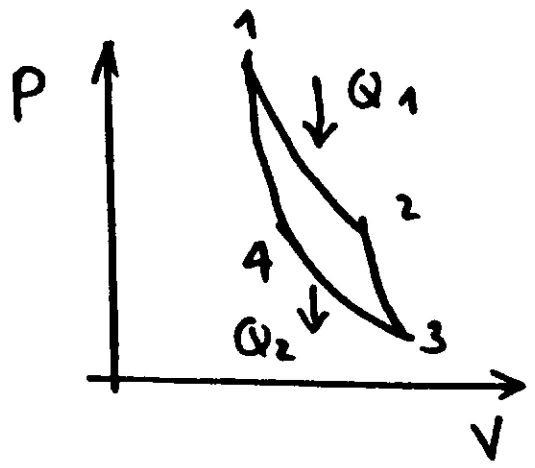
EKSAMEN VÅR 87.

OPPGAVE 1

Reversibel prosess: En prosess som er så langsom at systemet kan regnes for å være i termodynamisk likevekt under hele prosessen.

Adiabatisk prosess: En varmeisolert prosess ; $\Delta Q = 0$

CARNOT PROSESSEN.



1-2 : Isoterm prosess, $T = T_H$
2-3 : Adiabat : $\Delta Q = 0$
3-4 : Isoterm , $T = T_L$
4-1 : Adiabat ; $\Delta Q = 0$

$W = Q_1 + Q_2$ $Q_2 < 0$

Virkningsgrad def: $\eta = \dfrac{W}{Q_1} = \dfrac{Q_1 + Q_2}{Q_1}$

Temperaturavhengighet $\eta = \dfrac{T_H - T_L}{T_H}$

b) Vi anvender def. på virkningsgrad
Søkt er Q_2, kjent er W

$$\eta = \frac{Q_1 + Q_2}{Q_1} = \frac{W}{Q_1} = \frac{W}{W - Q_2}$$

$$\Rightarrow \quad -Q_2 = W \frac{1-\eta}{\eta}$$

Vi vet også verdien av η da vi kjenner T_H og T_L

$$\eta = \frac{T_H - T_L}{T_H} = \frac{520 - 290}{520} = 0.442$$

$$\underline{\underline{-Q_2}} = 10^9 \frac{1 - 0.442}{0.442} \approx \underline{\underline{1.262 \cdot 10^9 \text{ W}}}$$

Dette avgis til elvevannet.

Varmebalansen i elva:

$$|Q_2| = C_V \cdot \rho \cdot M \, \Delta T$$

$$\Rightarrow \quad \Delta T = \frac{|Q_2|}{C_V \cdot \rho \cdot M} \qquad \begin{aligned} C_V &= 4.2 \cdot 10^3 \text{ J/kg} \cdot \text{K} \\ &= 4.2 \cdot 10^6 \text{ J/m}^3 \cdot \text{K} \end{aligned}$$

$$\underline{\underline{\Delta T}} = \frac{1.26 \cdot 10^9}{4.2 \cdot 10^6 \cdot 40} = \underline{\underline{7.5 \text{ K}}}$$

c) Oppgitt $N = 1$ mol.
Gasslignningen blir da $pV = RT$.

Punkt 3. $T_3 = 293\,K$ $p_3 = 1.01 \cdot 10^5$ Pa

Fra gasslignningen:

$$\underline{\underline{V_3}} = \frac{RT_3}{p_3} = \frac{8.31 \cdot 293}{1{,}01 \cdot 10^5} = \underline{\underline{0.0241\ m^3}}$$
$$= \underline{\underline{24.1\ l}}$$

Punkt 2: $V_2 = V_3 = 24.1\ l$
$T_2 = 373\ K$

$$\frac{p_3}{T_3} = \frac{p_2}{T_2} \Rightarrow p_2 = p_3 \frac{T_2}{T_3}$$

$$p_2 = \frac{T_2}{T_3} p_3 = \frac{373}{293} \cdot 1.01 \cdot 10^5$$

$$\underline{\underline{p_2 = 1.29 \cdot 10^5\ Pa}}$$

Punkt 1: $\underline{\underline{T_1 = T_2 = 373\ K}}$
Fra adiabatligningen $TV^{\gamma-1} = \text{konst}$

$$V_1^{\gamma-1} T_1 = V_3^{\gamma-1} T_3$$

$$\Rightarrow V_1 = V_3 \left(\frac{T_3}{T_1}\right)^{\frac{1}{\gamma-1}} = 24.1 \left(\frac{293}{373}\right)^{\frac{1}{1.4-1}} = 13.2\ l$$

$$\underline{\underline{V_1 = 13.2\ l}}$$

$$p_1 V_1 = p_2 V_2 \Rightarrow \underline{\underline{p_1}} = p_2 \frac{V_2}{V_1} = 1.29 \cdot 10^5 \cdot \frac{24.1}{13.2} = \underline{2.35 \cdot 10^5\ Pa}$$

d) Vi baserer oss på varmebalansen

$\Delta Q_{12} = ?$ Gitt av $dQ = du + pdV$
$= C_v dT + pdV$

$dT = 0$ for en adiabat. Dette gir

$$\Delta Q_{12} = \int p dV = RT_2 \int \frac{dV}{V} = RT_2 \ln \frac{V_2}{V_1}$$

$$\Delta Q_{23} = C_v (T_3 - T_2)$$

$Q_{31} = 0$; adiabatisk prosess.

Dette gir

$$W = \Sigma \Delta Q = RT_2 \ln \frac{V_2}{V_1} + C_v (T_3 - T_2)$$

Nå er $C_v = \frac{R}{\gamma - 1}$
Dette gir derfor:

$$W = R \left(T_2 \ln \frac{V_2}{V_1} + \frac{1}{\gamma - 1} (T_3 - T_2) \right)$$

$$= 8.31 \left(373 \ln \frac{24.1}{13.2} + \frac{1}{1.4-1} (293 - 373) \right)$$

$$\underline{\underline{= 204.0 \, J}}$$

Virkningsgraden η er gitt av

$$\eta = \frac{W}{Q_{12}} = \frac{R \left(T_2 \ln \frac{V_2}{V_1} + \frac{1}{\gamma - 1} (T_3 - T_2) \right)}{RT_2 \ln \frac{V_2}{V_1}}$$

$$\eta = 1 + \frac{1}{\gamma-1}\,\frac{\frac{T_3}{T_2}-1}{\ln\frac{V_2}{V_1}}$$

Fra adiabatligningen fås

$$\frac{V_1}{V_2} = \left(\frac{T_3}{T_1}\right)^{\frac{1}{\gamma-1}}$$

$$\ln\frac{V_2}{V_1} = \frac{1}{\gamma-1}\ln\frac{T_1}{T_3}$$

Dette gir:

$$\underline{\underline{\eta = 1 - \frac{1-\frac{T_3}{T_2}}{\ln\frac{T_1}{T_3}}}} = \underline{\underline{0.11}}$$

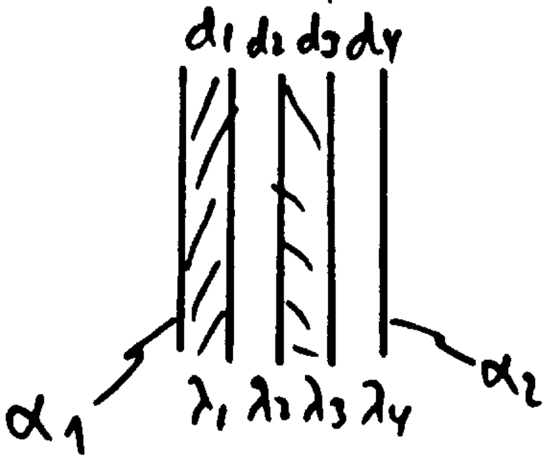
OPPGAVE 2

a) "Ohms" lov: $\dot{Q}R = \Delta T$

Her er: $\dot{Q}$ varmestrøm
R Resistans
ΔT Temperaturforskjell

Analogien gitt av: $\dot{Q} \leftrightarrow I$
$\Delta T \leftrightarrow V$
$R \leftrightarrow R$

For en plan lagdelt vegg gjelder:

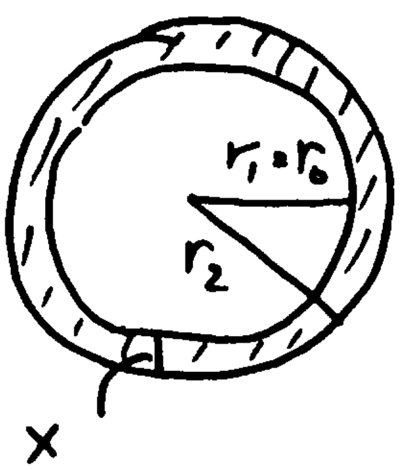


$$R = \frac{d_1}{\lambda_1 A} + \frac{d_2}{\lambda_2 A} + \frac{d_3}{\lambda_3 A} + \frac{1}{A\alpha_1} + \frac{1}{A\alpha_2}$$

ɔ: Seriekopling av motstander

λ er varmeledningsevne, A areal
α_1, α_2 varmeovergangstall.

b)



Fra formelsamling ses at termisk resistans for kuleskall er gitt av:

$$\underline{\underline{R_1}} = \frac{1}{4\pi\lambda}\left(\frac{1}{r_0} - \frac{1}{r_0+x}\right) = \underline{\underline{\frac{1}{4\pi\lambda}\,\frac{x}{r_0(r_0+x)}}}$$

c) For varmeovergang er ekvivalent resistans gitt som $R = \frac{1}{\alpha A}$

Dette gir i vårt tilfelle et bidrag

$$R_0 = \frac{1}{\alpha A} = \frac{1}{\alpha}\,\frac{1}{4\pi(r_0+x)^2}$$

Total resistans blir da

$$R_2 = R_1 + R_0$$
$$= \underline{\underline{\frac{1}{4\pi}\left\{\frac{x}{\lambda r_0(r_0+x)} + \frac{1}{\alpha}\,\frac{1}{(r_0+x)^2}\right\}}}$$

d) Vi må nå beregne $\frac{dR_2}{dx}$

$$\frac{dR_2}{dx} = \frac{1}{4\pi}\frac{d}{dx}\left(\frac{1}{\lambda}\left(\frac{1}{r_0} - \frac{1}{r_0+x}\right) + \frac{1}{\alpha}\frac{1}{(r_0+x)^2}\right)$$

$$= \frac{1}{4\pi}\left(\frac{1}{\lambda}\frac{1}{(r_0+x)^2} - \frac{2}{\alpha}\frac{1}{(r_0+x)^3}\right)$$

$\frac{dR_2}{dx} = 0$ gir:

$$r_0 + x_0 = \frac{2\lambda}{\alpha}$$

$$\underline{\underline{x_0 = \frac{2\lambda}{\alpha} - r_0}}$$

$$\underline{\underline{x_0}} = 2\,\frac{0.5}{3} - 0.25 = \underline{\underline{0.0833\,m = 8.33\,cm}}$$

Vi setter denne verdi for x_0 inn i total resistans: ($r_0 + x_0 = 2\lambda/\alpha$)

$$R_{2min} = \frac{1}{4\pi}\left(\frac{1}{\lambda}\left(\frac{1}{r_0} - \frac{\alpha}{2\lambda}\right) + \frac{1}{\alpha}\frac{\alpha^2}{4\lambda^2}\right)$$

$$\underline{\underline{= \frac{1}{4\pi}\left(\frac{1}{\lambda r_0} - \frac{\alpha}{4\lambda^2}\right)}}$$

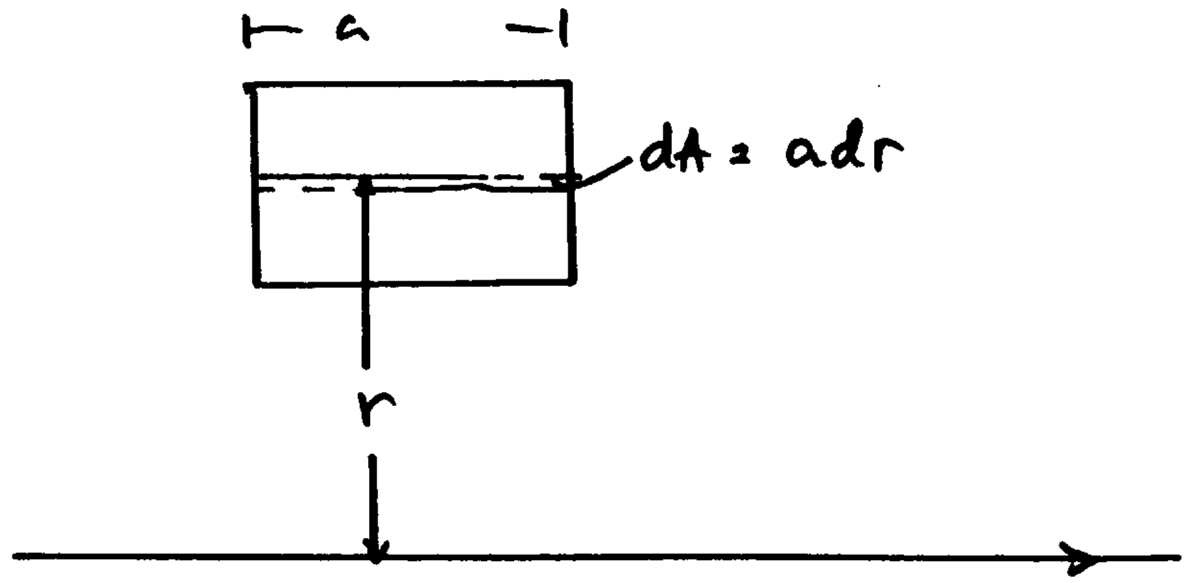
$$\underline{\underline{R_{2min}}} = \frac{1}{4\pi}\left(\frac{1}{0.5 \cdot 0.25} - \frac{3}{4 \cdot 0.5^2}\right) = \underline{\underline{0.398\ \frac{K \cdot s}{J}}}$$

Fra "Ohms" Lov:

$$\underline{\underline{\dot{Q}}} = \frac{\Delta T}{R_{2min}} = \frac{80}{0.398} = \underline{\underline{201\,W}}$$

OPPGAVE 3

a)



Magnetfeltet rundt en rett leder er gitt av

$$B = \frac{\mu_0 I_1}{2\pi r} \qquad \vec{B} \perp \text{ sløyfas plan}$$

Fluksen er da gitt av:

$$d\Phi = B_\perp dA = \frac{\mu_0 I_1}{2\pi r}\, a\,dr$$

$$\Phi = \frac{\mu_0 I_1 a}{2\pi} \int_{y_1}^{y_1+b} \frac{dr}{r} = \frac{\mu_0 I_1 a}{2\pi} \ln \frac{y_1+b}{y_1}$$

b) Kreftene fra ledningstykkene ⊥ på lederen opphever hverandre

Dette gir da

$$F = B I \cdot a$$

Netto kraft

$$F_N = I_2 a \left(\frac{\mu_0 I_1}{2\pi y_1} - \frac{\mu_0 I_1}{2\pi (y_1+b)} \right)$$

$$F_N = \frac{\mu_0}{2\pi} I_1 I_2 a \left(\frac{1}{y_1} - \frac{1}{y_1+b} \right)$$

Denne kraften fører til en forskyvning y_0

$$F_N = |k y_0| \qquad k = \text{fjærkonst.}$$

$$k y_0 = \frac{\mu_0}{2\pi} I_1 I_2 a \left(\frac{1}{y_1} - \frac{1}{y_1+b} \right)$$

$$I_2 = \frac{k y_0}{\frac{\mu_0}{2\pi} I_1 a \left(\frac{1}{y_1} - \frac{1}{y_1+b} \right)}$$

Innsatt tallverdier

$$I_2 = \frac{1 \cdot 10^{-3}}{\frac{4\pi \cdot 10^{-7}}{2\pi} 200 \cdot 0.1 \left(\frac{1}{0.1} - \frac{1}{0.2} \right)}$$

$$\underline{\underline{I_2 = 50\,A}}$$

c) Kraften er nå gitt av

$$F_N = \frac{\mu_0 I_0 I_2 a}{2\pi} \left(\frac{1}{y_1} - \frac{1}{y_1+b} \right) \cos\omega t$$

$$= F_0 \cos\omega t$$

Vi får således tvungne svingninger av systemet.

d) Utslagets amplitude er gitt av

$$y_a = \frac{F_0/m}{\sqrt{(\omega^2-\omega_0^2)^2+4\gamma^2\omega^2}}$$

$$\omega_0 = \sqrt{\frac{k}{m}} \qquad \gamma = \frac{R}{2m}$$

e) Ved resonans; svak demping fås

$\omega = \omega_0$;

$$\underline{\underline{\omega_0}} = \sqrt{\frac{k}{m}} = \sqrt{\frac{1}{0.04}} = \underline{\underline{5\,s^{-1}}}$$

$$y_a = \frac{F_0/m}{2\gamma\omega_0} = \frac{F_0/m}{2\cdot\frac{R}{2m}\sqrt{\frac{k}{m}}}$$

$$= \frac{F_0}{R\omega_0}$$

$$= \frac{\mu_0 I_0 I_2 a}{2\pi}\left(\frac{1}{y_1}-\frac{1}{y_1+b}\right)\frac{1}{R\omega_0}$$

$$= \frac{4\pi\cdot10^{-7}\cdot 50\cdot 50\;0.1}{2\pi}\left(\frac{1}{0.1}-\frac{1}{0.2}\right)\frac{1}{10^{-2}\cdot 5}$$

$$\underline{\underline{y_a = 5\cdot10^{-3}\,m = 5\,mm}}$$