \vec{0}$$

# curl

-- kan i vannstrøm demonstreres med et  
(infinitesimalt) skovlhjul:



# Curl

Eks. forrige time:

$$\vec{B}(x, y, z) = [y, -x, 0] = y\hat{i} - x\hat{j}$$

$$\vec{B}(r, \varphi, z) = -r\hat{\varphi}$$

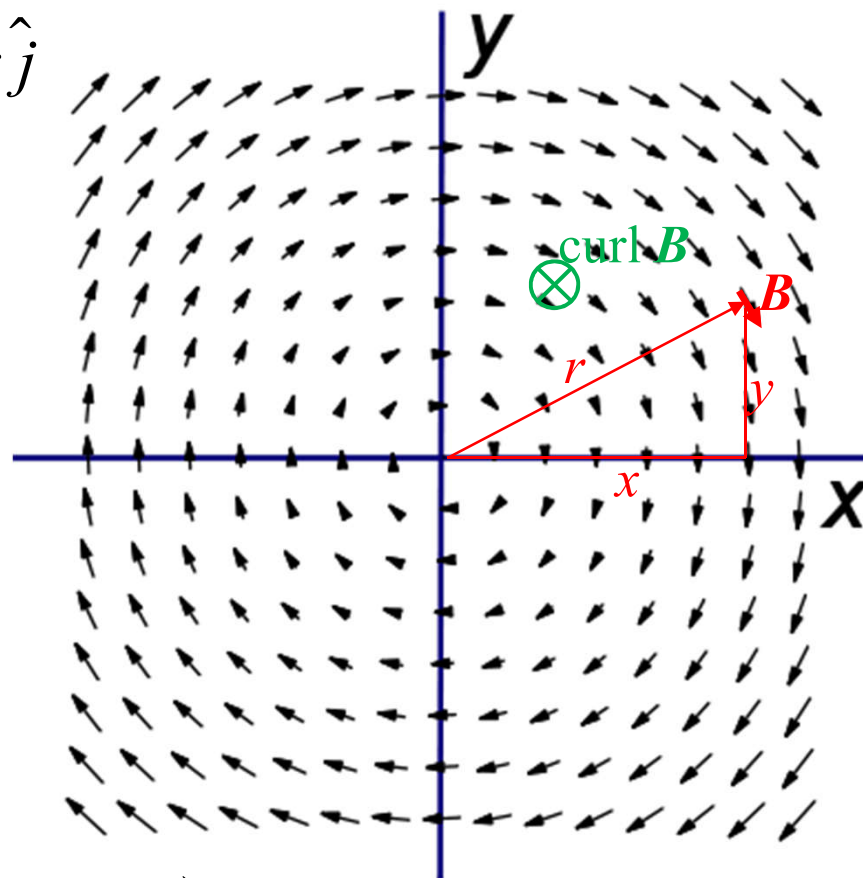
(konstanter utelatt, feil enheter for  $B$ )

Inni sylindrisk leder:

$$\vec{B}(r, \varphi, z) = -\frac{\mu_0}{2} J r \hat{\varphi}$$

$$\text{curl } \vec{B} = \mu_0 J \hat{k} = \mu_0 \vec{J}$$

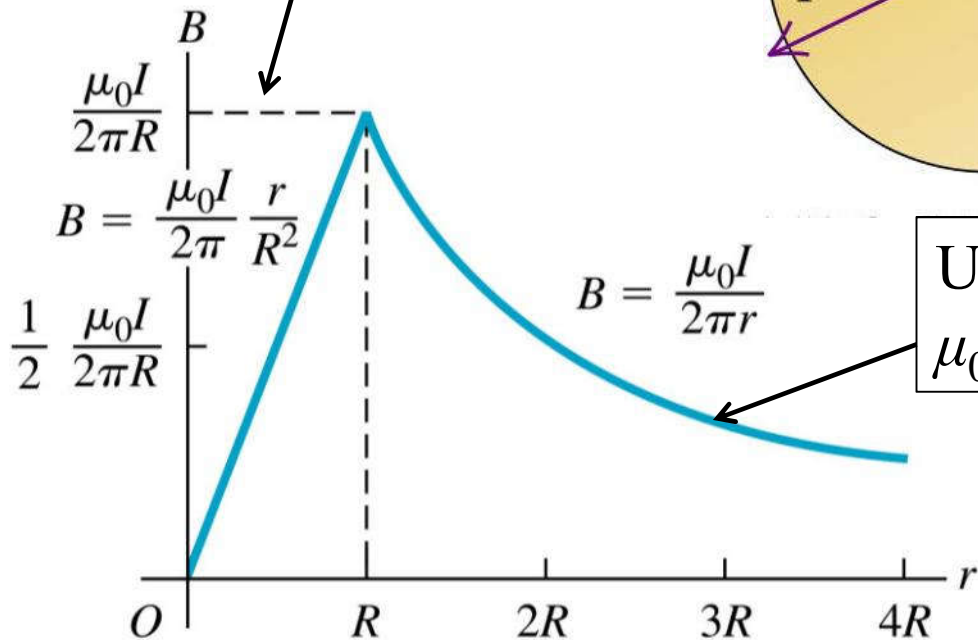
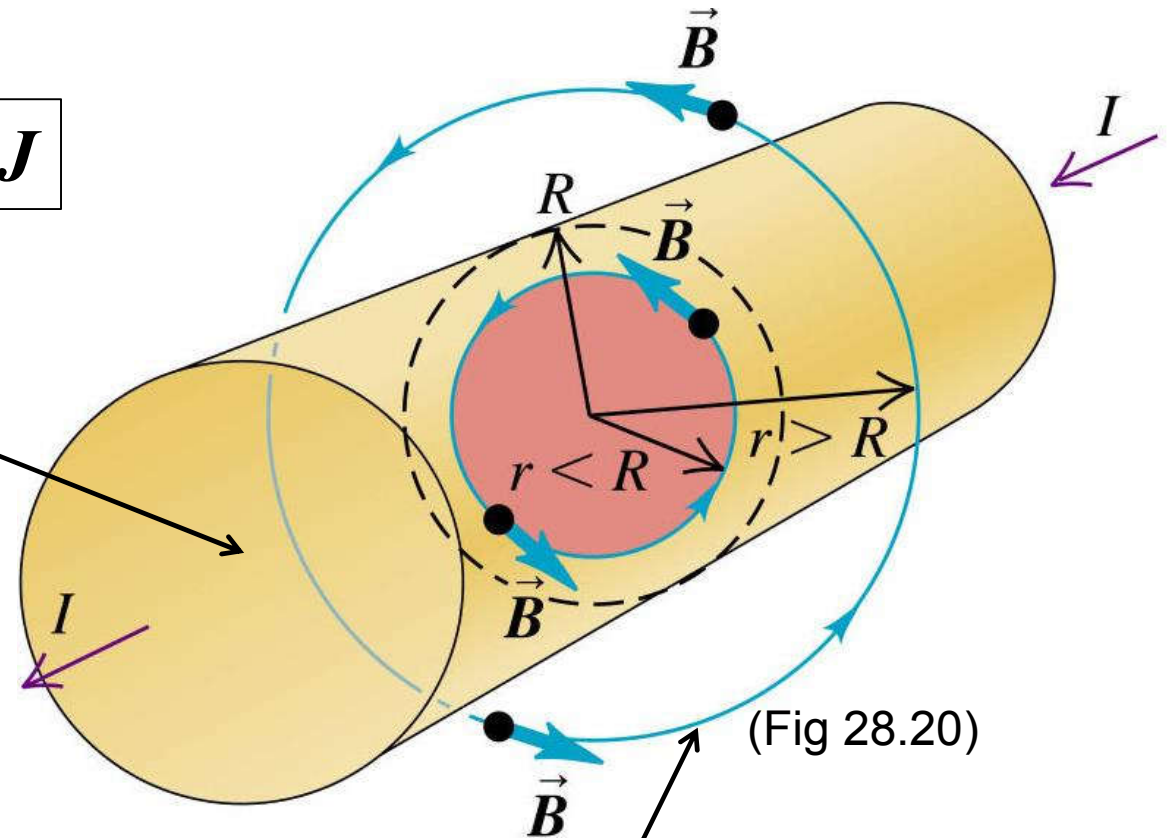
$$\text{curl } \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & -x & 0 \end{vmatrix} = 0 \cdot \hat{i} + 0 \cdot \hat{j} + \left( \frac{\partial(-x)}{\partial x} - \frac{\partial y}{\partial y} \right) \hat{k} = -2\hat{k}$$



# Eks. 5: Feltet inni og utafor en ledning

Ampere:  $\text{curl } \mathbf{B} = \mu_0 \mathbf{J}$

Inni:  
 $\mu_0 \mathbf{J} = \text{curl } \mathbf{B}$



Utafor:  
 $\mu_0 \mathbf{J} = \mathbf{0} = \text{curl } \mathbf{B}$

(Fig 28.21)

(Fig 28.20)

# Maxwells likninger i [Notat 4](#)

Statikk

$$E = D/\epsilon$$

$$H = B/\mu \text{ defineres straks}$$

Integralform

Differensialform

$$\oint \vec{D} \cdot d\vec{A} = Q \quad \text{Gauss' lov } \mathbf{D}$$

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad \text{Gauss' lov } \mathbf{B}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\oint \vec{H} \cdot d\vec{\ell} = I + \frac{\partial \Phi}{\partial t} \quad \text{Amperes lov}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\oint \vec{E} \cdot d\vec{\ell} = 0 - \frac{\partial \Phi_B}{\partial t} \quad \text{Faradays lov}$$

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$



# Kap 28: Magnetiske kilder

- **Elektrostatikk:**

Ladning  $q$  påvirkes av kraft  $q\mathbf{E}$  (Coulombs lov)

→ Definisjon  $E$ -felt

$E$ -feltet skapes fra ladninger (Coulombs lov)

- **Magnetostatikk:**

Ladning  $q$  i **bevegelse** påvirkes av kraft  $q\mathbf{v} \times \mathbf{B}$

→ Definisjon  $B$ -felt (Lorentzkrafta)

$B$ -feltet skapes fra ladninger i **bevegelse**

(Biot-Savarts lov)

- **Hjelpelover:**

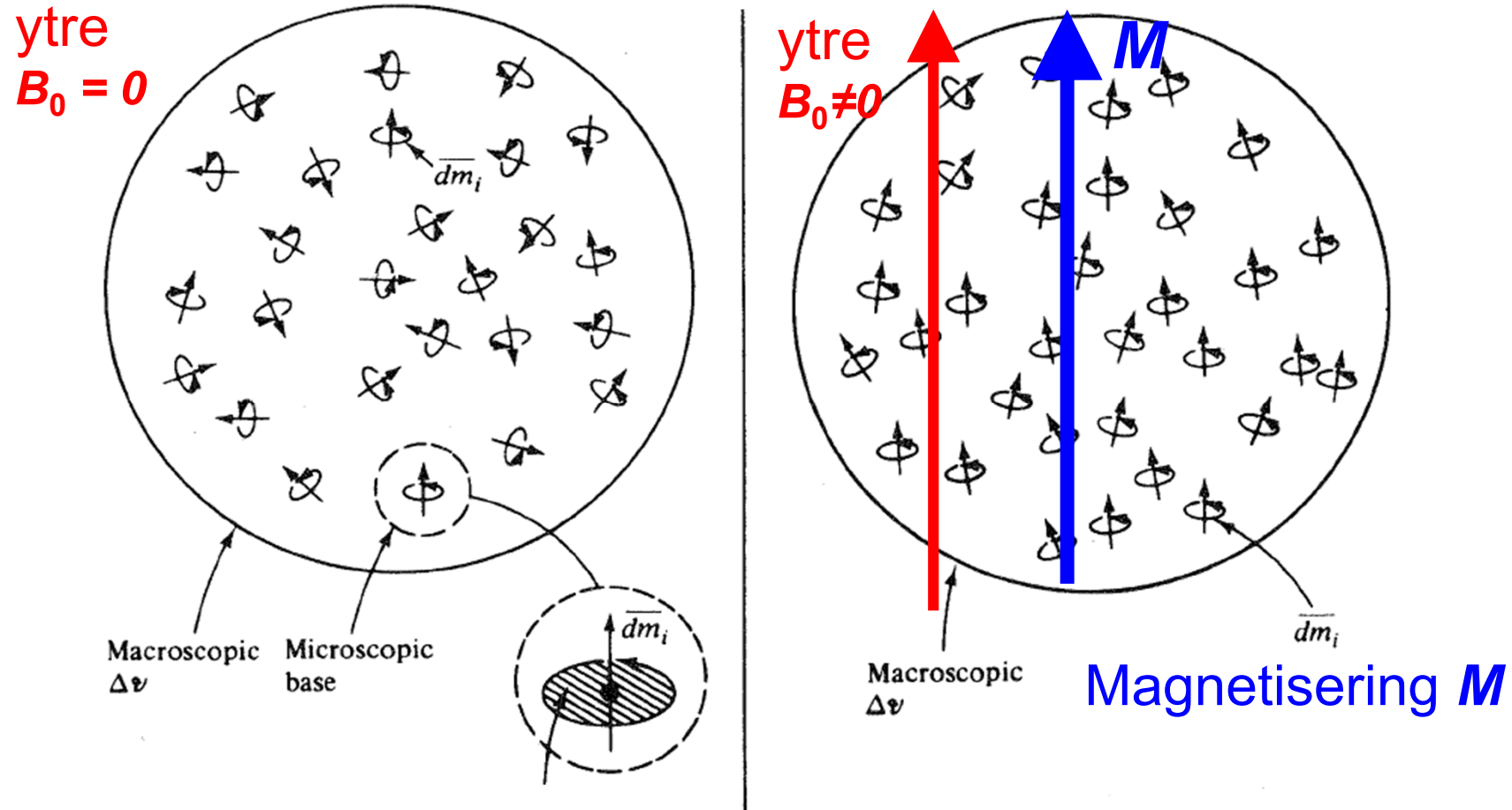
Elektrostatikk: Gauss' lov

Magnetostatikk: Amperes lov

- **Til slutt: Magnetiske materialer**

Ferromagnetisk materiale. Magnetisering.  $\mathbf{M}$ -vektor og  $\mathbf{H}$ -vektor

# Atomære magnetiske moment $\mu$ ( $= \overline{dm_i}$ ) i ytre magnetisk felt $B$



Paramagnetiske og ferromagnetiske:  
Innretting av magn.moment  $\mu$

# Tre typer magnetisk materiale:

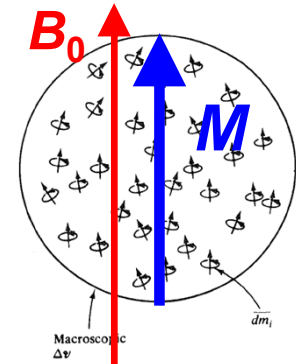
Type	Effekt	Årsak: Ytre $H_0$ .....
Dia- magnetisk	$B$ -felt ↓	induserer magn.mom. $\mu$ med $\mu \parallel (-H)$
Para- magnetisk	$B$ -felt ↑	innretter permanente $\mu$ med $\mu \parallel H$
Ferro- magnetisk	$B$ -felt ↑↑↑↑↑↑↑↑↑↑	innretter permanente $\mu$ med $\mu \parallel H$ <b>Mange</b>

# Hva vi har lært:

- Magnetisering, definisjon:

$$\mathbf{M} = \sum \boldsymbol{\mu} / \text{volum}$$

$$[\text{analogi: } \mathbf{P} = \sum \mathbf{p} / \text{volum} ]$$



- Magnetisk feltstyrke:

$$\mathbf{H} = \mathbf{B} / \mu_0 \quad (\text{i tomrom})$$

$$[ \mathbf{E} = \mathbf{D} / \varepsilon_0 ]$$

- Magnetisering, eksperimentelt:  $\mathbf{M} = \chi_m \mathbf{H}$      $[ \mathbf{P} = \chi_e \varepsilon_0 \mathbf{E} ]$

- Totalt  $\mathbf{B}$ -felt i magnetisk materiale:

$$\mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M} = \mu_0 \mu_r \mathbf{H}$$

$$[ \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_r \varepsilon_0 \mathbf{E} ]$$

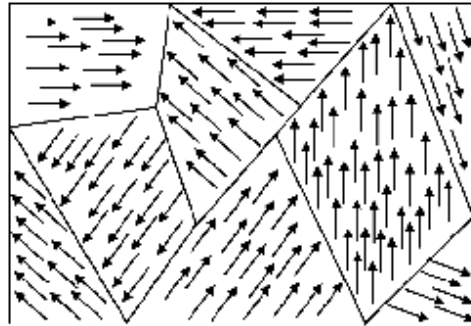
m.m.

$$B = \mu_0 \mu_r H$$

**TABLE 9-1 REPRESENTATIVE VALUES FOR PERMEABILITY  $\mu_r$  FOR SEVERAL MATERIALS**

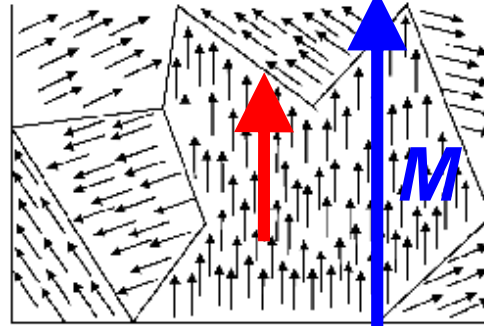
Material	Type	$\mu_r$
Bismuth	Diamagnetic	0.9999834
Silver	Diamagnetic	0.99998
Copper	Diamagnetic	0.999991
Vacuum	Nonmagnetic	1.00
Aluminum	Paramagnetic	1.00002
Nickel chloride	Paramagnetic	1.00004
Cobalt	Ferromagnetic	250
Nickel	Ferromagnetic	600
Mild steel	Ferromagnetic	2,000
Iron	Ferromagnetic	5,000
Mumetal	Ferromagnetic	100,000
Supermalloy	Ferromagnetic	800,000

# Ferromagnetiske materialer



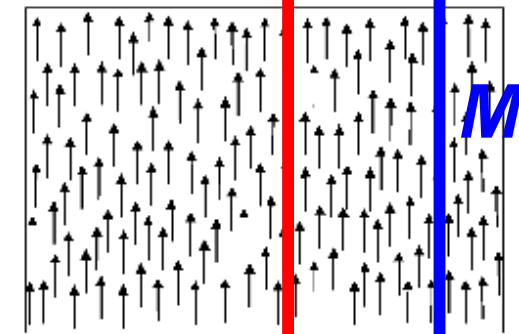
## Ytre $H = 0$ :

Magn.moment  $\mu$   
samordnet innenfor  
domener ( $\sim 100 \mu\text{m}$ )



## Middels $H$ -felt:

Domener med  
magn.moment  $\mu$  i  
samme retning som  $H$   
vokser i størrelse



## Sterke $H$ -felt:

Magnetisk  
moment innen  
domener roterer  
til  $\parallel H$

=> Metning

# IUPAC Periodic Table of the Elements

1																	18
1 <b>H</b> hydrogen 1.007 94(7)											2 <b>He</b> helium 4.002 602(2)						
3 <b>Li</b> lithium 6.941(2)	4 <b>Be</b> beryllium 9.012 182(3)											5 <b>B</b> boron 10.811(7)	6 <b>C</b> carbon 12.0107(8)	7 <b>N</b> nitrogen 14.0067(2)	8 <b>O</b> oxygen 15.9994(3)	9 <b>F</b> fluorine 18.998 4032(5)	10 <b>Ne</b> neon 20.1797(6)
11 <b>Na</b> sodium 22.989 769 28(2)	12 <b>Mg</b> magnesium 24.3050(8)											13 <b>Al</b> aluminium 26.981 538 6(8)	14 <b>Si</b> silicon 28.0855(3)	15 <b>P</b> phosphorus 30.973 762(2)	16 <b>S</b> sulfur 32.065(5)	17 <b>Cl</b> chlorine 35.453(2)	18 <b>Ar</b> argon 39.948(1)
19 <b>K</b> potassium 39.0983(1)	20 <b>Ca</b> calcium 40.078(4)	21 <b>Sc</b> scandium 44.955 912(6)	22 <b>Ti</b> titanium 47.887(4)	23 <b>V</b> vanadium 50.9415(1)	24 <b>Cr</b> chromium 51.9961(6)	25 <b>Mn</b> manganese 54.938 045(5)	26 <b>Fe</b> iron 55.845(2)	27 <b>Co</b> cobalt 58.933 195(5)	28 <b>Ni</b> nickel 58.6934(2)	29 <b>Cu</b> copper 63.546(3)	30 <b>Zn</b> zinc 65.409(4)	31 <b>Ga</b> gallium 69.723(1)	32 <b>Ge</b> germanium 72.64(1)	33 <b>As</b> arsenic 74.921 60(2)	34 <b>Se</b> selenium 78.96(3)	35 <b>Br</b> bromine 79.904(1)	36 <b>Kr</b> krypton 83.798(2)
37 <b>Rb</b> rubidium 85.4678(3)	38 <b>Sr</b> strontium 87.62(1)	39 <b>Y</b> yttrium 88.905 85(2)	40 <b>Zr</b> zirconium 91.224(2)	41 <b>Nb</b> niobium 92.906 38(2)	42 <b>Mo</b> molybdenum 95.94(2)	43 <b>Tc</b> technetium [98]	44 <b>Ru</b> ruthenium 101.07(2)	45 <b>Rh</b> rhodium 102.905 50(2)	46 <b>Pd</b> palladium 106.42(1)	47 <b>Ag</b> silver 107.8682(2)	48 <b>Cd</b> cadmium 112.411(8)	49 <b>In</b> indium 114.818(3)	50 <b>Sn</b> tin 118.710(7)	51 <b>Sb</b> antimony 121.760(1)	52 <b>Te</b> tellurium 127.60(3)	53 <b>I</b> iodine 126.904 47(3)	54 <b>Xe</b> xenon 131.293(6)
55 <b>Cs</b> caesium 132.905 451 9(2)	56 <b>Ba</b> barium 137.327(7)	57-71 <b>lanthanoids</b>	72 <b>Hf</b> hafnium 178.49(2)	73 <b>Ta</b> tantalum 180.947 88(2)	74 <b>W</b> tungsten 183.84(1)	75 <b>Re</b> rhenium 186.207(1)	76 <b>Os</b> osmium 190.23(3)	77 <b>Ir</b> iridium 192.217(3)	78 <b>Pt</b> platinum 195.084(9)	79 <b>Au</b> gold 196.966 569(4)	80 <b>Hg</b> mercury 200.59(2)	81 <b>Tl</b> thallium 204.383(2)	82 <b>Pb</b> lead 207.2(1)	83 <b>Bi</b> bismuth 208.980 40(1)	84 <b>Po</b> polonium [209]	85 <b>At</b> astatine [210]	86 <b>Rn</b> radon [222]
87 <b>Fr</b> francium [223]	88 <b>Ra</b> radium [226]	89-103 <b>actinoids</b>	104 <b>Rf</b> rutherfordium [261]	105 <b>Db</b> dubnium [262]	106 <b>Sg</b> seaborgium [266]	107 <b>Bh</b> bohrium [264]	108 <b>Hs</b> hassium [277]	109 <b>Mt</b> meitnerium [268]	110 <b>Ds</b> darmstadtium [271]	111 <b>Rg</b> roentgenium [272]							
			57 <b>La</b> lanthanum 138.905 47(7)	58 <b>Ce</b> cerium 140.116(1)	59 <b>Pr</b> praseodymium 140.907 65(2)	60 <b>Nd</b> neodymium 144.242(3)	61 <b>Pm</b> promethium [146]	62 <b>Sm</b> samarium 150.36(2)	63 <b>Eu</b> europium 151.964(1)	64 <b>Gd</b> gadolinium 157.25(3)	65 <b>Tb</b> terbium 158.925 35(2)	66 <b>Dy</b> dysprosium 162.500(1)	67 <b>Ho</b> holmium 164.930 32(2)	68 <b>Er</b> erbium 167.259(3)	69 <b>Tm</b> thulium 168.934 2(1)	70 <b>Yb</b> ytterbium 173.04(3)	71 <b>Lu</b> lutetium 174.967(1)
			89 <b>Ac</b> actinium [227]	90 <b>Th</b> thorium 232.038 06(2)	91 <b>Pa</b> protactinium 231.036 88(2)	92 <b>U</b> uranium 238.028 91(3)	93 <b>Np</b> neptunium [237]	94 <b>Pu</b> plutonium [244]	95 <b>Am</b> americium [243]	96 <b>Cm</b> curium [247]	97 <b>Bk</b> berkelium [247]	98 <b>Cf</b> californium [251]	99 <b>Es</b> einsteinium [252]	100 <b>Fm</b> fermium [257]	101 <b>Md</b> mendelevium [258]	102 <b>No</b> nobelium [259]	103 <b>Lr</b> lawrencium [262]

Key:  
atomic number  
Symbol  
name  
standard atomic weight

**Ferromagnetisk**

**Diamagnetisk**

**Paramagnetisk**



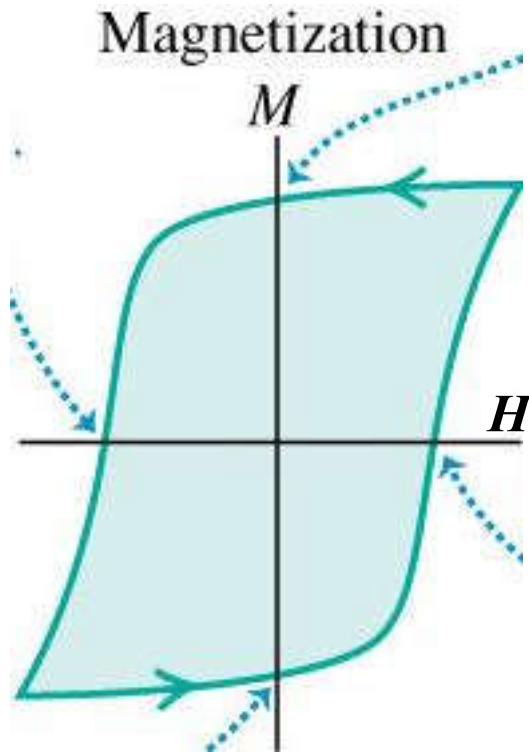
## Notes

- 'Aluminium' and 'caesium' are commonly used alternative spellings for 'aluminum' and 'caesium'.
- IUPAC 2005 standard atomic weights (mean relative atomic masses) are listed with uncertainties in the last figure in parentheses [M. E. Wieser, *Pure Appl. Chem.* 78, 2051 (2006)]. These values correspond to current best knowledge of the elements in natural terrestrial sources. For elements that have no stable or long-lived nuclides, the mass number of the nuclide with the longest confirmed half-life is listed between square brackets.
- Elements with atomic numbers 112 and above have been reported but not fully authenticated.

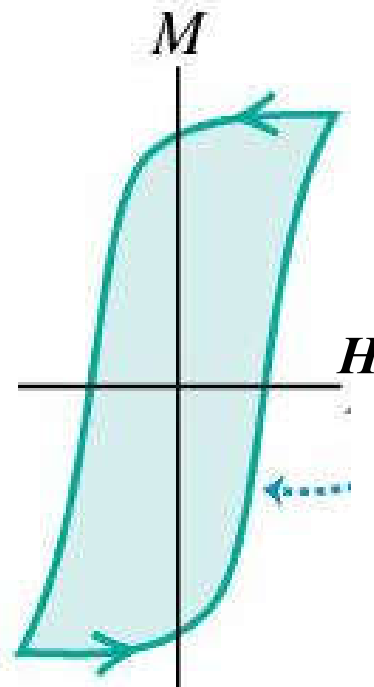
Copyright © 2007 IUPAC, the International Union of Pure and Applied Chemistry. For updates to this table, see [http://www.iupac.org/reports/periodic\\_table/](http://www.iupac.org/reports/periodic_table/). This version is dated 22 June 2007.



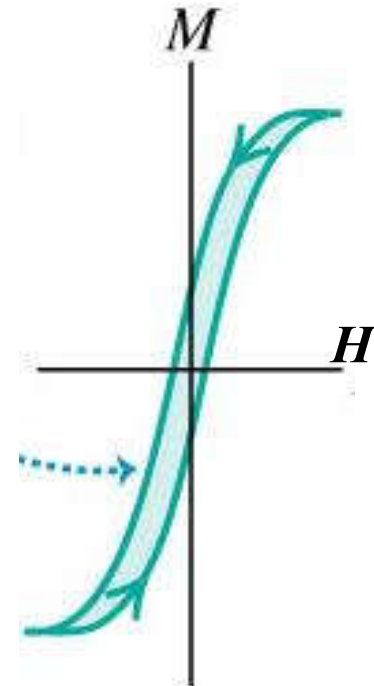
# Ulike grader av hysteresese i ferromagnetisk materiale



«Hardt» jern:  
permanentmagneter



av/på-magneter  
(eks. harddisk)



«Bløtt» jern:  
transformatorer



# Hva vi har lært:

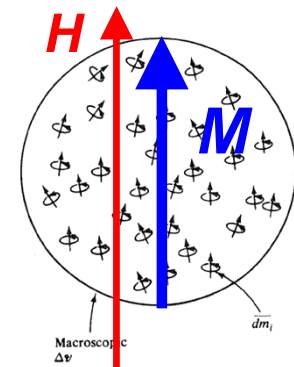
- Magnetisk feltstyrke:  $\mathbf{H} = \mathbf{B}/\mu_0$  (i tomrom)
- Magnetisering, definisjon:  $\mathbf{M} = \sum \boldsymbol{\mu} / \text{volum}$
- Magnetisering, eksperimentelt:  $\mathbf{M} = \chi_m \mathbf{H}$

- Totalt  $\mathbf{B}$ -felt i magnetisk materiale:

$$\begin{aligned}
 \mathbf{B} &= \mu_0 \mathbf{H} + \mu_0 \mathbf{M} \\
 &= \mu_0 \mathbf{H} + \mu_0 \chi_m \mathbf{H} \\
 &= \mu_0 \mu_r \mathbf{H},
 \end{aligned}$$

Kraftig forsterkning av  $\mathbf{B}$ -feltet i ferromagn. materiale

relativ permeabilitet:  $\mu_r = \chi_m + 1$

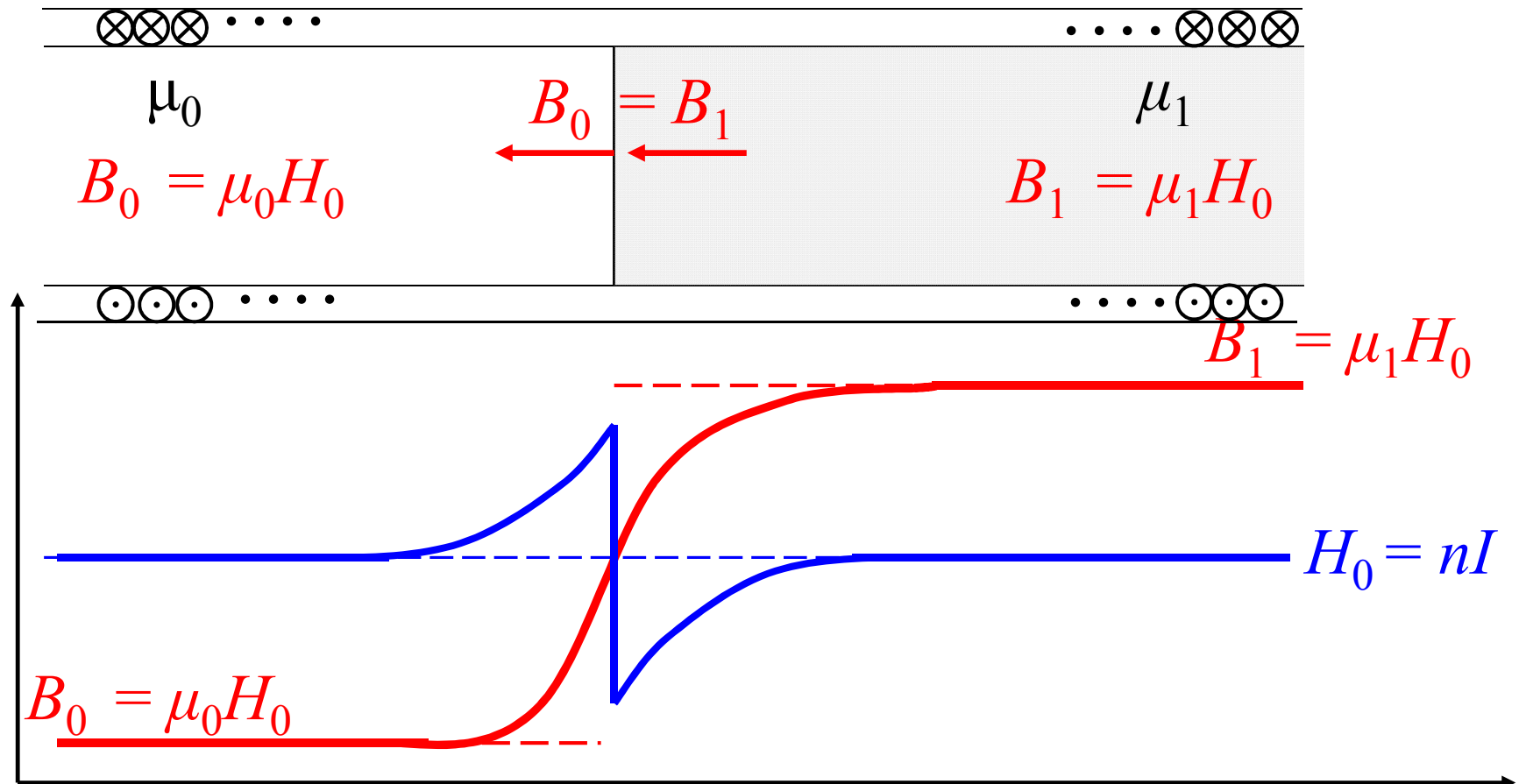


- Amperes lov på ny, enkel form:

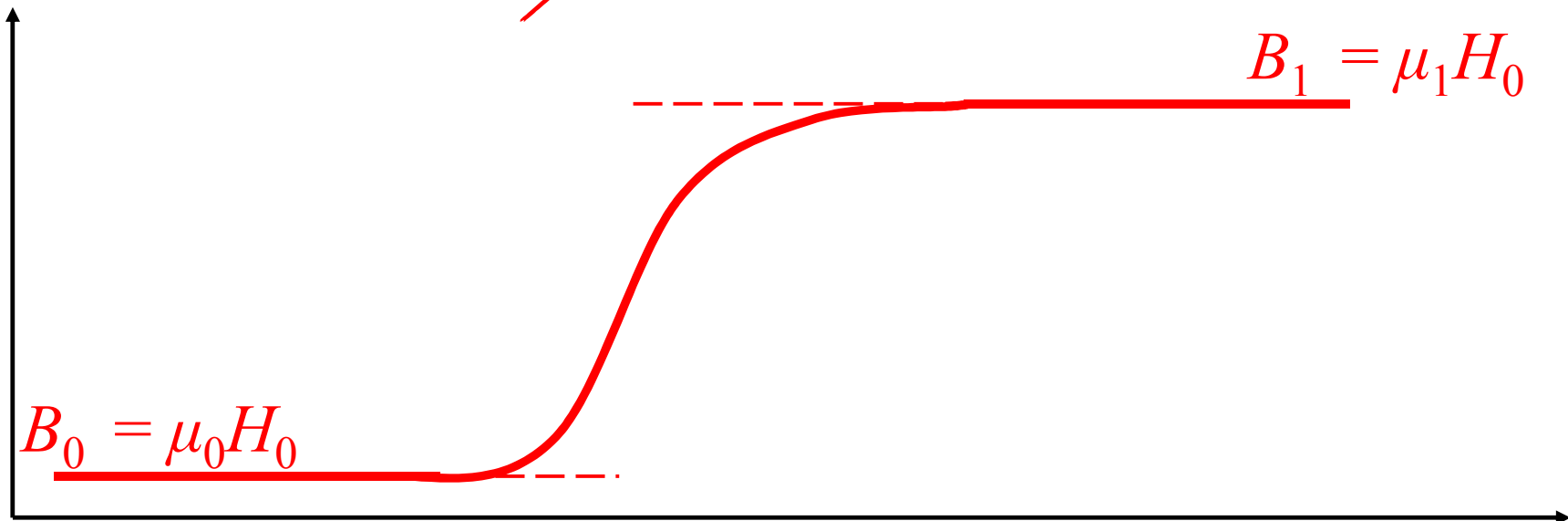
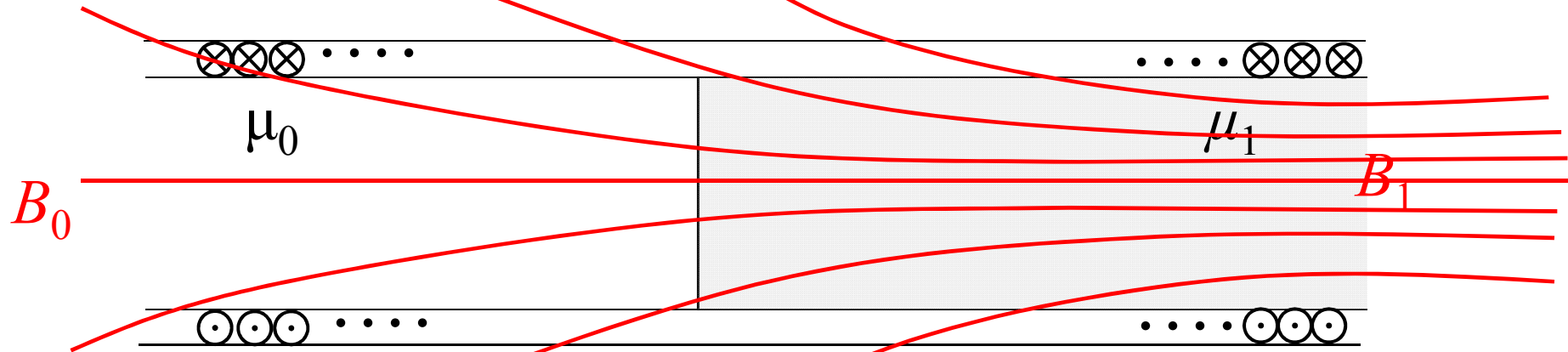
$$\int \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \quad \Leftrightarrow \quad \int \mathbf{H} \cdot d\mathbf{s} = I$$

$$\text{curl } \mathbf{B} = \mu_0 \mathbf{J} \quad \Leftrightarrow \quad \text{curl } \mathbf{H} = \mathbf{J}$$

# Eks. 6B. Halvfylt solenoide



# Eks. 6B. Halvfylt solenoide. Reelle feltlinjer



Kontinuitetskrav over grenseflater (skille i  $\mu_r$ ):

1)  $B_{\perp}$  kontinuerlig

2)  $H_{\perp}$  diskontinuerlig (faktor  $\mu_r$ )

3)  $H_{\parallel}$  kontinuerlig

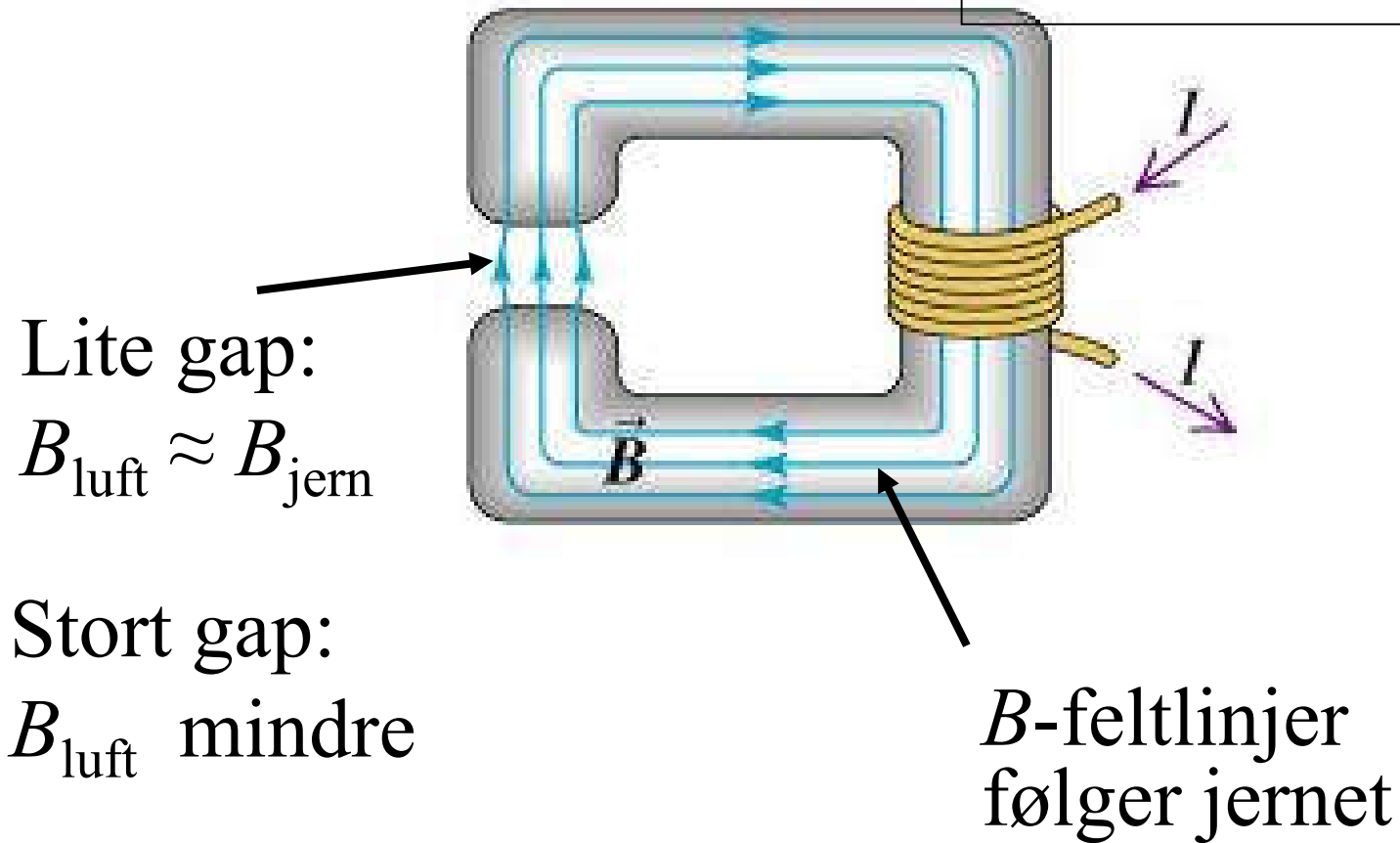
4)  $B_{\parallel}$  diskontinuerlig (faktor  $\mu_r$ )

Eks. 6C. Langsfylt solenoide.

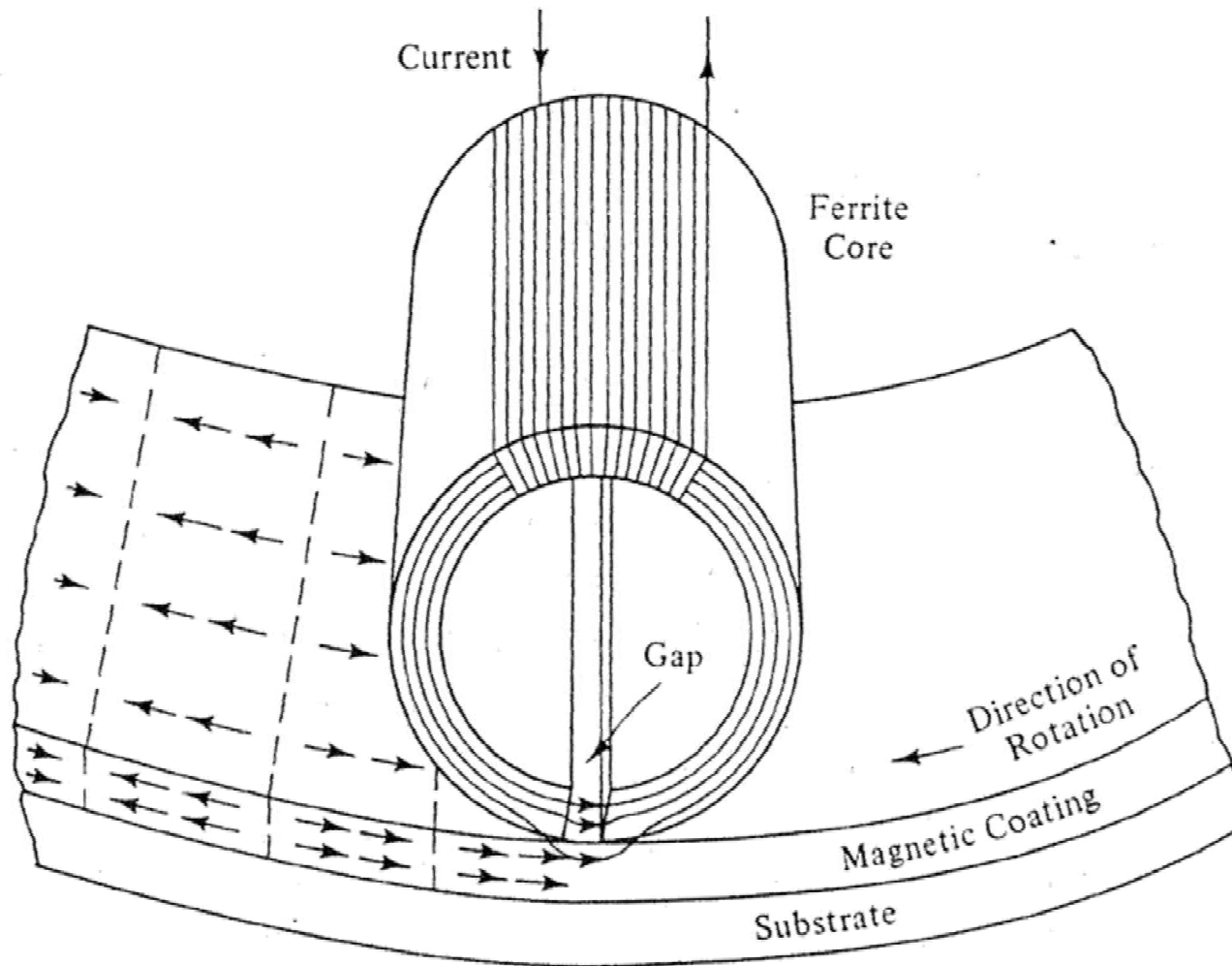
= Øving 12, opg 1  
og Eks. 2012 opg. 5.

# Eks. 7 Luftgap i magnet

= øving 12, opg 2.



# Magnetgap til bruk for å skrive på harddisk, videotape og lignende



# Kap. 28: Oppsummering: Kilde til magnetisk felt

- Bevegelse av ladninger er kilde for magnetfelt  $\mathbf{B}$

- Enkeltladning i bevegelse: 
$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\mathbf{v} \times \hat{r}}{r^2}$$

- Strøm i leder: 
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2}$$

- Magnetfelt  $\mathbf{B}$  kan finnes ved integrasjon over leder fra Biot-Savarts lov

-- eller ved bruk av:

- Amperes lov:

$$\int \mathbf{B} \cdot d\mathbf{s} = \mu_0 I$$

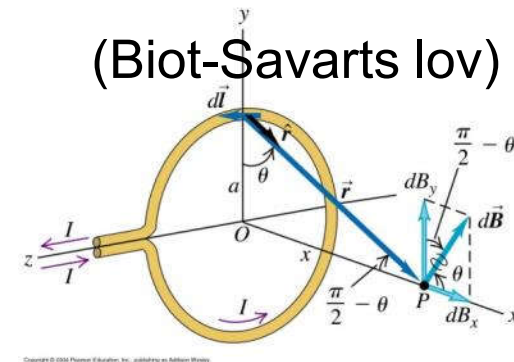
der  $I$  er strøm innenfor den lukkede integrasjonsvegen.

Differensialform:  $\text{curl } \mathbf{B} = \mu_0 \mathbf{J}$

$$\int \mathbf{H} \cdot d\mathbf{s} = I$$

$$\text{curl } \mathbf{H} = \mathbf{J}$$

- Viktige anvendelser: Rett leder, solenoide, m.m.



# Kap. 28: Oppsummering:

## Magnetiske materialer [mer i [Notat 2](#)]

- Materialer kan magnetiseres:  $\mathbf{M} = \chi_m \mathbf{H}$ 
  - Diamagnetiske:  $\chi_m$  liten, negativ
  - Paramagnetiske:  $\chi_m$  liten, positiv
  - Ferromagnetiske:  $\chi_m$  **stor** positiv
- Strømmer skaper magnetisk feltstyrke  $\mathbf{H}$  og flukstetthet:  
$$\mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M} = \mu_0 \mathbf{H} + \mu_0 \chi_m \mathbf{H} = \mu_0 \mu_r \mathbf{H}$$

.. altså avhengig av relativ permeabilitet  $\mu_r$  og dermed av materialet.
- I alle tidligere formler kan vi erstatte  $\mu_0$  med  $\mu = \mu_0 \mu_r$
- Kontinuitetskrav over grenseflater (skille i  $\mu_r$ ): [Mer i [Notat 6](#)]  
 $B_{\perp}$  kontinuerlig     $B_{\parallel}$  diskontinuerlig  
 $H_{\perp}$  diskontinuerlig     $H_{\parallel}$  kontinuerlig



# Maxwells likninger i [Notat 4](#)

Statikk

Integralform

$$\oint \vec{D} \cdot d\vec{A} = Q \quad \text{Gauss' lov } \mathbf{D}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad \text{Gauss' lov } \mathbf{B}$$

$$\oint \vec{H} \cdot d\vec{\ell} = I + \frac{\partial \Phi}{\partial t} \quad \text{Amperes lov}$$

$$\oint \vec{E} \cdot d\vec{\ell} = 0 - \frac{\partial \Phi_B}{\partial t} \quad \text{Faradays lov}$$

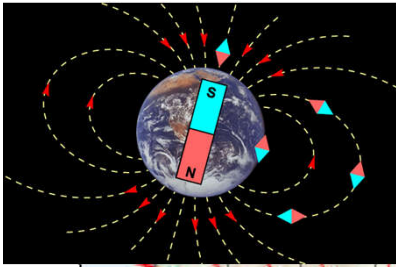
Differensialform

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

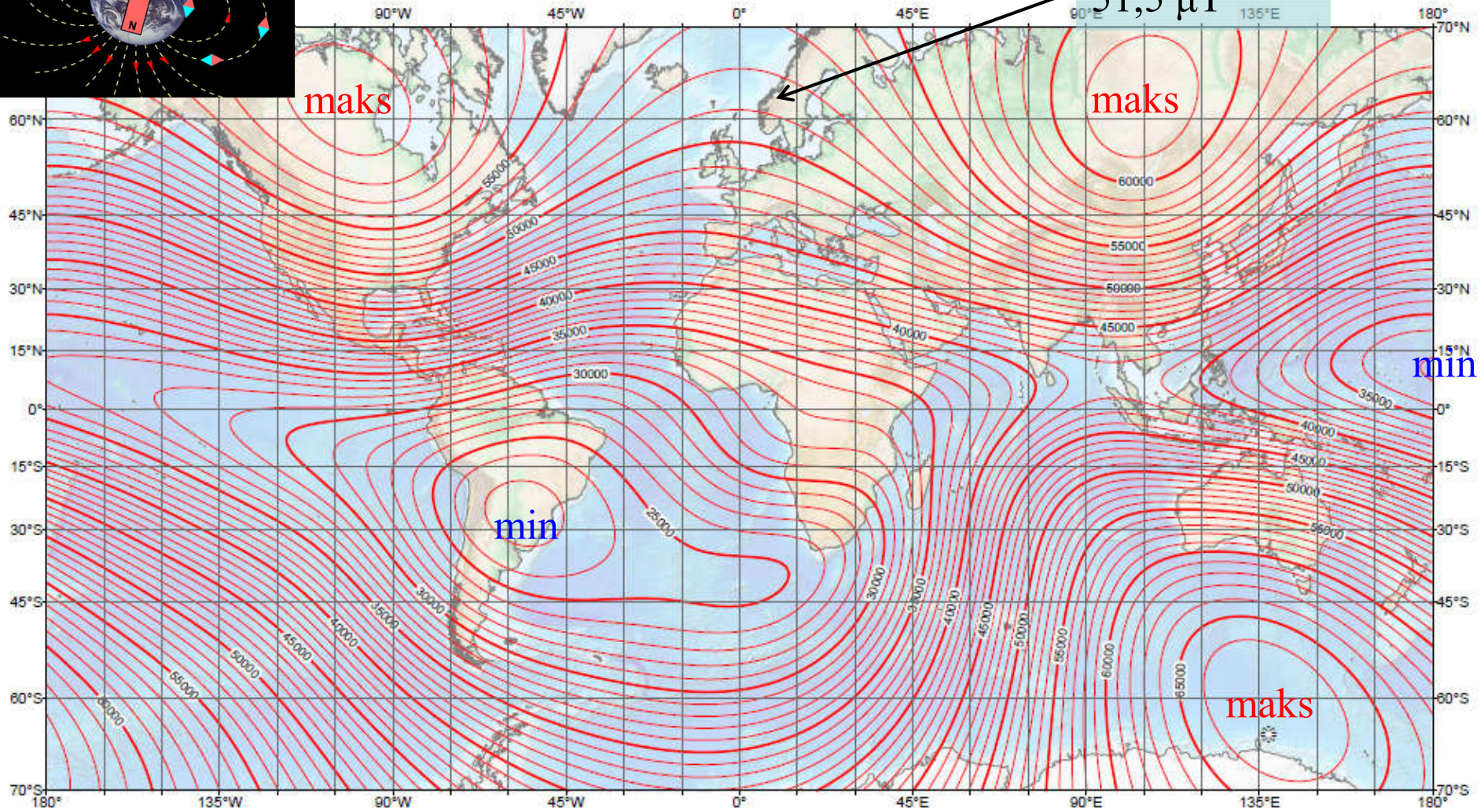
$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = 0 - \frac{\partial \vec{B}}{\partial t}$$



US/UK World Magnetic Model -- Epoch 2010.0  
Main Field **Total Intensity (F)**

Trondheim  
51,5  $\mu\text{T}$



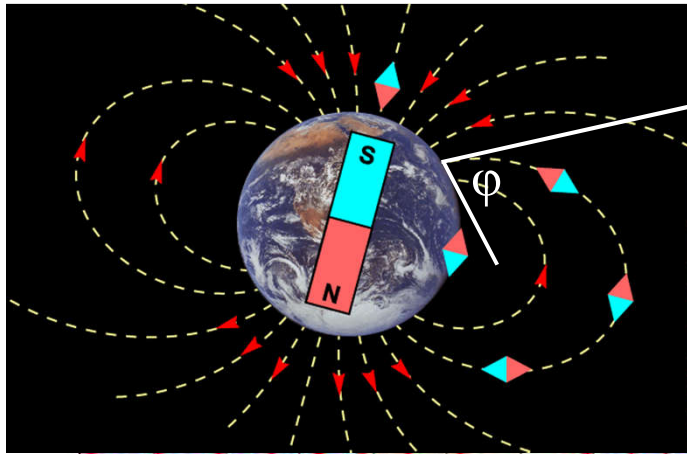
Main Field Total Intensity (F)  
Contour Interval: 1000 nT.  
Mercator Projection.  
📍 : Position of dip poles

Map developed by NOAA/NGDC & CIRES  
<http://ngdc.noaa.gov/geomag/WMM/>  
Map reviewed by NGA/BGS  
Published January 2010

*B*-felt i nT

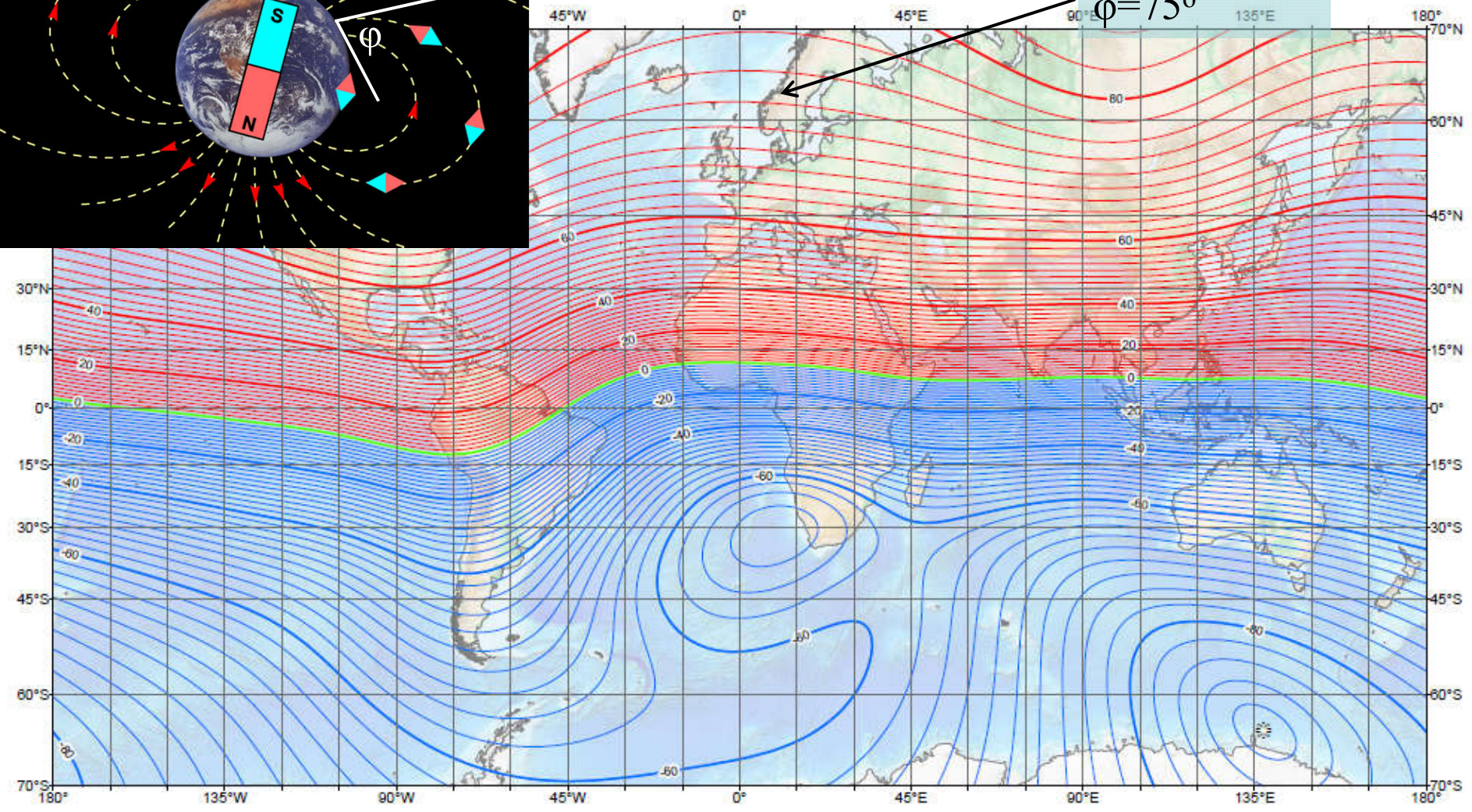
Fra: [en.wikipedia.org/wiki/Earth%27s\\_magnetic\\_field](http://en.wikipedia.org/wiki/Earth%27s_magnetic_field)





World Magnetic Model -- Epoch 2010.0  
 Main Field **Inclination (I)**

Trondheim  
 $\phi = 75^\circ$



**Main field Inclination (I)**  
 Contour interval: 2 degrees, red contours positive (down); blue negative (up); green zero line.  
 Mercator Projection.  
 ☉ : Position of dip poles

Map developed by NOAA/NGDC & CIRES  
<http://mgdc.noaa.gov/geomag/WWW/>  
 Map reviewed by NGA/BGS  
 Published January 2010

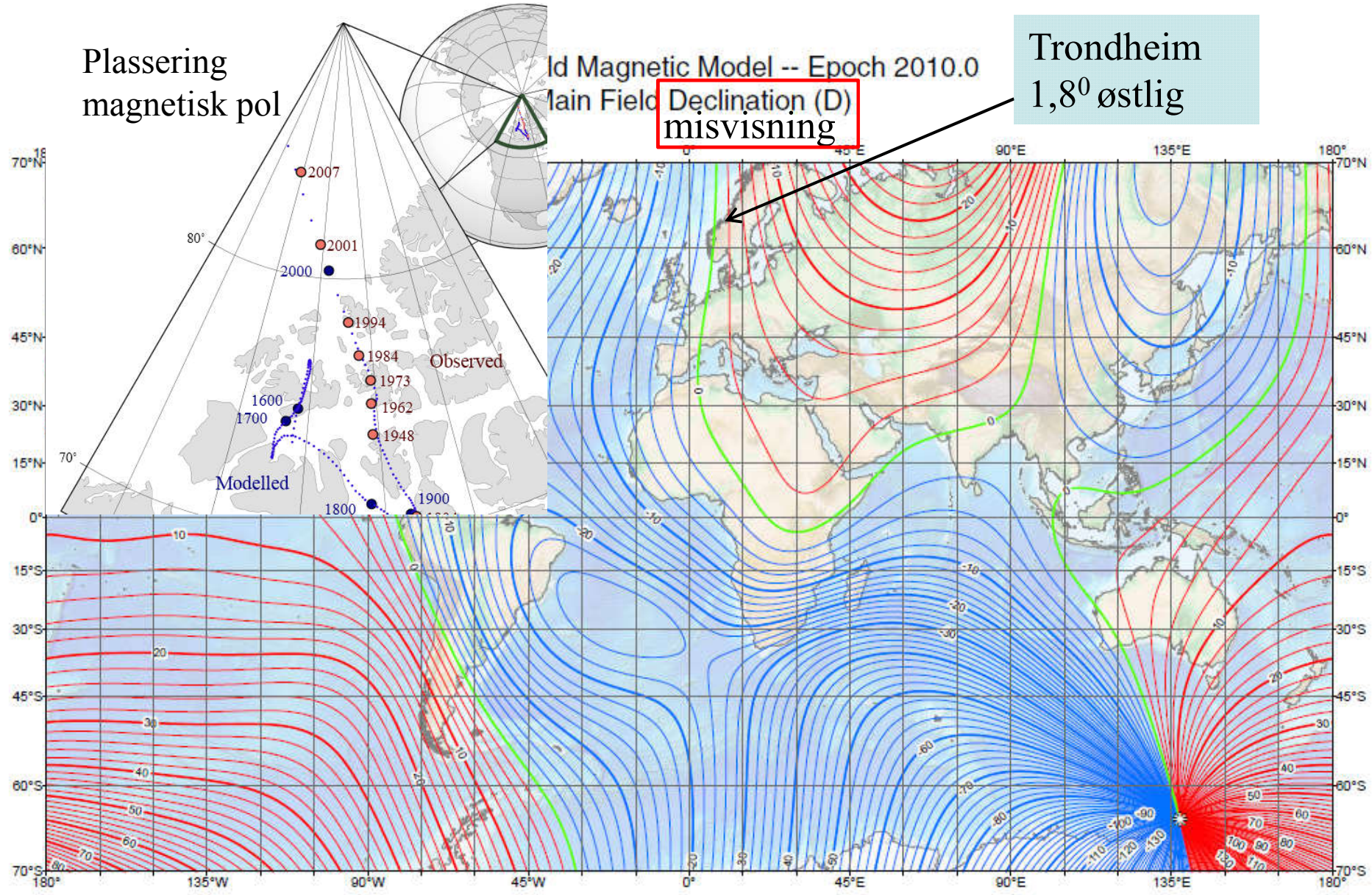
Fra: [en.wikipedia.org/wiki/Earth%27s\\_magnetic\\_field](http://en.wikipedia.org/wiki/Earth%27s_magnetic_field)



Plassering magnetisk pol

World Magnetic Model -- Epoch 2010.0  
Main Field Declination (D)  
misvisning

Trondheim  
1,8° østlig



Main field declination (D)  
Contour Interval: 2 degrees, red contours positive (east); blue negative (west); green (agonic) zero line.  
Mercator Projection.  
⊕, ⊙ : Position of dip poles

Map developed by NOAA/NGDC & CIRES  
<http://ngdc.noaa.gov/geomag/WMM/>  
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Fra: [en.wikipedia.org/wiki/Earth%27s\\_magnetic\\_field](http://en.wikipedia.org/wiki/Earth%27s_magnetic_field)