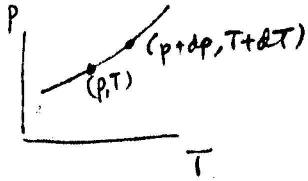
Løsningsskisse

Fag 715 62 Faseoverganger og kritiske fenomener

1.6.1979

Oppgave 1

Likevektsbetingelser: p, T, μ samme verdi i fasene.



$$\left.\begin{aligned} \mu_1(p+dp, T+dT) &= \mu_2(p+dp, T+dT) \\ \mu_1(p,T) &= \mu_2(p,T) \end{aligned}\right\} \Rightarrow$$

$$\left[\left(\frac{\partial \mu}{\partial p}\right)_T^1 - \left(\frac{\partial \mu}{\partial p}\right)_T^2\right] dp + \left[\left(\frac{\partial \mu}{\partial T}\right)_p^1 - \left(\frac{\partial \mu}{\partial T}\right)_p^2\right] dT = 0$$

Da $\left(\frac{\partial \mu}{\partial p}\right)_T = \frac{1}{N}\left(\frac{\partial G}{\partial p}\right)_T = \frac{V}{N}$ og $\left(\frac{\partial \mu}{\partial T}\right)_p = \frac{1}{N}\left(\frac{\partial G}{\partial T}\right)_p = -\frac{S}{N}$

fås

$$\underline{\frac{dp}{dT} = \frac{S_1 - S_2}{V_1 - V_2}}.$$

(S_i, V_i refererer seg til samme stoffmengde)

Når de "førstederiverte" S, V er like over faseovergangen må vi gå til neste orden. Mest direkte til målet fører:

$$\left.\begin{aligned} S_1(p+dp, T+dT) &= S_2(p+dp, T+dT) \\ S_1(p,T) &= S_1(p,T) \end{aligned}\right\} \Rightarrow$$

$$\left[\left(\frac{\partial S}{\partial p}\right)_T^1 - \left(\frac{\partial S}{\partial p}\right)_T^2\right] dp + \left[\left(\frac{\partial S}{\partial T}\right)_p^1 - \left(\frac{\partial S}{\partial T}\right)_p^2\right] dT = 0$$

Med $\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p \left(= -\frac{\partial^2 G}{\partial T \partial p}\right) = -V\alpha$

og $T\left(\frac{\partial S}{\partial T}\right)_p = C_p$

fås

$$\underline{\frac{dp}{dT} = \frac{c_p^1 - c_p^2}{T(\alpha^1 - \alpha^2)}} \qquad c_p = C_p / V$$

Oppgave 2

Med $\quad Z = \sum_{\{s_i\}} e^{-\beta H}$ $\quad$ fås

$$\left\langle \sum_i s_i \right\rangle = \frac{1}{\mu_0 \beta m} \frac{\partial \ln Z}{\partial \mathcal{H}} \qquad \text{og} \qquad \left\langle \left(\sum_i s_i\right)^2 \right\rangle = \frac{1}{Z} \frac{1}{(\mu_0 \beta m)^2} \frac{\partial^2}{\partial \mathcal{H}^2} Z$$

⟹

$$(\mu_0 \beta m)^2 \sum_i \sum_j \left(\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right) = \frac{1}{Z} \frac{\partial^2 Z}{\partial \mathcal{H}^2} - \frac{1}{Z^2} \left(\frac{\partial Z}{\partial \mathcal{H}} \right)^2 = \frac{\partial}{\partial \mathcal{H}} \left(\frac{1}{Z} \frac{\partial Z}{\partial \mathcal{H}} \right)$$

$$= \frac{\partial}{\partial \mathcal{H}} \left(\mu_0 \beta m \left\langle \sum_i s_i \right\rangle \right)$$

⟹

$$\sum_i \sum_j \Gamma(\vec{r}_i - \vec{r}_j) = \frac{1}{\mu_0 \beta m} \frac{\partial}{\partial \mathcal{H}} \left\langle \sum_i s_i \right\rangle$$

La $\mathcal{N}$ være antall spinn. Da er det magnetiske moment pr. spinn lik $\mathcal{M} = m \langle \sum_i s_i \rangle / \mathcal{N}$. For et makroskopisk system kan vi sette

$$\sum_i \sum_j \Gamma(\vec{r}_i - \vec{r}_j) = \mathcal{N} \sum_{\vec{r}} \Gamma(\vec{r}),$$

som gir

$$\boxed{m^2 \mu_0 \beta \sum_{\vec{r}} \Gamma(\vec{r}) = \left(\frac{\partial \mathcal{M}}{\partial \mathcal{H}} \right)_T = \chi_T} .$$

$\boxed{J=0}$

$$Z = \left(e^{\beta \mu_0 m \mathcal{H}} + e^{-\beta \mu_0 m \mathcal{H}} \right)^{\mathcal{N}} = \left[2 \cosh(\beta \mu_0 m \mathcal{H}) \right]^{\mathcal{N}}$$

$$\mathcal{M} = \frac{1}{\beta \mu_0 \mathcal{N}} \frac{\partial \ln Z}{\partial \mathcal{H}} = \underline{m \tanh(m \beta \mu_0 \mathcal{H})}$$

I dette tilfelle er $\langle s_i s_j \rangle = \langle s_i \rangle \langle s_j \rangle$ for $i \neq j$

$$\Gamma(\vec{r}) = \begin{cases} 1 - \langle s_i \rangle^2 = 1 - \mathcal{M}^2/m^2, & \vec{r} = 0 \\ 0 & \text{ellers} \end{cases}$$

Dermed

$$m^2 \mu_0 \beta \sum_{\vec{r}} \Gamma(\vec{r}) = m^2 \mu_0 \beta \left[1 - \tanh^2(m \beta \mu_0 \mathcal{H}) \right],$$

som stemmer med

$$\chi_T = \frac{\partial}{\partial \mathcal{H}} m \tanh(m \beta \mu_0 \mathcal{H}) = m^2 \beta \mu_0 \left[1 - \tanh^2(m \beta \mu_0 \mathcal{H}) \right].$$

Mean field.

Tilst. likn. $\mathcal{M} = m \tanh(m\mu_0 \mathcal{H}_{eff}\beta)$

som snues: $y = \tanh x = \frac{e^{2x}-1}{e^{2x}+1} \Rightarrow x = \frac{1}{2}\ln\frac{1+y}{1-y}$,

ɔ:

$$\frac{1}{2}\ln\frac{1+\mathcal{M}/m}{1-\mathcal{M}/m} = m\mu_0\beta\mathcal{H}_{eff} = m\mu_0\beta\left[\mathcal{H} + 6J\mathcal{M}/\mu_0 m^2\right]$$

eller,

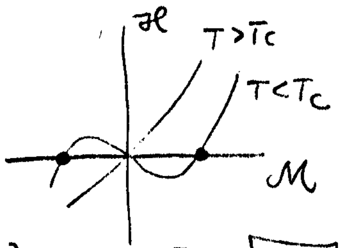
$$\underline{\mathcal{H} = (2m\mu_0\beta)^{-1}\ln\frac{m+\mathcal{M}}{m-\mathcal{M}} - 6J\mathcal{M}/\mu_0 m^2}.$$

For små $\mathcal{M}$ er høyre side ($\ln(1\pm x) = \pm x - \frac{x^2}{2} \pm \frac{x^3}{3} + \dots$)

$$\mathcal{H} = \frac{kT-6J}{m^2\mu_0}\mathcal{M} + \frac{kT}{3m^4\mu_0}\mathcal{M}^3 + \mathcal{O}(\mathcal{M}^5)$$

som gir Curie temperaturen

$$\underline{kT_c = 6J.}$$



Indekser: For $T=T_c$ er $\mathcal{H} = \frac{kT}{3m^4\mu_0}\mathcal{M}^3 + \mathcal{O}(\mathcal{M}^5)$, $\boxed{\delta=3}$

$$\frac{1}{\chi_T} = \left(\frac{\partial\mathcal{H}}{\partial\mathcal{M}}\right)_T = \frac{kT}{2m^2\mu_0}\left(\frac{1}{1+\mathcal{M}/m} + \frac{1}{1-\mathcal{M}/m}\right) - \frac{6J}{\mu_0 m^2} = \frac{kT-6J}{m^2\mu_0} = \frac{k(T-T_c)}{m^2\mu_0}$$

for $\mathcal{M}=0$ ($\mathcal{H}=0$). Dvs. $\chi_T = \frac{m^2\mu_0}{k(T-T_c)}$ for $T>T_c$ ɔ: $\boxed{\gamma=1}$.

Generelt for klassiske teorier (Bragg-Williams, van der Waals, Landau, mesten enhver analytisk tilstandsligning).

*

Korrelasjonsfunksjon:

$$\Gamma(\vec r) \sim \frac{F(r/\xi)}{r^{d-2+\eta}}, \quad r \text{ stor}, \quad T\approx T_c,$$

$$\xi \sim (T-T_c)^{-\nu} \qquad T\downarrow T_c$$

$$\chi_T \propto \sum_{\vec r}\Gamma(\vec r) \propto \int\frac{F(r/\xi)}{r^{d-2+\eta}}\,d\vec r = \xi^{2-\eta}\int\frac{F(R)\,d\vec R}{R^{d-2+\eta}}$$

ved å innføre dimensjonsløs avstand. Da $\xi\sim(T-T_c)^{-\nu}$ går $\sum_r\Gamma(\vec r)$ som $(T-T_c)^{-(2-\eta)\nu}$ ɔ: $\boxed{\gamma=(2-\eta)\nu}$.

Oppgave 3

$$g_s(\lambda^a t, \lambda^b h) = \lambda g_s(t,h)$$

$$\Rightarrow \quad \lambda^b \frac{\partial g_s(\lambda^a t, \lambda^b h)}{\partial(\lambda^b h)} = \lambda \frac{\partial g_s(t,h)}{\partial h}$$

$$\text{):} \quad m_s(\lambda^a t, \lambda^b h) = \lambda^{1-b} m_s(t,h)$$

$h = 0,\ t<0$

$$m_s(\lambda^a t, 0) = \lambda^{1-b} m_s(t,0)$$

$$m_s(t,0) \propto (-t)^\beta \quad \text{med} \quad \lambda^{a\beta} = \lambda^{1-b} \quad \boxed{\beta = \frac{1-b}{a}}$$

$t=0$

$$m_s(0, \lambda^b h) = \lambda^{1-b} m_s(0,h)$$

$$m_s(0,h) \propto h^{1/\delta} \quad \text{med} \quad \lambda^{b/\delta} = \lambda^{1-b} \quad \boxed{\delta = \frac{b}{1-b}}$$

$$\lambda^b \frac{\partial m_s(\lambda^a t, \lambda^b h)}{\partial(\lambda^b h)} = \lambda^{1-b} \frac{\partial m_s(t,h)}{\partial t}$$

$$\chi_T(\lambda^a t, \lambda^b h) = \lambda^{1-2b} \chi_T(t,h)$$

$$\chi_T(\lambda^a t, 0) = \lambda^{1-2b} \chi_T(t,0)$$

$$t<0 \quad \chi_T(t) \sim (-t)^{-\gamma'} \Rightarrow \lambda^{-a\gamma'} = \lambda^{1-2b} \quad \boxed{\gamma' = \frac{2b-1}{a}}$$

Eliminasjon av a og b:

$$\frac{\gamma'}{\beta} = \frac{2b-1}{1-b} = \frac{b}{1-b} - 1 = \delta - 1 \quad \text{):} \quad \boxed{\gamma' = \beta(\delta - 1)}$$