Oppg. 1

(a) Prøveløysing $x = A\cos(\omega t+\phi)$ gir $\ddot{x} = -\omega^2 x$. Innsetjing i differensiallikninga med $b=0$ og $F(t)\equiv 0$ gir då:

$$-m\omega^2 x + 0 + kx = 0 \qquad \omega^2 = \frac{k}{m} \equiv \omega_0^2$$

$$T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k}}$$

$$k = m\omega_0^2 = \underline{\underline{m(2\pi/T_0)^2}} = 5{,}00\,(2\pi/10{,}0)^2 = \underline{\underline{1{,}97\ \mathrm{N/m}}}$$

(b) Prøveløysing $x = Ce^{rt}$ gir $\dot{x} = rx$ og $\ddot{x} = r^2 x$, som innsett i differensiallikninga gir (etter divisjon med x):

$$mr^2 + br + k = 0$$

$$r^2 + 2\delta r + \omega_0^2 = 0 \qquad \delta = \frac{b}{2m} \qquad \omega_0^2 = \frac{k}{m}$$

$$r = -\delta \pm \sqrt{\delta^2-\omega_0^2} = -\delta \pm i\omega_d$$

$$\omega_d = \sqrt{\omega_0^2-\delta^2}$$

Løysinga kan skrivast

$$x = C_1 e^{r_1 t} + C_2 e^{r_2 t} = Ae^{-\delta t}\cos(\omega_d t+\phi)$$

der C_1 og C_2 (eventuelt A og ϕ) er to vilkårlege integrasjonskonstantar. [Med bruk av Eulers formlar kan ein eventuelt finna samanhengen mellom dei to setta av integrasjonskonstantar]

Oppgåveteksten må tolkast slik at vi har underdempa tilfelle, d.v.s. ω_d er reell, $(\delta < \omega_0)$

$$\delta^2 = \omega_0^2 - \omega_d^2 = (2\pi)^2\left(\frac{1}{T_0^2} - \frac{1}{T_d^2}\right)$$

$$\underline{\underline{b = 2m\delta = 4\pi m\left(\frac{1}{T_0^2} - \frac{1}{T_d^2}\right)^{1/2} = \frac{4\pi m}{T_0 T_d}(T_d+T_0)^{1/2}(T_d-T_d)^{1/2}}}$$

$$b = 4\pi\cdot 5{,}00\,(10{,}0^{-2} - 10{,}5^{-2})^{1/2} = 1{,}92\ \mathrm{kg/s} = 1{,}9\ \mathrm{Ns/m}$$

1 (c) A_0/F_0 er størst når $\omega|Z_m|$ er minst, dvs. når $\omega^2|Z_m|^2$ er minst. Set den deriverte av $\omega^2|Z|_m^2$ lik null:

$$0 = \frac{d}{d\omega^2}\left(b^2\omega^2 + m^2\omega^4 - 2k\cdot m\,\omega^2 + k^2\right) =$$

$$= b^2 + 2m^2\omega^2 - 2mk - 0$$

$$\omega^2 = \frac{2mk - b^2}{2m^2} = \frac{k}{m} - 2\left(\frac{b}{2m}\right)^2 = \omega_0^2 - 2\delta^2 = \omega_d^2 - \delta^2 =$$

$$= \omega_d^2 - (\omega_0^2 - \omega_d^2) = 2\omega_d^2 - \omega_0^2 = 4\pi^2\left(\frac{2}{T_d^2} - \frac{1}{T_0^2}\right)$$

Størst utsvingsamplitude for

$$\omega = \omega_1 = \underline{\underline{2\pi\left(\frac{2}{T_d^2} - \frac{1}{T_0^2}\right)^{1/2}}} = 2\pi\left(\frac{2}{10{,}5^2} - \frac{1}{10{,}0^2}\right) = \underline{\underline{0{,}567 \text{ rad/s}}}$$

Størst fart-amplitude når $|Z_m|$ er minst. Vi ser uten videre at $|Z_m|$ er minst når $(\omega m - k/\omega)^2 = 0$ dvs. når $\omega = \omega_2 = \sqrt{\frac{k}{m}} = \omega_0 = \frac{2\pi}{T_0} = \underline{\underline{0{,}628 \text{ rad/s}}}$

(d) $$v(t) = \dot{x} = \omega A_0 \cos(\omega t + \phi_0) = \frac{F_0}{|Z_m|}\cos(\omega t + \phi_0)$$

$$a(t) = \dot{v} = -\omega^2 \dot{x} = -\omega^2 A_0 \sin(\omega t + \phi_0) = -\frac{\omega F_0}{|Z_m|}\sin(\omega t + \phi_0)$$

$\omega = 0{,}800$ rad/s

$$|Z_m| = \sqrt{1{,}92^2 + (0{,}800\cdot 5{,}00 - 1{,}97/0{,}800)^2} = 2{,}46 \text{ N}\cdot\text{s/m}$$

$$x_{max} = A_0 = \frac{F_0}{\omega|Z_m|} = \frac{1{,}00}{0{,}800\cdot 2{,}46} = \underline{\underline{0{,}508 \text{ m}}}$$

$$v_{max} = \frac{F_0}{|Z_m|} = \frac{1{,}00}{2{,}46} = \underline{\underline{0{,}407 \text{ m/s}}}$$

$$a_{max} = \frac{\omega F_0}{|Z_m|} = \underline{\underline{0{,}325 \text{ m/s}^2}}$$

Oppg. 2

(a) $\sin\theta_m = \frac{m\lambda}{d} = \frac{2\lambda}{d}$ (i 2. orden, $m = 2$)

$\lambda_a = 589,0\,\text{nm}$ $\sin\theta_a = \frac{2\cdot 589\cdot 10^{-9}\,\text{m}}{2,500\cdot 10^{-6}\,\text{m}} = 0,4712$ $\underline{\theta_a = 0,4907\text{ rad}}$

$\lambda_b = 589,6\,\text{nm}$ $\sin\theta_b = \frac{2\cdot 589,6\cdot 10^{-9}\,\text{m}}{2,500\cdot 10^{-6}\,\text{m}} = 0,4717$ $\underline{\theta_b = 0,4912\text{ rad}}$

(b) $1 \geq |\sin\theta_m| = \frac{|m|\lambda_a}{d} = |m|\frac{589,0\cdot 10^{-9}\,\text{m}}{2,500\cdot 10^{-6}\,\text{m}} = |m|\cdot 0,2356$

$|m| < \frac{1}{0,2356} = 4,244$ $\Rightarrow$: $\underline{|m| \leq 4}$

$m = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

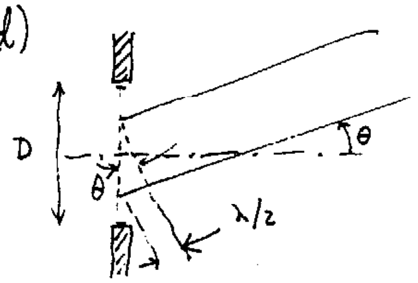
Berre 9 hovudmaksimum går an

(c) $R = \frac{\lambda}{\Delta\lambda} = Nm$ $\Delta\lambda$ minste bølgjelengdeskilnad som kan oppløysast

$$N = \frac{\lambda/\Delta\lambda}{m} \geq \frac{589,3/0,6}{2} = 491 \approx 0,5\cdot 10^3$$

Gitteret må ha minst $\underline{500}$ spalter

(d)



Fyrste nullpunkt i diffraksjonsmønsteret for ein vinkel θ, der lyset frå dei to tenkte halvopartane av spaltebreidda D har ein innbyrdes gangskilnad på $\lambda/2$

$\frac{D}{2}\sin\theta = \frac{\lambda}{2}$ $\sin\theta = \frac{\lambda}{D}$

Krav $\sin\theta > \sin\theta_b$ $\frac{\lambda}{D} > \frac{2\lambda}{d}$ $\underline{D < \frac{d}{2}}$

Oppg. 3

(a) $\psi_a(x) = A e^{-\alpha^2 x^2/2}$

$$\psi_a'(x) = -\alpha^2 x\, \psi_a(x)$$

$$\psi_a''(x) = -\alpha^2 \psi_a(x) + \alpha^4 x^2 \psi_a(x)$$

Innsett i Schr.-likninga og divisjon med $\psi_a(x)$:

$$-\frac{\hbar^2}{2m}\left(-\alpha^2 + \alpha^4 x^2\right) + \frac{1}{2} C x^2 = E$$

$$E = \alpha^2 \frac{\hbar^2}{2m} \qquad -\frac{\hbar^2}{2m}\alpha^4 x^2 + \frac{1}{2} C x^2 = 0$$

$$\alpha^4 = \frac{m}{\hbar^2} C = \frac{m^2}{\hbar^2}\frac{C}{m} = \frac{m^2}{\hbar^2}\omega^2$$

$$\boxed{\omega = \sqrt{C/m}} \qquad E = \frac{\hbar^2}{2m}\alpha^2 = \frac{\hbar^2}{2m}\frac{m}{\hbar}\omega = \frac{\hbar\omega}{2} \equiv E_a$$

$$\underline{\underline{\alpha = \sqrt{m\omega/\hbar}}} \qquad \underline{E_a = E_0} \qquad \nu : \underline{\nu = \nu_a = 0}$$

$$\psi_b(x) = B x e^{-\alpha^2 x^2/2} = x y \qquad y = B e^{-\alpha^2 x^2/2} \qquad y' = -\alpha^2 x y$$

$$\psi_b'(x) = y + x y' = y(1 - \alpha^2 x^2)$$

$$\psi_b''(x) = y(-2\alpha^2 x) + y'(1 - \alpha^2 x^2) = y(-3\alpha^2 x + \alpha^4 x^3)$$

Innsetjing i Schr.-likn og divisjon med y:

$$-\frac{\hbar^2}{2m}\left(-3\alpha^2 x + \alpha^4 x^3\right) + \frac{1}{2} C x^2 x = E x$$

$$\underbrace{\left(\frac{3}{2}\frac{\hbar^2}{m}\alpha^2 - E\right)}\, x + \underbrace{\left(-\frac{\hbar^2}{2m}\alpha^4 + \frac{1}{2} C\right)}_{\alpha^4 = \frac{mC}{\hbar^2}\ \text{(som ovanfor)}} x^3 = 0$$

$$E = \frac{3}{2}\frac{\hbar^2}{m}\alpha^2 = \frac{3}{2}\frac{\hbar^2}{m}\frac{m}{\hbar}\omega = \frac{3}{2}\hbar\omega \equiv E_b$$

$$E_b = E_1 \qquad \nu = \nu_b = 1$$

3 (b) Normalisering: $1 = \int_{-\infty}^{\infty} |\psi_a(x)|^2 dx = |A|^2 \int_{-\infty}^{\infty} (e^{-\alpha^2 x^2/2})^2 dx =$

$$= |A|^2 \int_{-\infty}^{\infty} e^{-\alpha^2 x^2} dx = |A|^2 \frac{1}{\alpha} \int_{-\infty}^{\infty} e^{-u^2} du = |A|^2 \frac{1}{\alpha} \sqrt{\pi}$$

$$\underline{|A| = \sqrt{\frac{\alpha}{\sqrt{\pi}}} = \left(\frac{m\omega}{\pi \hbar}\right)^{1/4}} = \left(\frac{2m\omega}{h}\right)^{1/4} \qquad \boxed{|e^{-iEt/\hbar}| = 1}$$

(c) $\langle x \rangle = \int_{-\infty}^{\infty} \Psi_a^*(x,t)\, x\, \Psi_a(x,t)\, dx = \int_{-\infty}^{\infty} x |\Psi(x,t)|^2 dx = \int_{-\infty}^{\infty} x\, |\psi_a(x)|^2 dx =$

$$= \frac{|A|^2}{\alpha^2} \int_{-\infty}^{\infty} u e^{-u^2} du = \underline{0} \quad \text{p.g.a. symmetrien}$$

(integranden er ein odde funksjon).

$$(\Delta x)^2 = \langle (x - \langle x \rangle)^2 \rangle = \langle (x-0)^2 \rangle = \langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 |\psi_a(x)|^2 dx =$$

$$= \frac{|A|^2}{\alpha^3} \int_{-\infty}^{\infty} u^2 e^{-u^2} du = \frac{2|A|^2}{\alpha^3} \int_0^{\infty} u^2 e^{-u^2} du = \frac{2|A|^2}{\alpha^3} \frac{\sqrt{\pi}}{4} =$$

$$= \left(\frac{m\omega}{\pi\hbar}\right)^{1/2} \left(\frac{\hbar}{m\omega}\right)^{3/2} \frac{\sqrt{\pi}}{2} = \frac{\hbar}{2m\omega}$$

$$\underline{\Delta x = \left(\frac{\hbar}{2m\omega}\right)^{1/2}}$$

(d) $p_{op} = -i\hbar\, \partial/\partial x \qquad p_{op}^2 = -\hbar^2 \frac{\partial^2}{\partial x^2}$

$$\langle p^2 \rangle = \int_{-\infty}^{\infty} \Psi_a^*(x,t) (-i\hbar)^2 \frac{\partial^2}{\partial x^2} \Psi_a(x,t)\, dx = -\hbar^2 \int_{-\infty}^{\infty} \psi_a^*(x) \frac{d^2}{dx^2} \psi_a(x)\, dx$$

då $|\exp\{-iE_a t/\hbar\}| = 1$.

$$\frac{d^2}{dx^2} \psi_a(x) = (-\alpha^2 + \alpha^4 x^2)\, \psi_a(x)$$

$$\langle p^2 \rangle = -\hbar^2 \int_{-\infty}^{\infty} (-\alpha^2 + \alpha^4 x^2) |\psi_a(x)|^2 dx = -2\hbar^2 \int_0^{\infty} (-\alpha^2 + \alpha^4 x^2) |A|^2 \cdot$$

$$\cdot e^{-\alpha^2 x^2} dx = -2\hbar^2 |A|^2 \alpha \int_0^{\infty} (-1 + u^2)\, e^{-u^2} du =$$

$$= -2\hbar^2 |A|^2 \alpha \left(-\tfrac{1}{2}\sqrt{\pi} + \tfrac{1}{4}\sqrt{\pi}\right) = \hbar^2 |A|^2 \alpha \sqrt{\pi}/2 = \hbar^2 \left(\frac{m\omega}{\pi\hbar}\right)^{1/2} \left(\frac{m\omega}{\hbar}\right)^{1/2} \frac{\sqrt{\pi}}{2} =$$

$$= \hbar^2 \frac{\alpha^2}{2}$$

$$= \underline{\tfrac{1}{2} m\hbar\omega}$$

e)

Forventningsverdien av kinetisk energi

$$\underline{\langle E_k \rangle} = \frac{\langle p^2 \rangle}{2m} = \frac{1}{2} h\nu m / 2m = \underline{\frac{1}{4} h\nu = \frac{1}{4}\hbar\omega}$$

Forventningsverdien av potensiell energi

$$\langle U_p \rangle = \frac{1}{2} K \langle x^2 \rangle = \frac{1}{2} K \cdot \frac{1}{2} \frac{\hbar}{m\omega}$$

$$K = m\omega^2$$

$$\underline{\langle U_p \rangle} = \underline{\frac{1}{4}\hbar\omega}$$

Forventningsverdien av total energi

$$\langle E \rangle = \langle E_k \rangle + \langle U_p \rangle = \frac{1}{4}\hbar\omega + \frac{1}{4}\hbar\omega$$

$$\underline{\langle E \rangle} = \underline{\frac{1}{2}\hbar\omega}$$

2(f) Det er gitt at $\Psi(x,t)$ er normalisert. Altså er $\Psi^*(x,t)\Psi(x,t)$ sannsynstettleiken. Då blir forventingsverdien for partikkelposisjonen:

$$\boxed{\begin{array}{c} n, m = 1, 2 \\ c_n = c_m = c_1 = c_2 = 1/\sqrt{2} \end{array}}$$

$$\langle x \rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)\, x\, \Psi(x,t)\, dx =$$

$$= \int_{-\infty}^{\infty} \sum_m c_m^* \psi_m^*(x) e^{iE_m t/\hbar}\, x \sum_n c_n \psi_n(x) e^{-iE_n t/\hbar}\, dx =$$

$$= \sum_m \sum_n c_m^* c_n e^{-i(E_n - E_m)t/\hbar} \int_{-\infty}^{\infty} \psi_m^*(x)\, x\, \psi_n(x)\, dx$$

$\psi_0(x) = A_0 \exp\{-x^2 m\omega/\hbar\}$ er jamn funksjon og $x\psi_0(x)$ er odde

$\psi_1(x) = 2A_1 \{m\omega/\hbar\}^{1/2} x \exp\{-x^2 m\omega/\hbar\}$ er odde og $x\psi_1(x)$ er jamn.

$$\left.\begin{array}{l} \int_{-\infty}^{\infty} \psi_0^*(x)\, x\, \psi_0(x)\, dx = 0 \\ \int_{-\infty}^{\infty} \psi_1^*(x)\, x\, \psi_1(x)\, dx = 0 \end{array}\right\} \text{ då integrandane er odde funksjonar}$$

$$\int_{-\infty}^{\infty} \psi_0^*(x)\, x\, \psi_1(x)\, dx = \left(\int_{-\infty}^{\infty} \psi_1^*(x)\, x\, \psi_0(x)\, dx\right)^* = 2\int_0^{\infty} \psi_0^*(x)\, x\, \psi_1(x)\, dx =$$

$$= 2A_0^* A_1 \int_0^{\infty} e^{-x^2 m\omega/2\hbar}\, x \left\{\frac{m\omega}{\hbar}\right\}^{1/2} x\, 2\, e^{-x^2 m\omega/2\hbar}\, dx$$

$$= 2 A_0^* A_1 \left(\frac{\hbar}{m\omega}\right) \int_0^{\infty} u^2 e^{-u^2}\, du = 2 A_0^* A_1 \frac{\hbar}{m\omega} \frac{\sqrt{\pi}}{4} = \frac{1}{2} A_0^* A_1 \frac{\hbar\sqrt{\pi}}{m\omega}$$

Av dei fire ledda i dobbeltsummen $\sum_{m=0}^{1} \sum_{n=0}^{1} \cdots$ er altså to null, og dei to resterande er kompleks konjugerte til kvarandre.

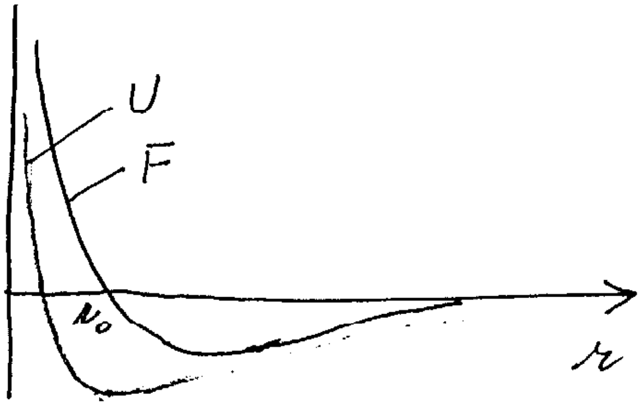
$$\langle x \rangle = c_0^* c_1 e^{-i(E_1 - E_0)t/\hbar} \frac{1}{2} A_0^* A_1 \frac{\hbar\sqrt{\pi}}{m\omega} + (\text{tilsv. kompl. konj.}) =$$

$$= |c_0| \cdot |c_1| e^{-i[(E_1 - E_0)t/\hbar + \gamma_0 - \gamma_1]} \frac{1}{2} A_0 A_1 \frac{\hbar\sqrt{\pi}}{m\omega} + (\text{kompl. konj.}) =$$

$$= \underline{2|c_0| \cdot |c_1| A_0 A_1 \frac{\hbar\sqrt{\pi}}{m\omega} \cos(\omega t + \gamma_0 - \gamma_1)} \qquad \gamma_0 = \gamma_1 = 0$$

Merk at $E_1 - E_0 = (1 + \frac{1}{2})\hbar\omega - \frac{1}{2}\hbar\omega = \hbar\omega$ der ω er den naturlege vinkelfrekvensen til oscillatoren.

3 (g)



F_{min} nå. $\frac{dF}{dr} = \frac{2a}{r^3} - \frac{3b}{r^4} = 0 \quad r_m = \frac{3b}{2a}$

$F_{min} = F(r_m) = -\frac{a}{\left(\frac{3b}{2a}\right)^2} + \frac{b}{\left(\frac{3b}{2a}\right)^3} = -\frac{4}{27}\frac{a^3}{b^2}$

$= 2{,}84 \cdot 10^{-12}$ N

h)

$$F = -\frac{a}{r_0^2} + \frac{b}{r_0^3} = 0$$

$$\Rightarrow \underline{r_0} = \frac{b}{a} = \frac{8{,}10 \cdot 10^{-39}}{1{,}08 \cdot 10^{-28}}\,m = \underline{7{,}5 \cdot 10^{-11}\,m}$$

r_0 er bindingslengden.

$$dU = -F\,dr$$

Systemets energi

$$U = -\int_{\infty}^{r} F\,dr = -\int_{\infty}^{r}\left(-\frac{a}{r^2}\right)dr - \int_{\infty}^{r}\frac{b}{r^3}\,dr$$

$$U = -\left[\frac{a}{r}\right]_{\infty}^{r} - \left[-\frac{b}{2}\frac{1}{r^2}\right]_{\infty}^{r}$$

$$U = -\frac{a}{r} + \frac{b}{2r^2}$$

$r = r_0$

$$U_0 = -\frac{a}{r_0} + \frac{b}{2r_0^2} = -\frac{a}{\left(\frac{b}{a}\right)} + \frac{b}{2\left(\frac{b}{a}\right)^2} = -\frac{a^2}{2b}$$

$$\underline{U_0} = -\frac{(1{,}08 \cdot 10^{-28})^2}{2 \cdot 8{,}10 \cdot 10^{-39}}\,J = \underline{-7{,}20 \cdot 10^{-19}\,J}$$

U_0 er bindingsenergien (entalpien).

3 i) $b = -\left(\frac{dF}{dr}\right)_{r=r_0}$

$$\frac{dF}{dr} = +\frac{2a}{r^3} - \frac{3b}{r^4}$$

$$b = -\frac{2a}{r_0^3} + \frac{3b}{r_0^4} = -\frac{2a}{\left(\frac{b}{a}\right)^3} + \frac{3b}{\left(\frac{b}{a}\right)^4} = \frac{a^4}{b^3}$$

$$\underline{b} = \frac{(1{,}08\cdot 10^{-28})^4}{(8{,}10\cdot 10^{-39})^3}\ \text{N/m} = \underline{256\ \text{N/m}}$$

$$\omega = \sqrt{\frac{b}{m}}$$

der m er redusert masse

$$m = \frac{m_H \cdot m_H}{m_H + m_H} = \frac{1}{2} m_H = \frac{1}{2}\cdot 1{,}673\cdot 10^{-27}\ \text{kg}$$

frekvens

$$\nu = \frac{1}{2\pi}\sqrt{\frac{b}{m}} = \frac{1}{2\pi}\sqrt{\frac{256}{\frac{1}{2}\cdot 1{,}673\cdot 10^{-27}}}\ \text{Hz}$$

$$\underline{\nu} = \underline{8{,}80\cdot 10^{13}\ \text{Hz}} \qquad \omega = 2\pi\nu = 5{,}5\cdot 10^{14}\ \text{rad/s}$$

(j) $(n+\frac{1}{2})h\nu + U_0 = 0$ $\qquad h\nu = \hbar\omega$

$$n = -\frac{U_0}{h\nu} - \frac{1}{2} = \frac{7{,}20\cdot 10^{-19}}{6{,}625\cdot 10^{-34}\cdot 8{,}80\cdot 10^{13}} - 0{,}5$$

$$n = 12{,}3 - 0{,}5 = 11{,}8$$

$$\underline{n} = \underline{12}$$

Oppg. 4

(a) $pV = nRT$ $\qquad pV^\gamma = p_1 V_1^\gamma = p_2 V_2^\gamma$

$$W_{ad} = \int_{V_1}^{V_2} p\,dV = p_1 V_1^\gamma \int_{V_1}^{V_2} V^{-\gamma} dV = p_1 V_1^\gamma \left[\frac{1}{-\gamma+1} V^{-\gamma+1}\right]_{V_1}^{V_2} =$$

$$= \frac{p_1 V_1^\gamma}{\gamma - 1}\left(V_1^{-\gamma+1} - V_2^{-\gamma+1}\right) = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1} =$$

$$= \frac{p_1 V_1}{\gamma - 1}\left(1 - \frac{p_2 V_2}{p_1 V_1}\right) = \frac{p_1 V_1}{\gamma - 1}\left(1 - \left(\frac{p_2}{p_1}\right)^{1 - 1/\gamma}\right)$$

$$W_{ad} = \frac{4{,}4 \cdot 10^6 \cdot 20}{1{,}40 - 1}\left(1 - \left(\frac{3{,}6}{4{,}4}\right)^{1 - 1/1{,}40}\right) = 12{,}3 \cdot 10^6\,\mathrm{J} = \underline{12\,\mathrm{MJ}}$$

(b) $\dfrac{p_1 V_1}{T_1} = \dfrac{p_2 V_2}{T_2}$ $\qquad p_1 V_1^\gamma = p_2 V_2^\gamma$

$$T_2 = T_1 \frac{p_2 V_2}{p_1 V_1} = T_1 \left(\frac{p_2}{p_1}\right)^{1 - 1/\gamma} = 293\left(\frac{3{,}6}{4{,}4}\right)^{1 - 1/1{,}4} = 276{,}7\,\mathrm{K}$$

$$\underline{T_2 = 277\,\mathrm{K}}$$

(c) $$W_{it} = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} p_1 V_1 \frac{1}{V} dV = p_1 V_1 \ln\frac{V_2}{V_1} = \underline{p_1 V_1 \ln\frac{p_1}{p_2}}$$

$$W_{it} = 4{,}4 \cdot 10^6 \cdot 20 \ln\frac{4{,}4}{3{,}6} = 17{,}7 \cdot 10^6\,\mathrm{J} = \underline{18\,\mathrm{MJ}}$$

(d) For ein ideell gass avheng den indre energien berre av temperaturen. Altså er $\Delta U = 0$ for ein isoterm prosess. Etter fyrste hovudsetning får vi då

$$Q_{it} = W_{it} + \Delta U_{it} = W = W_{it} = \underline{p_1 V_1 \ln\frac{p_1}{p_2}} = 18\,\mathrm{MJ}$$

4 (e) Innvendig radius i tanken: $a = \left(\frac{3V_1}{4\pi}\right)^{1/3} = \left(\frac{3 \cdot 20}{4\pi}\right)^{1/3} = 1{,}68\ \text{m}$

Varmestraumen: $\frac{dQ_2}{dt} = k\, \frac{4\pi a^2}{b} \Delta T = 40 \frac{\text{J}}{\text{s}\cdot\text{m}\cdot\text{K}}\ \frac{4\pi (1{,}68\ \text{m})^2}{0{,}019\ \text{m}} (293 - 276{,}7)\,\text{K}$

$$\frac{dQ_2}{dt} = \frac{40 \cdot 4\pi\, 1{,}68^2 \cdot 16{,}3}{0{,}019} = 1{,}22 \cdot 10^6\ \text{J/s}$$

(f) Dersom adiabatisk: $t_{ad} = \frac{W_{ad}}{P_1} = \frac{12{,}3 \cdot 10^6\ \text{J}}{200 \cdot 10^3\ \text{W}} = 61\ \text{s}$

Dersom isoterm: $t_{it} = \frac{W_{it}}{P_1} = \frac{17{,}7 \cdot 10^6\ \text{J}}{200 \cdot 10^3\ \text{W}} = \underline{\underline{88\ \text{s}}}$

Dersom prosessen var adiabatisk ville temperaturen gå ned til T_2 og på slutten av prosessen ville det då vera eit varmetap $1{,}22 \cdot 10^6\ \text{J/s} \approx 1\ \text{MJ/s} = 1\ \text{MW}$
I starten då varmetapet var null (før temperaturfallet)
Som eit overslag reknar vi at varmetapet aukar lineart. Varmetapet totalt ville då på 61 s vera

$$\frac{1{,}22 \cdot 10^6\ \text{J/s} \cdot 61\ \text{s}}{2} = 37 \cdot 10^6\ \text{J}$$

Dette er meir enn den varmemengda som er rekna ut under ($\overset{d}{e}$) ville vera nødvendig (~~ville vera nødvendig~~) for å gjera prosessen isoterm. Vi kan altså konkludera med at prosessen ikkje kan vera adiabatisk eller tilnærma adiabatisk. Ser vi isolert på denne gassekspansjonen, er det meir energi å henta (18 MJ mot 12 MJ) i isotermt tilfelle enn adiabatisk. Sett einast på dette, vil det ikkje vera lurt med varmeisolering. (Noko anna er at det likevel ville vera lurt med varmeisolering – om det ikkje var for dyrt – når vi skal vurdera både energifylling og energitapping totalt på denne trykkluft-tanken.)