LØSNINGSFORSLAG

EKSAMEN I FAG 70540 FYSIKK. Avd III (Bygg)

Oppgave 1

i) Harmonisk oscillator: $x(t) = A\cos(\omega_0 t + \delta)$

100 svingninger pr. min:

$\Rightarrow T_0 = 60/100 = 0{,}6\,s \quad ; \quad \underline{\underline{\omega_0}} = 2\pi/T_0 = 2\pi/0{,}6 = \underline{\underline{10{,}4720\,s^{-1}}}$

ii) a) $F_\lambda = -\lambda\dot{x}$

99 svingninger pr. min $\Rightarrow \underline{T = 60/99\,s = 0{,}6061\,s}$

$\omega = (\omega_0^2 - \gamma^2)^{1/2} \Rightarrow \gamma = \sqrt{\omega_0^2 - \omega^2}$

$\underline{\omega} = 2\pi/T = \underline{10{,}3667\,s^{-1}}$

$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0$

løsning: $x(t) = Ae^{-\gamma t}\sin(\omega t + \delta)$

Begynnelsesbetingelser: $x(0) = A_0$

$\dot{x}(0) = 0$

$\Rightarrow x(0) = A\sin\delta = A_0$

$\dot{x} = A(\omega\cos(\omega t + \delta)e^{-\gamma t} - \gamma e^{-\gamma t}\sin(\omega t + \delta))$

$\Leftrightarrow \dot{x}(0) = 0 = A(\omega\cos\delta - \gamma\sin\delta)$

$\Leftrightarrow \tan\delta = \omega/\gamma$

$\sin\delta = \frac{\tan\delta}{\sqrt{1+\tan^2\delta}} = \frac{\omega/\delta}{\sqrt{1+(\omega/\delta)^2}} = \frac{\omega}{\sqrt{\omega^2+\gamma^2}}$

Fra $\omega = \sqrt{\omega_0^2 - \gamma^2} \Rightarrow \omega_0 = \sqrt{\omega^2 + \gamma^2}$

$\Rightarrow \sin\delta = \omega/\omega_0$

$\Rightarrow A_0 = A \cdot \omega/\omega_0$ og $\underline{\underline{A = A_0\,\omega_0/\omega}}$

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b) $\gamma = A/2m \Rightarrow \lambda = 2m\gamma$

$\gamma = \sqrt{\omega_0^2 - \omega^2}$

$\Rightarrow \underline{\lambda = 2m\sqrt{\omega_0^2 - \omega^2}} = 6\pi\eta r$

$m = 4/3\, \pi r^3 \rho$ $\qquad \rho = 7{,}8 \cdot 10^3\ \mathrm{kg/m^3}$

$\qquad r = 0{,}01\ \mathrm{m}$

$\Rightarrow \underline{m} = 4/3 \cdot \pi \cdot (0{,}01\ \mathrm{m})^3 \cdot (7{,}8 \cdot 10^3\ \mathrm{kg/m^3}) = 0{,}0327\ \mathrm{kg}$

$\underline{\lambda} = 2 \cdot 0{,}0327\ \mathrm{kg}\sqrt{10{,}4720^2 - 10{,}3666^2}\ \mathrm{s^{-1}} = \underline{0{,}09692\ \mathrm{kg/s}}$

$\underline{\underline{\eta}} = \frac{\lambda}{6\pi r} = \frac{0{,}09692\ \mathrm{kg/s}}{6\pi \cdot 0{,}01\ \mathrm{m}} = \underline{\underline{0{,}514\ \mathrm{kg/sm}}}$

c) i) $A e^{-\gamma t} = A_0/10 \Leftrightarrow e^{-\gamma t} = A_0/A \cdot 10$

$\Rightarrow -\gamma t = \ln(A_0/A \cdot 10) = \ln\left(\frac{\omega}{10\omega_0}\right)$

$\Rightarrow t = -\frac{1}{\gamma}\ln\left(\frac{\omega}{10\omega_0}\right)$

$= -\frac{1}{\sqrt{\omega_0^2 - \omega^2}}\ln\left(\frac{\omega}{10\omega_0}\right) = \frac{1}{\sqrt{\omega_0^2 - \omega^2}}\ln\left(\frac{10\omega_0}{\omega}\right)$

$\Rightarrow t = \frac{1}{\sqrt{10{,}4720^2 - 10{,}3666^2}}\ln\left(\frac{10 \cdot 10{,}4720}{10{,}3666}\right)$

$\Rightarrow \underline{\underline{t = 1{,}56\ \mathrm{s}}}$

c) Liten dempning ⇒ ser kun på ytterstillingene og når de er minket til ca 1/10 i energi, dvs. ser kun på potensiell energi:

$E_{pot} = \frac{1}{2}kx^2$

Maksimal potensiell energi: $E_{max}$

(3)

$$E_{pot} = \frac{1}{2} k \cdot A^2 (e^{-\gamma t})^2 \cos^2(\omega t + \delta)$$

$$E_{max} = \frac{1}{2} k A^2 e^{-2\gamma t} \overset{\text{bet.}}{=} \frac{1}{2} k\, x(0)^2 \cdot \frac{1}{10}$$

$$\Leftrightarrow \frac{1}{2} k A^2 e^{-2\gamma t} = \frac{1}{10} \cdot \frac{1}{2} k A_0^2$$

$$\Leftrightarrow A^2 e^{-2\gamma t} = \frac{1}{10} \cdot A_0^2 = \frac{1}{10} \cdot A^2 \cdot \frac{\omega^2}{\omega_0^2}$$

$$\Leftrightarrow e^{-2\gamma t} = \frac{1}{10} \cdot \omega^2/\omega_0^2$$

$$\Leftrightarrow -2\gamma t = \ln\left[\omega^2/10\omega_0^2\right]$$

$$\Leftrightarrow t = 1/2\gamma \ln\left[10\omega_0^2/\omega^2\right] = m/\lambda \ln\left[10\omega_0^2/\omega^2\right]$$

$$= 0{,}0293/0{,}08657 \ln\left[10(10{,}4720)^2/10{,}3666^2\right]$$

$$\Leftrightarrow \underline{\underline{t = 3{,}45\,s}}$$

d) $\lambda = 2m\sqrt{\omega_0^2 - \omega^2}$

Kritisk dempning: $\omega_0^2 = \left[\lambda/2m\right]^2$ ; $\gamma = \omega_0$

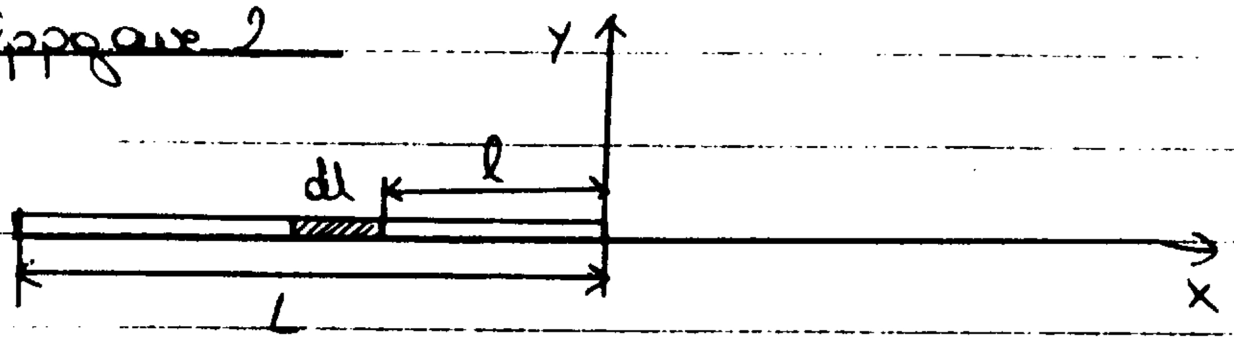
$$\lambda = 2\sqrt{mk} = \lambda_c$$

$$\Rightarrow \lambda_c = 2\sqrt{m\omega_0^2 m} = 2m\omega_0 = 2 \cdot 0{,}0327 \cdot 10{,}4720 = 0{,}6849$$

$$\underline{\eta} = \lambda/6\pi r = 0{,}6849/6 \cdot \pi \cdot 0{,}01 = \underline{\underline{3{,}65\ \text{kg/sm}}}$$

## Oppgave 2

a)




$$dV = \frac{\lambda\, dl}{4\pi\epsilon_0}\,\frac{1}{x+l}$$

$$V = \int_0^L \frac{\lambda\, dl}{4\pi\epsilon_0 (x+l)} = \frac{\lambda}{4\pi\epsilon_0}\ln\left|\frac{x+L}{x}\right|$$

$$\Leftrightarrow\ V = \frac{10^{-5}\,C/m}{4\pi\epsilon_0}\ln\left|\frac{1+0{,}50}{1}\right| = 9\cdot10^{9}\cdot10^{-5}\ln\left|\frac{1+0{,}50}{1}\right|$$

$$\Leftrightarrow\ \underline{\underline{V = 3{,}65\cdot10^{4}\,V}} \qquad [V\to 0 \text{ når } x\to\infty \quad OK]$$

b) E-felt langs aksen:

$$dE = \frac{\lambda\, dl}{4\pi\epsilon_0 (x+l)^2}$$

$$\Leftrightarrow\ E = \int_0^L \frac{\lambda\, dl}{4\pi\epsilon_0 (x+l)^2} = \frac{\lambda}{4\pi\epsilon_0}\,\frac{L}{x(x+L)} = 10^{-5}\cdot 9\cdot10^{9}\cdot\frac{0{,}5}{1(1+0{,}5)}$$

$$\Leftrightarrow\ \underline{\underline{E = 3{,}0\cdot10^{4}\,V/m}}$$

c) $\underline{\underline{E}} = -\frac{\partial V}{\partial x} = \underline{\underline{\frac{\lambda}{4\pi\epsilon_0}\,\frac{L}{x(x+L)}}}$ OK

d) $W = q\cdot V = 5\cdot10^{-6}\,C\,(V_x - V_\infty) = 5\cdot10^{-6}\cdot 3{,}65\cdot10^{4}\,J$

$$\Leftrightarrow\ \underline{\underline{W = 0{,}183\,J}}$$

e) $E'$ = felt fra punktladningene:

$$E' = \frac{Q}{4\pi\epsilon_0 x^2} + \frac{Q}{4\pi\epsilon_0 (x+L)^2} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{x^2} + \frac{1}{(x+L)^2}\right]$$

$$E' = 10^{-5} \cdot 9 \cdot 10^{9} \left[ \frac{1}{1^2} + \frac{1}{(1+0,5)^2} \right] V/m = 1,63 \cdot 10^{5}\, V/m$$

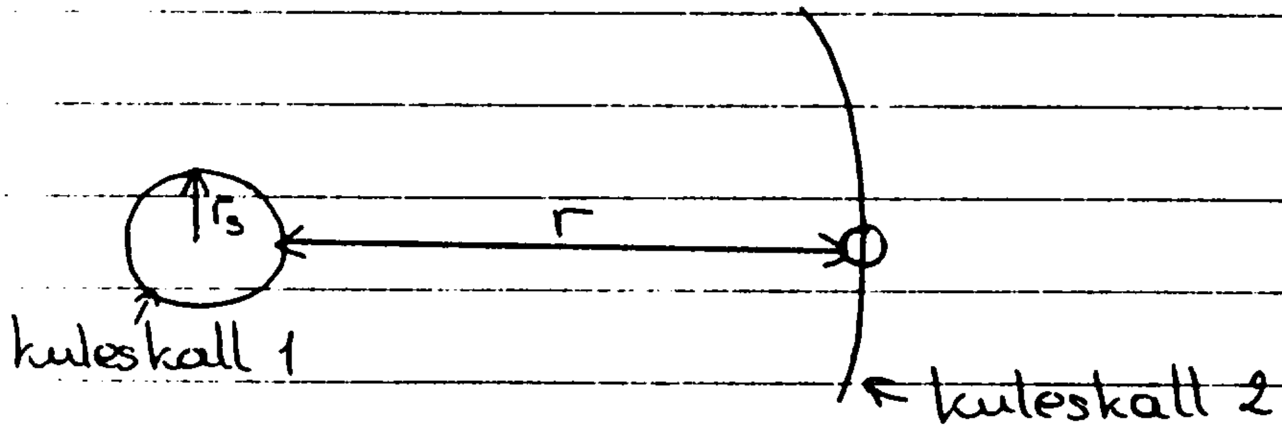
$$E_{tot} = E + E' = \left[ 3,0 \cdot 10^{4} + 1,63 \cdot 10^{5} \right] V/m$$

$$\Leftrightarrow \underline{\underline{E_{tot} = 1,93 \cdot 10^{5}\, V/m}}$$

## Oppgave 3

a) Energistrømtettheten nær sola: $j_s$

jorda: $j_j$




Total energistrøm gjennom skall 1 og 2 den samme.

$$j_s \cdot 4\pi r_s^2 = j_j \cdot 4\pi (r + r_s)^2 \approx j_j 4\pi \cdot r^2$$

$$\Leftrightarrow j_j = \sigma T_s^4 \left( \frac{r_s}{r} \right)^2 = 5,70 \cdot 10^{-8} (6000)^4 \left( \frac{7,00 \cdot 10^{8}}{1,50 \cdot 10^{11}} \right)^2$$

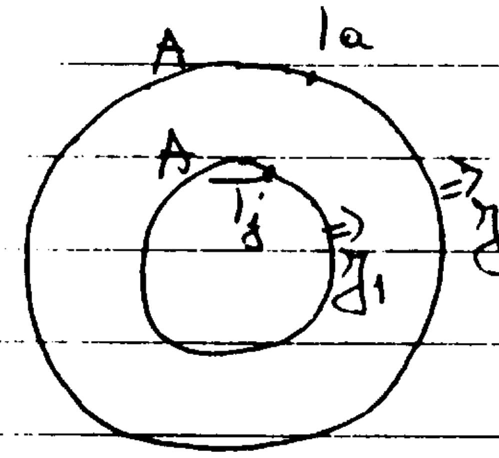
$$\Leftrightarrow \underline{\underline{j_j = 1,609 \cdot 10^{3}\, W/m^2}}$$

b) $J_s = \pi r_j^2 \cdot j_j (1 - \text{refl}) = \pi \cdot (6,40 \cdot 10^{6}\, m)^2 (1 - 0,370) \cdot 1,609 \cdot 10^{3}\, W/m^2$

$$\Leftrightarrow \underline{\underline{J_s = 1,304 \cdot 10^{17}\, W}}$$

(6)

c)




forskjellig frekvens i stråling fra jord og sol.

$T_K = 0$

$$J_1 = J_2 \Rightarrow \underbrace{A\sigma(T_j^4 - T_a^4)}_{\text{netto inn til atm. fra jorda}} = \underbrace{A\sigma T_a^4}_{\text{ut fra atm.}} = J_{TOT}$$

$$\Rightarrow 2T_a^4 = T_j^4 \quad \left[\Leftrightarrow T_a = \left(\tfrac{1}{2}\right)^{1/4} T_j\right]$$

$$T_a^4 = \frac{J_{TOT}}{\sigma A} = \frac{T_j^4}{2}$$

$$\Leftrightarrow T_j^4 = \frac{2 J_{TOT}}{\sigma A} \Leftrightarrow T_j = \left[\frac{J_S + J_N + J_M}{2\pi\sigma r_j^2}\right]^{1/4} \text{ q.e.d.}$$

d) $$T_j(0) = \left[\frac{J_S + J_N}{2\pi\sigma r_j^2}\right]^{1/4}$$

$$= \left[\frac{1{,}304\cdot 10^{17} + 3{,}50\cdot 10^{13}}{2\pi\cdot 5{,}7\cdot 10^{-8}(6{,}4\cdot 10^6)^2}\right]^{1/4} = \underline{\underline{307{,}0753\ K}}$$

$$\text{ny } T_j(0) = \left[\frac{1{,}304\cdot 10^{17} + 3{,}50\cdot 10^{13} + 7\cdot 10^{12}}{2\pi\cdot 5{,}7\cdot 10^{-8}(6{,}4\cdot 10^6)^2}\right]^{1/4} = 307{,}0794\ K$$

$$\underline{\Delta T_j = 4{,}11\cdot 10^{-3}\ K}$$

Tilsvarende atmosfaretemp: $T_a = \left(\frac{1}{2}\right)^{1/4} \cdot T_j$

$\Leftrightarrow \quad T_a = \left(\frac{1}{2}\right)^{1/4} \cdot 307{,}0794\,K$

$\Leftrightarrow \quad \underline{\underline{T_a = 258{,}23\,K}}$

e) $\Delta T_{j\max} = 1K.$

$\Rightarrow \quad J_M = (T_j)^4 \cdot 2\pi\sigma\, r_j^2 - J_S - J_N$

$= (308{,}0753\,K)^4 \cdot 2 \cdot \pi \cdot 5{,}7 \cdot 10^{-8}\,W/m^2K^4 \cdot (6{,}4 \cdot 10^6)^2 \cdot m^2$
$\div\ 1{,}304 \cdot 10^{17} - 3{,}50 \cdot 10^{13}$

$\Leftrightarrow \quad \underline{\underline{J_M = 1{,}707 \cdot 10^{15}\ W}}$