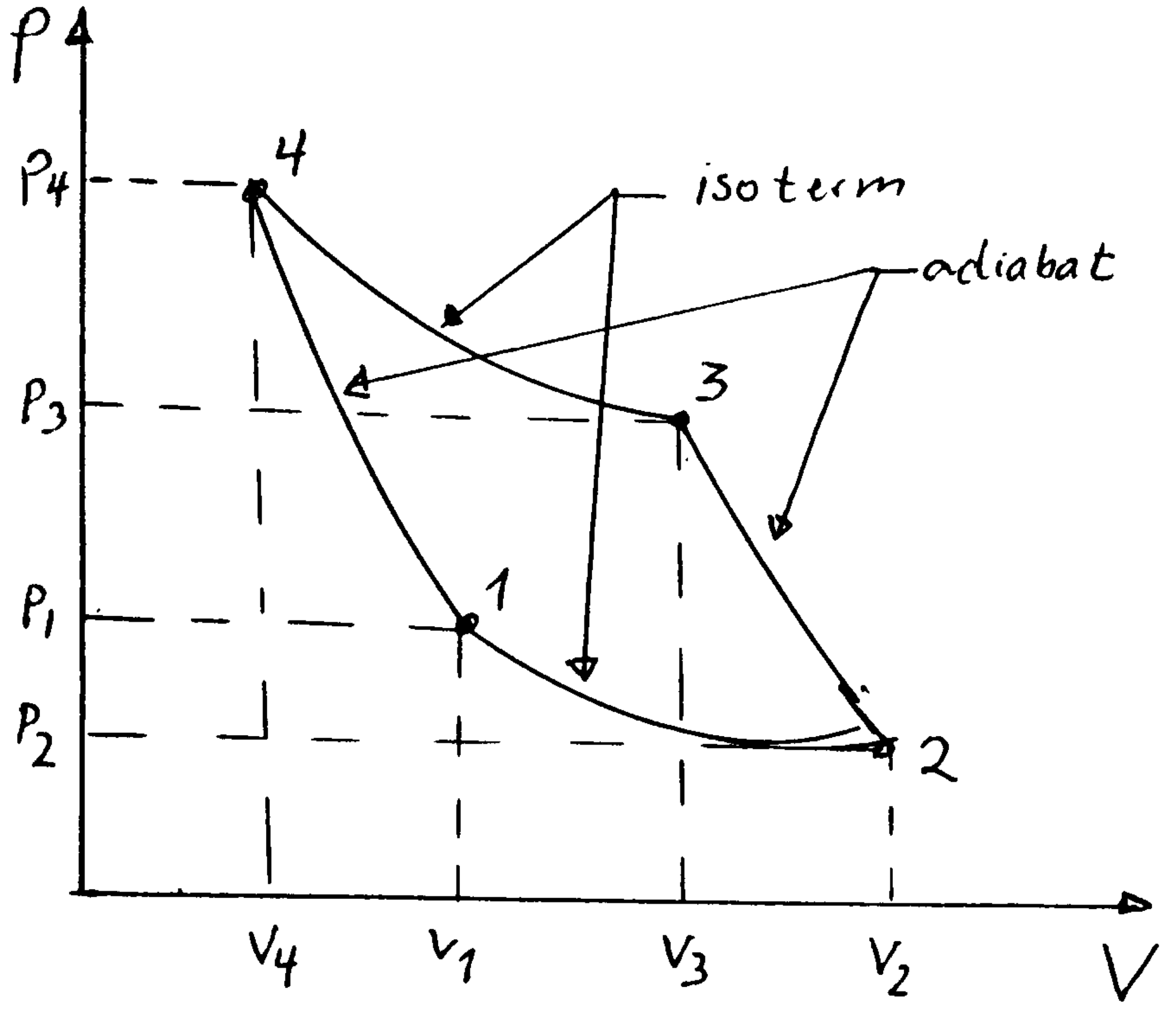
EKSAMEN I FYSIKK, Avd III (Bygg), 24. august 1989

LØSNINGSFORSLAG

Oppgave 1

a)



Gasslign. : $pV = mRT$

$$m = \frac{p_1 V_1}{R T_1}$$

b) Isoterm; $\Delta U = 0 \Rightarrow$

$$Q_1 = W_{12} = \int_{V_1}^{V_2} p\,dV = mRT_1 \int_{V_1}^{V_2} \frac{dV}{V} = mRT_1 \ln\left(\frac{V_2}{V_1}\right) = p_1 V_1 \ln\left(\frac{V_2}{V_1}\right)$$

$$Q_3 = W_{34} = \int_{V_3}^{V_4} p\,dV = mRT_3 \ln\left(\frac{V_4}{V_3}\right) = p_1 V_1 \frac{T_3}{T_1} \ln\left(\frac{V_4}{V_3}\right)$$

Vi har $W_{23} = -W_{41} \Rightarrow$ Totalt tilført arbeid:

$$W = W_{12} + W_{34} = \underline{\underline{p_1 V_1 \left[\ln\left(\frac{V_2}{V_1}\right) + \frac{T_3}{T_1} \ln\left(\frac{V_4}{V_3}\right) \right]}}$$

c) Pumpens effektfaktor:

$$\epsilon = \frac{Q_3}{W} = \frac{p_1 V_1 \frac{T_3}{T_1} \ln\left(\frac{V_4}{V_3}\right)}{p_1 V_1 \left[\ln\left(\frac{V_2}{V_1}\right) + \frac{T_3}{T_1} \ln\left(\frac{V_4}{V_3}\right) \right]} = \frac{T_3 \ln\left(\frac{V_4}{V_3}\right)}{T_1 \ln\left(\frac{V_2}{V_1}\right) + T_3 \ln\left(\frac{V_4}{V_3}\right)} \qquad *)$$

Vi har:

$$\left.\begin{aligned} T_1 V_2^{\gamma-1} &= T_3 V_3^{\gamma-1} \\ T_3 V_4^{\gamma-1} &= T_1 V_1^{\gamma-1} \end{aligned}\right\} \quad \frac{T_1 V_2^{\gamma-1}}{T_1 V_1^{\gamma-1}} = \frac{T_3 V_3^{\gamma-1}}{T_3 V_4^{\gamma-1}}$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4}$$ innsatt i ligning *):

$$\epsilon = \frac{T_3 \ln\left(\frac{V_1}{V_2}\right)}{T_1 \ln\left(\frac{V_2}{V_1}\right) + T_3 \ln\left(\frac{V_1}{V_2}\right)} = \frac{T_3 \ln\left(\frac{V_1}{V_2}\right)}{T_3 \ln\left(\frac{V_1}{V_2}\right) - T_1 \ln\left(\frac{V_1}{V_2}\right)} = \underline{\underline{\frac{T_3}{T_3 - T_1}}}$$

d) Reell virkningsgrad:

$$\epsilon_r = 0{,}55 \cdot \epsilon = 0{,}55 \frac{T_3}{T_3 - T_1} = 0{,}55 \frac{343}{343 - 280} = \underline{3{,}0}$$

Sentralvarmeanlegget mottar effekten P som er 90% av pumpens avgitte effekt P_3.

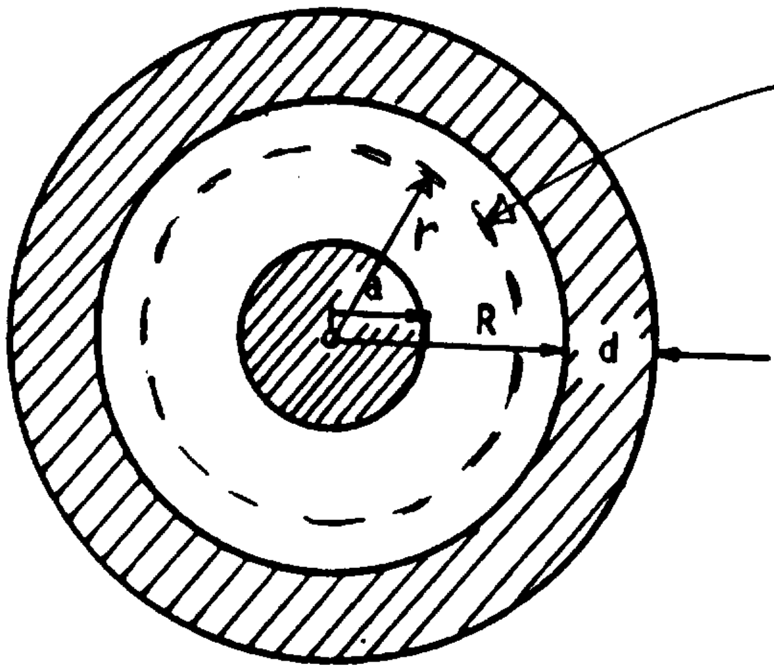
$$P_3 = \frac{P}{0{,}9} = \frac{1{,}8 \cdot 10^7}{0{,}9} W = \underline{2 \cdot 10^7 W}$$

$$\varepsilon_r = \frac{P_3}{P_0} = \frac{P}{0{,}9 \cdot P_0} = 3$$

$$\Rightarrow P_0 = \frac{P}{3 \cdot 0{,}9} = \frac{1.8 \cdot 10^7}{2{,}7} W = \frac{2}{3} \cdot 10^7 W = \underline{\underline{0{,}67 \cdot 10^7 W}}$$

$$\varepsilon_{syst} = \frac{P}{P_0} = \frac{3 \cdot 1.8 \cdot 10^7}{2 \cdot 10^7} = \underline{\underline{2{,}7}}$$

Oppgave 2



Gauss-flate: Sylinder med radius r og lengde l

Gauss sats:

$$\oint \vec{E}\,d\vec{A} = \frac{q}{\epsilon}$$

a) i) $a < r < R$

$$\oint \vec{E}\,d\vec{A} = E(r)\int dA = E(r)\cdot 2\pi r l = \frac{\lambda_1 l}{\epsilon_0}$$

$$\Rightarrow \qquad E(r) = \underline{\underline{\frac{\lambda_1}{2\pi\epsilon_0 r}}}$$

ii) $r > R + d$

$$E(r)\cdot 2\pi r\cdot l = \frac{(\lambda_1+\lambda_2)l}{\epsilon_0}$$

$$\Rightarrow \qquad E(r) = \underline{\underline{\frac{\lambda_1+\lambda_2}{2\pi\epsilon_0 r}}}$$

b) i) I et metall må det elektriske feltet være lik null. I motsatt fall har vi en ström i lederen

ii) Ladning pr. meter i indre sylindervegg = λ_2^{indre}

——— 〃 ——— ytre ——〃—— = λ_2^{ytre}

Gauss-flaten ligger i ytre sylinder.
Da ladningen fordeler seg på sylinder-veggene gir Gauss sats:

$$E(r)\cdot 2\pi r l = \frac{(\lambda_1 + \lambda_2^{indre})\,l}{\epsilon}$$

For at $E(r) = 0$, må vi ha:

$$\underline{\underline{\lambda_2^{indre} = -\lambda_1}}$$

$$\lambda_2^{ytre} = \lambda_2 - \lambda_2^{indre} = \underline{\underline{\lambda_2 + \lambda_1}}$$

c) Amperes lov $\oint \vec{B}\,d\vec{l} = \mu I$

i) $r < a$

$$\oint \vec{B}\,d\vec{l} = B(r)\oint dl = B(r)\cdot 2\pi r = \mu r \cdot i_1 \frac{r^2}{a^2}$$

$$\Rightarrow \underline{\underline{B(r) = \frac{\mu r i_1}{2\pi} \frac{r}{a^2}}}$$

ii) $a < r < R$

$$B \cdot 2\pi r = \mu_0 i_1$$

$$\Rightarrow \underline{\underline{B(r) = \frac{\mu_0 i_1}{2\pi r}}}$$

iii) $r > R + d$

$$B(r)\cdot 2\pi r = \mu_0 (i_1 + i_2)$$

$$\Rightarrow \underline{\underline{B(r) = \frac{\mu_0 (i_1 + i_2)}{2\pi r}}}$$

iv) Brøkdel av strømmen i_2 innenfor sløyfa med radius r er:

$$\frac{r^2 - R^2}{(R+d)^2 - R^2}$$

$R < r < R+d$

$$\underline{\underline{B(r) = \frac{\mu_0}{2\pi r}\left(i_1 + i_2 \frac{r^2 - R^2}{(R+d)^2 - R^2}\right)}}$$

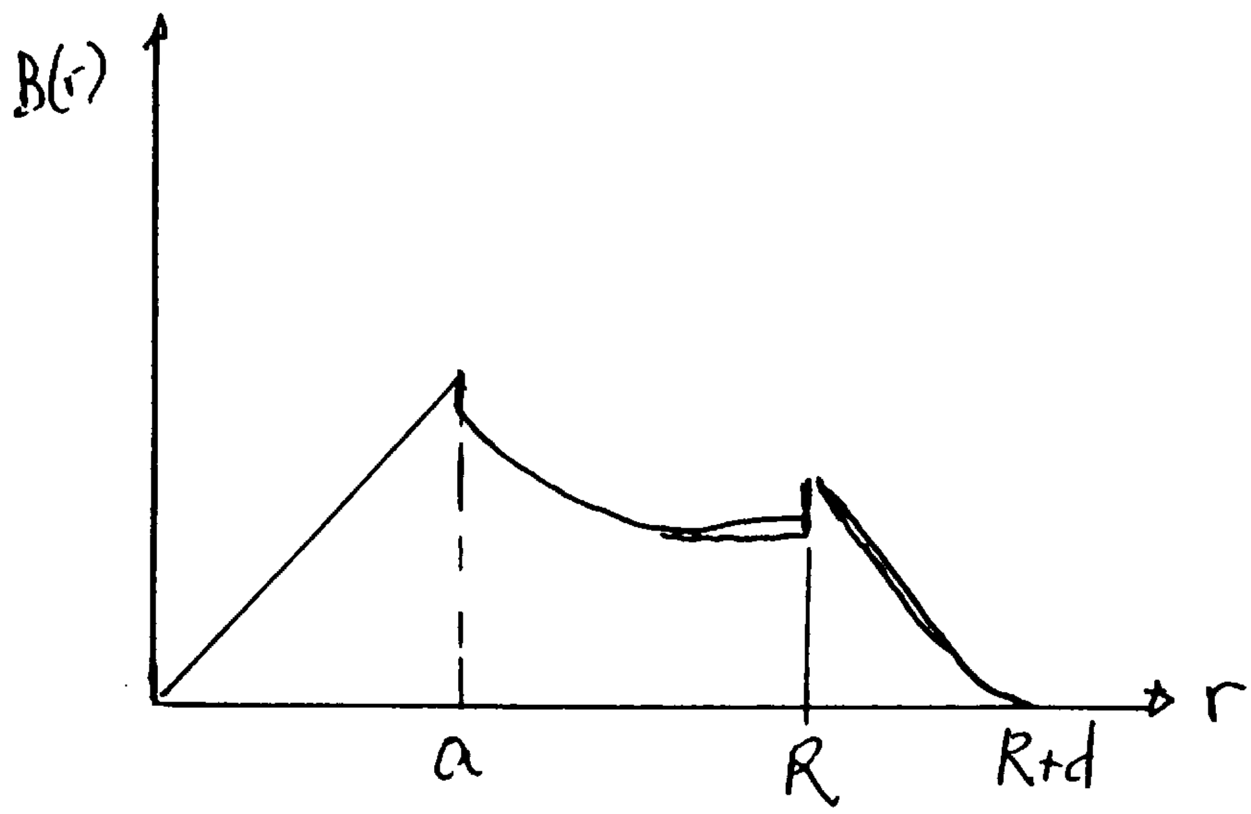
For $i_1 = -i_2$:

$r < a$: $B(r) = \frac{\mu_r i_1}{2\pi a^2} r$

$a < r < R$: $B(r) = \frac{\mu_0 i_1}{2\pi} \frac{1}{r}$

$R < r < R+d$: $B(r) = \frac{\mu_0 i_1}{2\pi r}\left(1 - \frac{r^2 - R^2}{(R+d)^2 - R^2}\right)$

$r > R+d$: $B(r) = 0$



Oppgave 3

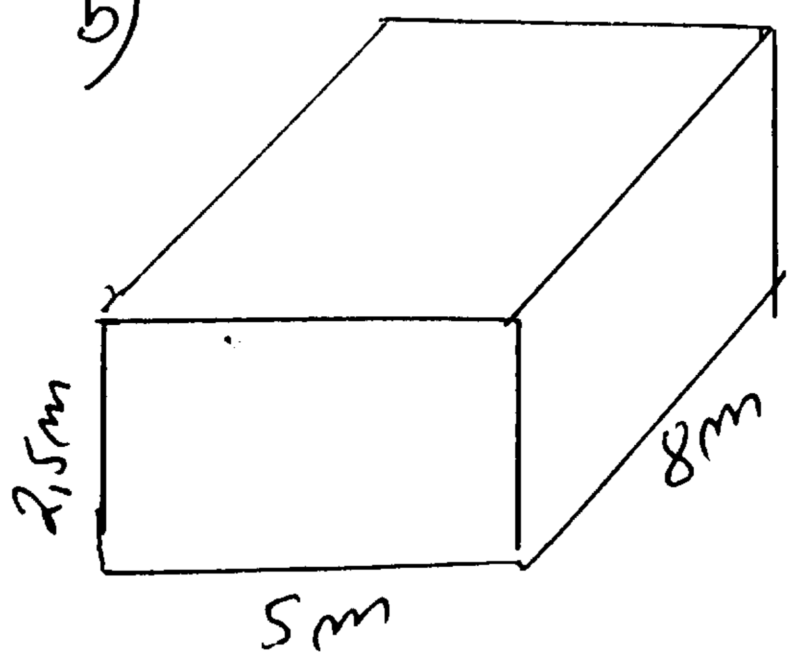
a) Varmeledning: $j_e = \lambda \dfrac{T_2 - T_1}{d}$

Varmestråling: $j_s = \sigma (T_2^4 - T_1^4) \approx 4\sigma T_1^3 (T_2 - T_1)$

$$j_e = 0{,}024 \, \frac{290-265}{0{,}012} = \underline{\underline{50 \ W/m^2}}$$

$$j_s = 5{,}67 \cdot 10^{-8} (290^4 - 265^4) = \underline{\underline{121{,}4 \ W/m^2}}$$

b)



Veggareal: 26m × 2,5m = 65 m²

Gulv + tak: 2×5×8 m² = 80 m²

Vindusareal: 10% av 65 m² = 6,5 m²

⇒ Veggareal = $\underline{58{,}5 \ m^2}$

Varmetap pr. m² i vegg/tak:

$$j_e = \frac{T_2 - T_1}{\frac{b_1}{\lambda_1} + \frac{b_2}{\lambda_2} + \frac{b_1}{\lambda_1}} = \frac{290-265}{\frac{2{,}5\cdot 10^{-2}}{0{,}14} + \frac{0{,}15}{0{,}047} + \frac{2{,}5\cdot 10^{-2}}{0{,}14}}$$

$$= \frac{25}{3{,}548} = \underline{\underline{7{,}046 \ W/m^2}}$$

Totalt tap vegg/tak:

$$J_{VT} = \cancel{j_e} \times areal = 7{,}046 \cdot (80 + 58{,}5) = 975{,}9\ W$$

Totalt tap vinduer:

$$J_V = (j_S + j_e) \times areal = (121{,}4 + 50) \cdot 6{,}5 = 1114{,}1\ W$$

$$J_{Total} = J_{VT} + J_V = 975{,}9 + 1114{,}1 = 2090\ W$$

Brøkdel vindustap: $\dfrac{1114{,}1}{2090} = \underline{\underline{53{,}3\ \%}}$

c) Inkluderer tap vegg/luft:

$$j_e = \frac{T_2 - T_1}{\frac{b_1}{\lambda_1} + \frac{b_2}{\lambda_2} + \frac{b_1}{\lambda_1} + \frac{2}{h}} = \frac{25}{3{,}548 + 0{,}2} = \underline{6{,}67\ W/m^2}$$

Reduksjon i varmestrømstetthet:

$$7{,}046 - 6{,}670 = \underline{\underline{0{,}376\ W/m^2}}$$

som i % blir $\dfrac{0{,}376}{7{,}046} = 0{,}053 \Rightarrow \underline{\underline{5{,}3\%}}$

d) Varmestrøm i venstre mellomrom:

$$j_1 = 4\sigma T_m^3 (T_m - T_1) + \frac{T_m - T_1}{d/\lambda}$$
$$= \left(4\sigma T_m^3 + \frac{\lambda}{d}\right)(T_m - T_1)$$

Varmestrøm i høyre mellomrom:

$$j_2 = 4\sigma T_m^3 (T_2 - T_m) + \frac{T_2 - T_m}{d/\lambda}$$
$$= \left(4\sigma T_m^3 + \frac{\lambda}{d}\right)(T_2 - T_m)$$

Ved likevekt er $j_1 = j_2 \Rightarrow T_m - T_1 = T_2 - T_m$

$$\Rightarrow \underline{\underline{T_m = \frac{T_1 + T_2}{2}}} \quad \text{q.e.d.}$$

Varmestrømtetthet, stråling:

$$j_s = \sigma(T_2^4 - T_m^4) \simeq 4\sigma T_2^3 (T_2 - T_m) = 4\sigma T_2^3 \left(T_2 - \frac{T_1 + T_2}{2}\right)$$
$$= 4\sigma T_2^3 \left(\frac{T_2 - T_1}{2}\right)$$

Dette er halvparten av strømtettheten vi ville fått med dobbeltvindu. Altså

$$j_s = \frac{121{,}4}{2}\ \mathrm{W/m^2} = \underline{60{,}7\ \mathrm{W/m^2}}$$

Varmestrøm ved ledning blir å regne som om det midterste vindu var borte (p.g.a. at glasset er en god leder sammenlignet med luft

$$j_e = \lambda \frac{T_2 - T_1}{d} = 0{,}024 \frac{290 - 265}{0{,}020} = \underline{30\ \mathrm{W/m^2}}$$

Totalt: $j = j_s + j_e = 60{,}7 + 30 \approx \underline{90{,}7\ \mathrm{W/m^2}}$