Oppgave 1

①

a) Et enatomig gassmolekyl har 3 frihetsgrader hver med energi $\frac{1}{2}kT$

$\Rightarrow$ translatorisk kinetisk energi $\frac{3}{2}kT$

Indre energi for N molekyler

$$U = N \cdot \frac{3}{2}kT = n N_A \cdot \frac{3}{2}kT$$

n antall mol molekyler, N_A Avogadros konstant

$$U = \frac{3}{2}nRT$$

idet $R = N_A k$

Varmekapasitet ved konstant volum prosess

$$\underline{C_V} = \frac{dU}{dT} = \underline{\frac{3}{2}nR}$$

b) I alle tre tilfeller er forandring i indre energi

$$\Delta U = C_V \Delta T = \frac{3}{2}nR(T_2 - T_1)$$

$$\underline{\Delta U} = \frac{3}{2} \cdot 6{,}0 \cdot 8{,}3 \cdot (400-200)\,J = \underline{15000\,J = 15\,kJ}$$

i Ved konstant volum er $\underline{W=0}$

$$\Rightarrow \quad \underline{Q} = \Delta U = \underline{15\,kJ} \quad \text{etter 1. lov}$$

ii Ved konstant trykk

$$W = p(V_2 - V_1) = nRT_2 - nRT_1 = nR(T_2 - T_1)$$

$$\underline{W} = 6{,}0 \cdot 8{,}3 \cdot (400-200)\,J = 10000\,J = \underline{10\,kJ}$$

Tilført varme ②

$\underline{Q} = \Delta U + W = 15\,kJ + 10\,kJ = \underline{25\,kJ}$

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Ved adiabatisk prosess er $Q = 0$

$\underline{W} = -\Delta U = \underline{-15\,kJ}$ etter 1. lov

c) Prosess $p = aV$

Utført arbeid

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} aV\,dV = \frac{1}{2}a(V_2^2 - V_1^2)$$

Ideell gass lign. $pV = nRT$

$$aV \cdot V = nRT \Rightarrow aV^2 = nRT$$

$$\Rightarrow \underline{W} = \underline{\frac{1}{2}nR(T_2 - T_1)}$$

Varmekapasitet

$$C = \frac{Q}{\Delta T} = \frac{\Delta U + W}{\Delta T} = C_V + \frac{W}{\Delta T}$$

$$C = \frac{3}{2}nR + \frac{\frac{1}{2}nR\Delta T}{\Delta T} = \frac{3}{2}nR + \frac{1}{2}nR$$

$$\underline{C} = 2nR = 2 \cdot 6{,}0 \cdot 8{,}3\,J/K = \underline{100\,J/K}$$

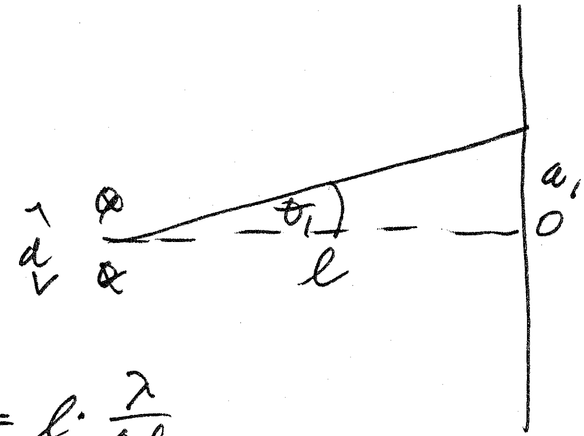
d) Ved fri ekspansjon er $Q = 0$ og $W = 0$

$\underline{\Delta U} = U(2V_1, T_1) - U(V_1, T_1) = Q - W = \underline{0}$

Temperaturen etter ekspansjonen den samme som før ekspansjonen.

Oppgave 2

b)



$$\tan\theta_1 \simeq \sin\theta_1 = \frac{a_1}{l}$$

$$\Rightarrow a_1 = l\sin\theta_1 = l\cdot\frac{\lambda}{2d}$$

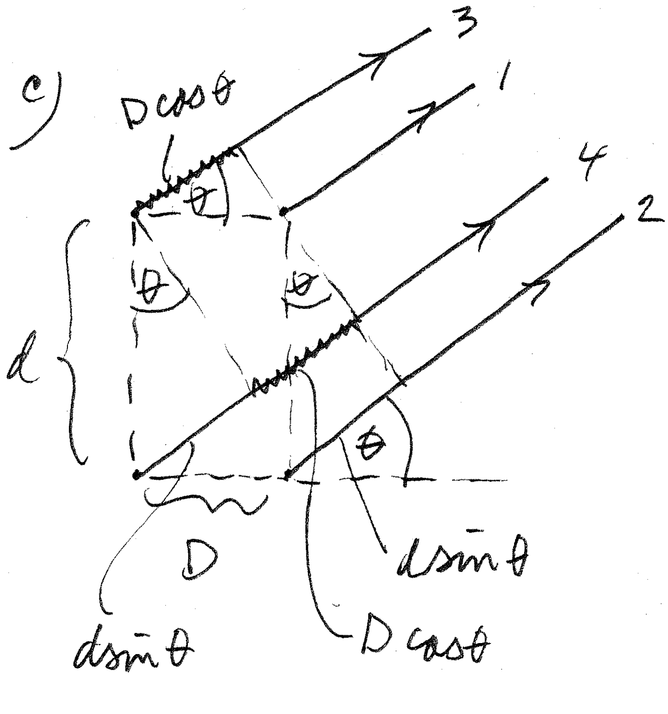
idet null intensitet $d\sin\theta = (n+\frac{1}{2})\lambda$

$n=0$ for nærmeste mørke stripe

$$\lambda = \frac{c}{f}$$

$$\Rightarrow \underline{a_1} = \frac{lc}{2df} = \frac{1{,}0\cdot 3{,}0\cdot 10^{8}}{2\cdot 5{,}0\cdot 10^{-6}\cdot 5{,}0\cdot 10^{14}}\ \text{m} = \underline{0{,}060\ \text{m}}$$

c)



For strålene 1, 2, 3 og 4

	gangforskjell	faseforskjell
Velger	$g_1 = 0$	0
	$g_2 = d\sin\theta$	$\delta_2 = \frac{2\pi}{\lambda} d\sin\theta$
	$g_3 = D\cos\theta$	$\delta_3 = \frac{2\pi}{\lambda} D\cos\theta$
	$g_4 = d\sin\theta + D\cos\theta$	$\delta_4 = \frac{2\pi}{\lambda}(d\sin\theta + D\cos\theta)$
		$= \delta_2 + \delta_3$

Resultant

$$E_{res} = E_0 \sin\omega t + E_0 \sin(\omega t + \delta_2) + E_0 \sin(\omega t + \delta_3) + $$
$$+ E_0 \sin(\omega t + \delta_4)$$

$$E_1 = E_0 \sin\omega t + E_0 \sin(\omega t + \delta_2)$$

$$\sin A + \sin B = 2\cos\frac{A-B}{2} \sin\frac{A+B}{2}$$

$$\Rightarrow E_1 = 2E_0 \cos\frac{\delta_2}{2} \sin\left(\omega t + \frac{\delta_2}{2}\right)$$

$$E_2 = E_0 \sin(\omega t + \delta_3) + E_0 \sin(\omega t + \delta_4)$$

$$\sin A + \sin B = 2\cos\frac{A-B}{2} \sin\frac{A+B}{2}$$

$$\Rightarrow E_2 = 2E_0 \cos\frac{\delta_3 - \delta_4}{2} \sin\left(\omega t + \frac{\delta_3 + \delta_4}{2}\right)$$

Nå er $\delta_4 = \delta_2 + \delta_3$

$$\Rightarrow E_2 = 2E_0 \cos\frac{\delta_2}{2} \sin\left(\omega t + \frac{2\delta_3 + \delta_2}{2}\right)$$

$$\Rightarrow E_{res} = E_1 + E_2$$

$$E_{res} = 2E_0 \cos\frac{\delta_2}{2}\left[\sin\left(\omega t + \frac{\delta_2}{2}\right) + \sin\left(\omega t + \frac{2\delta_3 + \delta_2}{2}\right)\right]$$

$$\sin A + \sin B = 2\cos\frac{A-B}{2}\sin\frac{A+B}{2}$$

$$E_{res} = 4E_0 \cos\frac{\delta_2}{2}\cos\frac{\delta_3}{2}\sin\left(\omega t + \frac{\delta_2 + \delta_3}{2}\right)$$

Amplitude

$$E_\theta = 4E_0 \cos\frac{\delta_2}{2}\cos\frac{\delta_3}{2}$$

$$E_\theta^2 = 16E_0 \cos^2\frac{\delta_2}{2}\cos^2\frac{\delta_3}{2}$$

Intensitet

$$\underline{Y = 16 Y_1 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)\cos^2\left(\frac{\pi D \cos\theta}{\lambda}\right)}$$

da $\delta_2 = \frac{2\pi}{\lambda} d \sin\theta$ og $\delta_3 = \frac{2\pi}{\lambda} D \cos\theta$

d) Mørk stripe for $\theta = 0$

$$\cos^2\left(\frac{\pi D}{\lambda}\cos 0\right) = 0$$

$$\frac{\pi D}{\lambda} = (n+1)\frac{\pi}{2} \qquad n = 0, 1, 2, \ldots$$

$$\Rightarrow \underline{D = (n+1)\frac{\lambda}{2}}$$

Oppgave 3.

a) i $$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+U_0\psi=E\psi$$

ii Løsning

$$\psi=A\sin kx+B\cos kx$$

der $k=\sqrt{\frac{2m(E-U_0)}{\hbar^2}}$

Når $x=0$ er $\psi=0$

$$A\sin k\cdot 0+B\cos k\cdot 0=0$$

$$\Rightarrow B=0$$

Når $x=L$ er $\psi=0$

$$A\sin kL=0$$

$$\Rightarrow kL=n\pi \qquad n=1,2,\ldots$$

$$k=\frac{n\pi}{L}$$

Løsning $\psi=A\sin\frac{n\pi x}{L}$

Innsatt i Schrødingerlign.

$$\frac{d\psi}{dx}=A\cdot\frac{n\pi}{L}\cos\frac{n\pi x}{L}$$

$$\frac{d^2\psi}{dx^2}=-A\left(\frac{n\pi}{L}\right)^2\sin\frac{n\pi x}{L}$$

ii Bølgefunksjonen $\psi = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$

Sannsynlighetstettheten $\psi^*\psi = |\psi|^2 = \frac{2}{L} \sin^2 \frac{n\pi x}{L}$

Ved den mest sannsynlige posisjon, må sannsynlighetstettheten være maks.

$$\frac{d}{dx}(\psi^*\psi) = \frac{2}{L}\left(\frac{n\pi}{L}\right) 2 \sin\frac{n\pi x}{L} \cos\frac{n\pi x}{L}$$

$$= \frac{2}{L}\left(\frac{n\pi}{L}\right) \sin\frac{2n\pi x}{L}$$

$$\frac{d^2}{dx^2}(\psi^*\psi) = \frac{4}{L}\left(\frac{n\pi}{L}\right)^2 \cos\frac{2n\pi x}{L}$$

$$\frac{d}{dx}(\psi^*\psi) = 0 \text{ når } \frac{2n\pi x}{L} = m\pi \quad m = 0, 1, 2, \ldots$$

$$\Rightarrow x = \frac{L}{2} \cdot \frac{m}{n}$$

$n = 1$ $\quad x = \frac{L}{2} m$

$m = 0$, 2. deriv pos.: min

$m = 1$, 2 — " — neg: max

$m = 2$, 2 — " — pos: min

altså $\underline{x = \frac{L}{2}}$

$n = 2$ $\quad x = \frac{L}{2} \frac{m}{2} = \frac{L}{4} m$

$m = 0$, 2 deriv. pos. min

$m = 1$, 2 — " — neg. max

$m = 2$, 2 — " — pos: min

$m = 3$, 2 — " — neg: max

$m = 4$, 2 — " — pos: min

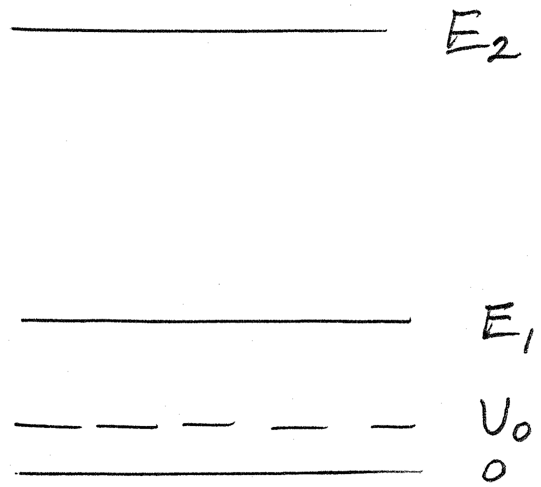
altså

$\underline{x = \frac{L}{4}}$ og $\underline{x = \frac{3L}{4}}$

$$-\frac{\hbar^2}{2m}(-A)\left(\frac{n\pi}{L}\right)^2 \sin\frac{n\pi x}{L} + V_0 A \sin\frac{n\pi x}{L} = E_n A \sin\frac{n\pi x}{L}$$

$$\Rightarrow \quad \underline{E_n} = \underline{\frac{\hbar^2\pi^2}{2mL^2} n^2 + V_0}$$

<u>iii</u>



<u>iv</u> Normering

$$\int_0^\infty |\psi|^2 dx = 1$$

$$A^2 \int_0^L \sin^2\frac{n\pi x}{L} dx = 1$$

$$A^2 \cdot \frac{L}{2} = 1 \quad \Rightarrow A = \sqrt{\frac{2}{L}}$$

$$\underline{\psi(x)} = \underline{\sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L}}$$

b) <u>i</u> Forventningsverdien

$$\langle x \rangle = \int_0^L \psi^* x \psi \, dx = \frac{2}{L}\int_0^L x \sin^2\frac{n\pi x}{L} dx$$

$$\underline{\langle x \rangle} = \frac{2}{L}\cdot\frac{L^2}{4} = \underline{\frac{L}{2}}$$

c) i $E_n = \frac{\hbar^2\pi^2}{2mL^2} n^2 + U_0$

$$E_1 = \frac{\hbar^2\pi^2}{2mL^2} + U_0$$

$$E_2 = \frac{\hbar^2\pi^2}{2mL^2} \cdot 4 + U_0$$

Strålingens energi

$$\Delta E = E_2 - E_1 = 3 \cdot \frac{\hbar^2\pi^2}{2mL^2}$$

$$= 3 \cdot \frac{(1{,}055 \cdot 10^{-34})^2 \cdot \pi^2}{2 \cdot 9{,}109 \cdot 10^{-31} \cdot (0{,}300 \cdot 10^{-9})^2} \text{ J}$$

$$\underline{\Delta E} = 3 \cdot 6{,}70 \cdot 10^{-19} \text{ J} = \underline{2{,}01 \cdot 10^{-18} \text{ J}} = \underline{12{,}5 \text{ eV}}$$

ii $hf = \Delta E$

Strålingens frekvens

$$\underline{f} = \frac{\Delta E}{h} = \frac{2{,}01 \cdot 10^{-18}}{6{,}625 \cdot 10^{-34}} \text{ Hz} = \underline{3{,}03 \cdot 10^{15} \text{ Hz}}$$

Strålingens bølgelengde

$$\underline{\lambda} = \frac{c}{f} = \frac{3{,}00 \cdot 10^{8}}{3{,}03 \cdot 10^{15}} \text{ m} = \underline{1{,}00 \cdot 10^{-7} \text{ m}}$$