<u>facit</u> slik som jeg hadde ment.

I

$$\lambda = \frac{h}{mv} = \left(\frac{h^2}{2meV}\right)^{1/2}$$

i CGS : (1 volt $= \frac{1}{300} \cdot$ CGS potensial enhet)

$$\lambda = \left[\frac{6,623\cdot 10^{-27}}{2\cdot 9,106\cdot 10^{-28}\cdot 4,8\cdot 10^{-10}} \cdot \frac{300}{V(\text{volt})}\right]^{1/2} \text{ centimeter}$$

$$= \left[\quad '' \quad\quad '' \quad\right]\cdot 10^{8}\ \text{Å}$$

$$= \left[\frac{43,8641\cdot 10^{-54}\cdot 300\cdot 10^{16}}{87,434\cdot 10^{-38}\ V(\text{volt})}\right]^{1/2} = \sqrt{\frac{150}{V(\text{volt})}}$$

$\therefore$ <u>konst = 150</u>

i SI - enheter

$$\lambda = \left[\frac{(6,623\cdot 10^{-34})^2}{2\cdot 9,106\cdot 10^{-31}\cdot 1,6\cdot 10^{-19}\ V}\right]^{1/2} \text{ meter}$$

$$= \left[\frac{43,8641\cdot 10^{-68}}{29,1392\cdot 10^{-50}\ V}\cdot 10^{20}\right]^{1/2} \text{ Ångstrøm}$$

$$= \underline{\sqrt{\frac{150}{V(\text{volt})}}}\ .$$

$\times$

Facit II :

Impulsbølgefunktionen er

a) $\phi(p,0) = \frac{1}{\sqrt{2\pi\hbar}} \int \psi(x,0)\, dx \cdot e^{-\frac{i}{\hbar}px} = (2\pi\Delta x^2)^{-1/4} \left(\frac{4\pi\Delta x^2}{2\pi\hbar}\right)^{1/2} \cdot e^{-\frac{p^2}{\hbar^2/\Delta x^2}}$

$w(p,0) = |\phi(p)|^2 = (2\pi\Delta x^2)^{-1/2} \frac{4\pi\Delta x^2}{2\pi\hbar} \cdot e^{-\frac{p^2}{\hbar^2/\Delta x^2}\cdot 2}$

$\therefore \quad \underline{\underline{\Delta p^2 = \frac{1}{4}\hbar^2/\Delta x^2}} \qquad \underline{\underline{w(p) = (2\pi\Delta p^2)^{-1/2} e^{-\frac{p^2}{2\Delta p^2}}}}$

b) $\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}} \int \phi(p,0) \cdot e^{-\frac{it}{\hbar}\frac{p^2}{2m}} dp \cdot e^{\frac{ix}{\hbar}p}$

$\int e^{-p^2\left(\frac{\Delta x^2}{\hbar^2} + \frac{it}{2m\hbar}\right) + \frac{ix}{\hbar}p} = \left(\frac{\pi\hbar^2}{\Delta x^2 + \frac{it\hbar}{2m}}\right)^{1/2} \cdot e^{-\frac{x^2}{4\left(\Delta x^2 + \frac{it\hbar}{2m}\right)}}$

$\frac{1}{\Delta x^2 + \frac{it}{2m}\hbar} = \frac{\Delta x^2 - \frac{it\hbar}{2m}}{\Delta x^4 + \frac{t^2\hbar^2}{4m^2}} = \frac{1 - \text{Imag}}{\Delta x^2 + \frac{\Delta p^2}{m^2} t^2} = \frac{1 - \text{Imag}}{\Delta x^2(t)}.$

$\Rightarrow w(x,t) = \left[2\pi\Delta x^2(t)\right]^{-1/2} \cdot e^{-\frac{x^2}{2\Delta x^2(t)}}$

hvor $\Delta x^2(t) = \Delta x^2 + \left(\frac{\Delta p}{m}\right)^2 \cdot t^2$

c) $\psi(x,0) = (2\pi\Delta x^2)^{-1/4} \cdot e^{-\frac{(x-x_0)^2}{4\Delta x^2} + \frac{i}{\hbar}p_0 x}$ for example.

(almindeligere $\psi = f(x-x_0) \cdot e^{\frac{i}{\hbar}p_0 x}$

med f symmetrisk: $|f|^2(z) = |f|^2(-z)$)

d) Kan regnes ut som før men enklere ved å foreta en Galilei transformasjon $x = x_0 + \frac{p_0}{m}t$ av a) b) $\Rightarrow$

$w(p) = N\, e^{-\frac{(p-p_0)^2}{2\Delta p^2}}, \quad w(x,t) = N' e^{-\frac{\left(x - x_0 - \frac{p_0}{m}t\right)^2}{4\Delta x^2(t)}}$

e) $\frac{d}{dt}\langle p^n\rangle = \left\langle \frac{i}{\hbar}\left[H, p^n\right]\right\rangle = 0$ for $H = \frac{p^2}{2m}$

f) $\psi = \rho\, e^{i\delta(x)}, \quad \frac{\hbar}{i}\nabla\psi = \frac{\hbar}{i}(\nabla\rho + i\rho\nabla\delta) \cdot e^{i\delta}$

$\langle v\rangle = \frac{1}{m}\frac{\hbar}{i}\left\{\int \rho\nabla\rho + i\int \rho^2\nabla\delta\right\}$

første ledd $= 0$. Skal vi få bevegelse må vi ha δ med

δ er et slags hastighetspotensial.

III

b) $\psi(x;t+\Delta t) = \int K(x,\xi;\Delta t)\,d\xi\,\psi(\xi,t)$

$$= e^{-\frac{i}{\hbar}V(x)\Delta t}\int K^0\,d\xi\left[\sum_{n=0}\frac{(\xi-x)^n}{n!}\frac{\partial^n\psi}{\partial\xi^n}\Big|_{\xi=x}\right]$$

$$= \;''\;\left[\psi(x,t)+\frac{i\hbar}{2m}\frac{\partial^2\psi}{\partial x^2}\Delta t + O(\Delta t)^2\right]$$

$$= \psi(x,t) + \frac{i}{\hbar}\left(\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} - V\psi\right)\Delta t + O(\Delta t)^2$$

v.s. $\psi(x,t)+\Delta t\cdot\frac{\partial\psi}{\partial t} + O(\Delta t)^2 = $ h.s.

$$\psi(x,t+\Delta t) - \psi(x,t) =$$
$$\Delta t\,\frac{\partial\psi}{\partial t} + O(\Delta t)^2 = -\frac{i}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V\psi\right)\Delta t + O(\Delta t)^2$$

dvs $i\hbar\frac{\partial\psi}{\partial t} = H\psi$

c) K^0: Entr $(i\hbar\frac{\partial}{\partial t} - H^0)K^0 = 0$ for $t>0$ (1)

$K^0 = 0$ $t<0$ (2)

og $K^0 \to \delta(r-r_0)$ når $t\to t_0$ (3)

Eller: (1') $(i\hbar\frac{\partial}{\partial t} - H_0)K^0 = \delta(r-r_0)\,\delta(t-t_0)$

(2') $K^0 = 0$ for $t<t_0$

$$K^0 = \int a(k,\omega)\,dk\,e^{ik(R-\omega T)}\,d\omega \qquad R = r-r_0 \quad T = t-t_0$$

(1') $\Rightarrow$ $\hbar\int\left[\omega - \frac{k^2\hbar}{2m}\right]a(k,\omega)\exp.$ $\Rightarrow$ $a = \left(\frac{1}{2\pi}\right)^4\frac{1}{\omega-\frac{k^2\hbar}{2m}}$

$$(2') \Rightarrow K = \lim_{\varepsilon\to 0}\left(\frac{1}{2\pi}\right)^4\int d^3k\,e^{ik\cdot R}\int_{-\infty}^{\infty}\frac{d\omega\,e^{-i\omega T}}{\omega+i\varepsilon-\frac{k^2\hbar}{2m}}$$

$\int d\omega$ gir faktor $(-2\pi i)\,e^{ik\cdot R}\cdot e^{-i\frac{k^2\hbar}{2m}T}$ for $T>0$ ellers 0.

$\int e^{ik\cdot R}\,d\Omega(\hat{k}) = 4\pi\frac{\sin kR}{kR}$ $\qquad \int d^3k\,\exp = \frac{4\pi}{R}\int_0^\infty k\,dk\,\sin kR\,e^{-\frac{i\hbar T}{2m}k^2}$

$-\frac{4\pi}{R}\frac{d}{dR}\cdot\frac{1}{2}\int_{-\infty}^{\infty}e^{ikR}\;''\;'' = \left(\frac{2\pi m}{i\hbar T}\right)^{3/2}e^{\frac{im}{2\hbar}\frac{R^2}{T}}$

q.e.d.