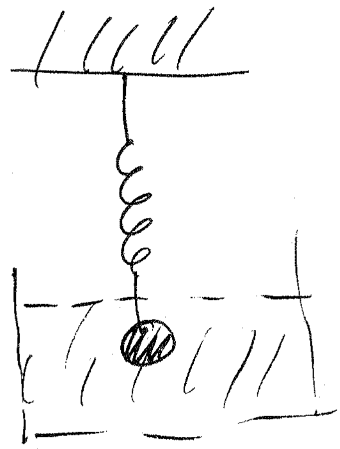
Oppgave 1

a) Frekvens $f_0 = \frac{48}{16}\,Hz = 3{,}00\,Hz$

Sirkelfrekvens $\underline{\omega_0} = 2\pi f_0 = 2\pi \cdot 3{,}00\ s^{-1} = \underline{18{,}8\ s^{-1}}$

b)



$$m\frac{d^2x}{dt^2} = -b\frac{dx}{dt} - kx$$

der k er fjærens kraft-konstant.

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

med løsning

$$x(t) = A e^{-\alpha t}\sin(\omega t + \delta)$$

der $\alpha = \frac{b}{2m}$ og sirkelfrekvensen for den dempede svingning $\omega = \sqrt{\omega_0^2 - \alpha^2}$

Frekvens for den dempede svingning

$$f = \frac{10}{3{,}60}\,Hz = 2{,}78\,Hz$$

Sirkel frekvens $\omega = 2\pi f = 2\pi \cdot 2{,}78\ s^{-1} = 17{,}5\ s^{-1}$

$$\alpha = \sqrt{\omega_0^2 - \omega^2} = \sqrt{18{,}8^2 - 17{,}5^2}\ s^{-1} = 6{,}87\ s^{-1}$$

②

$$b = 2m\alpha = 2\cdot\frac{4}{3}\pi r^3\cdot\alpha$$

$$b = \frac{2\cdot 4\pi\, 0{,}00500^3\cdot 6{,}87}{3}\ \mathrm{kg\,s^{-1}} = 19{,}28\cdot 10^{-3}\ \mathrm{kg\,s^{-1}}$$

Viskositetskoeffisient

$$\underline{\eta} = \frac{b}{6\pi r} = \frac{19{,}28\cdot 10^{-3}}{6\pi\cdot 0{,}00500}\ \mathrm{Pa\cdot s} = \underline{0{,}205\ \mathrm{Pa\cdot s}}$$

iii)

$$\omega^2 = \omega_0^2 - \alpha^2$$

Oppgitt $x(0) = A_0$

$$\left(\frac{dx}{dt}\right)_{t=0} = 0$$

$$\frac{dx}{dt} = \omega A e^{-\alpha t}\cos(\omega t + \delta) - \alpha A e^{-\alpha t}\sin(\omega t + \delta)$$

I $\left(\frac{dx}{dt}\right)_{t=0} = \omega A\cos\delta - \alpha A\sin\delta = 0$

II $x(0) = A\sin\delta = A_0$

Innsatt II i I: $\omega A\cos\delta - \alpha A_0 = 0$

$$\Rightarrow A\cos\delta = \frac{\alpha A_0}{\omega}$$

II $A\sin\delta = A_0$

Ved kvadrering og addering

$$A^2 = A_0^2\cdot\frac{\omega^2 + \alpha^2}{\omega^2} = A_0^2\cdot\frac{\omega_0^2 - \alpha^2 + \alpha^2}{\omega^2} = A_0^2\,\frac{\omega_0^2}{\omega^2}$$

$$\underline{A = A_0 \frac{\omega_0}{\omega}}$$

$$\tan\delta = \frac{A_0}{\frac{\alpha A_0}{\omega}} = \frac{\omega}{\alpha} = \frac{17{,}5}{6{,}87}$$

$$\underline{\delta = 68{,}6^\circ}$$

c) Ved kritisk demping er $\alpha = \omega_0$

$$b = 2m\alpha = 2m\omega_0 = 2 \cdot \frac{4}{3}\pi r^3 \varrho \omega_0$$

$$b = \frac{24\pi \cdot 0{,}00500^3 \cdot 2{,}68 \cdot 10^3 \cdot 18{,}8}{3}\ \mathrm{kg\,s^{-1}}$$

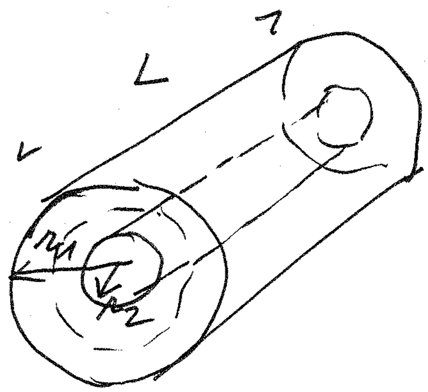
$$b = 52{,}8 \cdot 10^{-3}\ \mathrm{kg\,s^{-1}}$$

Viskositetskoeffisient

$$\underline{\eta} = \frac{b}{6\pi r} = \frac{52{,}8 \cdot 10^{-3}}{6\pi \cdot 0{,}00500}\ \mathrm{Pa \cdot s} = \underline{0{,}560\ \mathrm{Pa \cdot s}}$$

④

Oppgave 2



a) $$\left(\frac{dQ}{dt}\right)_{ledn} = -kA\,\frac{dT}{dr}$$

$$A = 2\pi r L$$

$$\frac{dQ}{dt} = -k\,2\pi r L\,\frac{dT}{dr}$$

kaller $j = \left(\frac{dQ}{dt}\right)_{ledn}$

$$j = -k\,2\pi r L\,\frac{dT}{dr}$$

$$\frac{dr}{r} = -\left(\frac{1}{j}\right)2\pi k L\,dT$$

$$\int_{r_2}^{r_1}\frac{dr}{r} = -\left(\frac{1}{j}\right)2\pi k L\int_{T_2}^{T_1} dT$$

$$\ln\frac{r_1}{r_2} = -\left(\frac{1}{j}\right)2\pi k L\,(T_1 - T_2)$$

$$\Rightarrow \quad \underline{j} = \left(\frac{dQ}{dt}\right)_{ledn} = \underline{2\pi k L\,\frac{T_2 - T_1}{\ln\left(\frac{r_1}{r_2}\right)}}$$

b) Varmestrøm vegg-luft

$$\left(\frac{dQ}{dt}\right)_{konv} = \alpha 2\pi r_1 L (T_1 - T_0)$$

der $A = 2\pi r_1 L$

Ved stasjonære forhold er varmestrømmen gjennom veggen lik varmestrømmen vegg-luft

$$\left(\frac{dQ}{dt}\right)_{ledn} = \left(\frac{dQ}{dt}\right)_{konv}$$

$$\Rightarrow \quad \frac{2\pi k (T_2 - T_1)}{\ln\left(\frac{r_1}{r_2}\right)} = \alpha 2\pi r_1 L (T_1 - T_0)$$

$$\Rightarrow \quad T_1 = \frac{T_2 + T_0 \cdot \frac{\alpha r_1}{k} \ln\left(\frac{r_1}{r_2}\right)}{1 + \frac{\alpha r_1}{k} \ln\left(\frac{r_1}{r_2}\right)}$$

$$T_1 = \frac{100 + 20 \cdot \frac{10 \cdot 0{,}020}{0{,}50} \ln \frac{0{,}020}{0{,}010}}{1 + \frac{10 \cdot 0{,}020}{0{,}50} \ln \frac{0{,}020}{0{,}010}} \ {}^\circ C$$

$$\underline{T_1 = 82{,}6\ {}^\circ C}$$

Varmestrømmen

$$\frac{dQ}{dt} = \alpha 2\pi r_1 L (T_1 - T_0) = 10 \cdot 2\pi \cdot 0{,}020 \cdot 1{,}0 \cdot (82{,}6 - 20)\ W$$

$$\underline{\frac{dQ}{dt} = 78{,}7\ W}$$

c) Da $\left(\frac{dQ}{dt}\right)_{ledn} = \left(\frac{dQ}{dt}\right)_{konv} = \dot{J}$

$$\Rightarrow \quad T_2 - T_1 = \frac{\dot{J}\left(\ln\left(\frac{r_1}{r_2}\right)\right)}{2\pi k L}$$

$$T_1 - T_0 = \frac{\dot{J}}{\alpha\, 2\pi r_1 L}$$

Ved addisjon

$$T_2 - T_0 = \frac{\dot{J} \ln\left(\frac{r_1}{r_2}\right)}{2\pi k L} + \frac{\dot{J}}{\alpha 2\pi r_1 L}$$

$$\Rightarrow \quad \dot{J} = \frac{(T_2 - T_0)\, 2\pi L}{\frac{\ln\left(\frac{r_1}{r_2}\right)}{k} + \frac{1}{\alpha r_1}}$$

radikand $R = \frac{\ln\left(\frac{r_1}{r_2}\right)}{k} + \frac{1}{\alpha r_1}$

$\dot{J}$ maks når $\frac{dR}{dr_1}$ min

$$\frac{dR}{dr_1} = \frac{1}{k}\frac{1}{r_1} - \frac{1}{\alpha r_1^2}$$

$$\frac{dR}{dr_1} = 0 \Rightarrow \underline{r_1} = \frac{k}{\alpha} = \frac{0{,}50}{10}\, m = \underline{0{,}050\, m}$$

$$\left[\ \frac{d^2R}{dr_1^2} = \frac{\alpha^2}{k^3} > 0 \text{ for } r_1 = \frac{k}{\alpha}\ \right]$$

Varmestrøm ved $r_i = 0{,}050\,m$

$$\dot{q} = \frac{(T_2 - T_0)\,2\pi L}{\frac{\ln\left(\frac{r_i}{r_{ii}}\right)}{k} + \frac{1}{\alpha r_i}}$$

$$\underline{\dot{q}} = \frac{80 \cdot 2\pi \cdot 1{,}0}{\frac{1{,}609}{0{,}50} + \frac{1}{10 \cdot 0{,}05}}\ W = \underline{96{,}3\ W}$$

$$T_i - T_0 = \dot{q}\,\frac{1}{\alpha 2\pi r_i L}$$

$$T_i = T_0 + \frac{\dot{q}}{\alpha 2\pi r_i L} = 20\ ^\circ C + \frac{96{,}3}{10 \cdot 2\pi \cdot 0{,}050 \cdot 1{,}0}\ ^\circ C$$

$$\underline{T_i} = \underline{50{,}7\ ^\circ C}$$

d) Når veggtykkelsen øker, øker også rørets ytre overflate.

Varmestrøm på grunn av ledning vil avta med økende veggtykkelse.

Varmestrøm på grunn av konveksjon vil øke med økende overflate.

Kombinasjonen av de to prosessene vil avgjøre om varmestrømmen øker eller minker når veggtykkelsen øker.

Oppgave 3.

(8)

a) $$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}-\frac{c}{x}\psi=E\psi \qquad x>0$$

b) $x>0$

$$\psi=Axe^{-\alpha x}$$

$$\frac{d\psi}{dx}=Ae^{-\alpha x}-\alpha Axe^{-\alpha x}$$

$$\frac{d^2\psi}{dx^2}=-\alpha Ae^{-\alpha x}-\alpha Ae^{-\alpha x}+\alpha^2Axe^{-\alpha x}$$

$$=\alpha A(\alpha x-2)e^{-\alpha x}$$

Innsatt i Schrödingerligningen

$$-\frac{\hbar^2}{2m}\alpha A(\alpha x-2)e^{-\alpha x}-\frac{c}{x}Axe^{-\alpha x}=EAe^{-\alpha x}$$

$$Ae^{-\alpha t}\left[-\left(\frac{\hbar^2\alpha^2}{2m}+E\right)x+\left(\frac{\hbar^2\alpha}{m}-c\right)\right]=0$$

Ligningen skal gjelde for alle x

$$\Rightarrow \quad \frac{\hbar^2\alpha^2}{2m}-c=0 \quad \Rightarrow \quad \underline{\alpha}=\underline{\frac{mc}{\hbar^2}}$$

$$\frac{\hbar^2\alpha^2}{2m}+E=0 \quad \Rightarrow \quad \underline{E}=-\underline{\frac{\hbar^2\alpha^2}{2m}}$$

eller innsatt α $\quad \underline{E}=-\underline{\frac{mc^2}{2\hbar^2}}$

$$c = \frac{e^2}{4\pi\varepsilon_0} = \frac{(1{,}60\cdot 10^{-19})^2}{4\pi\cdot 8{,}85\cdot 10^{-12}}\ Nm^2 = 2{,}30\cdot 10^{-28}\ Nm^2$$

$$\alpha = \frac{mc}{\hbar^2} = \frac{9{,}11\cdot 10^{-31}\cdot 2{,}30\cdot 10^{-28}}{(1{,}05\cdot 10^{-34})^2}\ m^{-1}$$

$$\underline{\alpha} = \underline{1{,}90\cdot 10^{10}\ m^{-1}}$$

$$E = -\frac{mc^2}{2\hbar^2} = -\frac{9{,}11\cdot 10^{-31}(2{,}30\cdot 10^{-28})^2}{2\cdot(1{,}05\cdot 10^{-34})^2}\ J$$

$$\underline{E} = \underline{-2{,}19\cdot 10^{-18} J} \qquad (-13{,}6\,eV)$$

c) Normering

$$\int_0^\infty \psi^* \psi \, dx = 1$$

$$\psi^*\psi = Axe^{-\alpha x}\cdot Axe^{-\alpha x} = A^2x^2e^{-2\alpha x}$$

$$\Rightarrow \quad A^2\int_0^\infty x^2 e^{-2\alpha x}dx = 1$$

$$A^2\cdot\frac{1\cdot 2}{(2\alpha)^2} = 1$$

$$\underline{A} = \underline{2\alpha^{\frac{3}{2}}} = 2(1{,}90\cdot 10^{10})^{\frac{3}{2}}\ m^{-\frac{3}{2}}$$

$$= 5{,}24\cdot 10^{15}\ m^{-3/2}$$

d) Sannsynlighetstetthet

$$\underline{\psi^*\psi} = \underline{A^2 x^2 e^{-2\alpha x}}$$

e) Den mest sannsynlige posisjon når $\psi^*\psi$ er maks.

$$\frac{d(\psi^*\psi)}{dx} = 2A^2 x e^{-2\alpha x} - 2\alpha A^2 x^2 e^{-2\alpha x}$$

$$= 2A^2(x - \alpha x^2) e^{-2\alpha x}$$

$$\frac{d(\psi^*\psi)}{dx} = 0 \quad \text{når} \quad x - \alpha x^2 = 0$$

Maks når $x = \underline{x_0} = \underline{\frac{1}{\alpha}}$

$$x_0 = \frac{1}{1{,}90 \cdot 10^{10}}\,m = 5{,}26 \cdot 10^{-11}\,m$$

Midlere posisjon, forventningsverdien

$$\langle x \rangle = \int_0^\infty \psi^* x \psi\, dx = \int_0^\infty x \psi^*\psi\, dx = A^2 \int_0^\infty x^3 e^{-2\alpha x}\, dx$$

$$= A^2 \cdot \frac{1 \cdot 2 \cdot 3}{(2\alpha)^4} = 4\alpha^3 \cdot \frac{6}{16\alpha^4}$$

$$\underline{\langle x \rangle} = \underline{\frac{3}{2\alpha}}$$

$$\langle x \rangle = \frac{3}{2 \cdot 1{,}90 \cdot 10^{10}}\,m = 7{,}89 \cdot 10^{-11}\,m$$

Ved vendepunktet x_0 er den totale energi potensiell

$$E = U(x_0)$$

$$-\frac{\hbar^2\alpha^2}{2m} = -\frac{c}{x_0}$$

$$\Rightarrow \quad x_0 = \frac{2mc}{\hbar^2\alpha^2} = \frac{2m\cdot\alpha\hbar^2}{\hbar^2\alpha^2}$$

$$\underline{x_0} = \underline{\frac{2}{\alpha}}$$

$$x_0 = \frac{2}{1{,}90\cdot 10^{15}}\,m = 10{,}5\cdot 10^{16}\,m$$

f)