UNIVERSITETET I TRONDHEIM
NORGES TEKNISKE HØGSKOLE
INSTITUTT FOR FYSIKK

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Examination, course 74 355 Nuclear physics
Saturday May 27, 1995
Time: 09.00–13.00

Allowed to use: (Alternative B): Officially approved pocket calculator.
Rottmann, *Mathematische Formelsammlung.*
Barnett and Cronin, *Mathematical Formulae.*
Øgrim og Lian, *Størrelser og enheter i fysikk og teknikk.*

Some constants:			
The elementary charge:	e	$=$	1.60×10^{-19} C
The speed of light in vacuum:	c	$=$	3.00×10^{8} m/s
The reduced Planck's constant:	$\hbar$	$=$	1.05×10^{-34} J s
		$=$	197 (MeV/c) fm
Boltzmann's constant:	k_B	$=$	1.38×10^{-23} J/K
		$=$	8.62×10^{-5} eV/K
Avogadro's constant:	N_A	$=$	6.02×10^{23} /mol
The neutron mass:	m_n	$=$	939.566 MeV/c^2
The atomic mass unit:	u	$=$	931.494 MeV/c^2
1 barn:	b	$=$	10^{-28} m^2

Problem 1:

The scattering amplitude for elastic scattering at the scattering angle θ may be written as

$$f(\theta) = \frac{1}{2\mathrm{i}k} \sum_{\ell=0}^{\infty} (2\ell+1) \left(\eta_\ell \,\mathrm{e}^{2\mathrm{i}\delta_\ell} - 1 \right) P_\ell(\cos\theta) \; .$$

Here $k = |\boldsymbol{k}|$, where $\boldsymbol{p} = \hbar\boldsymbol{k}$ is the incoming momentum.
We assume rotational symmetry about $\boldsymbol{k}$, and we ignore the intrinsic spin of the particles.
P_ℓ is the Legendre polynomial corresponding to the angular momentum ℓ, and δ_ℓ is the phase shift of elastic scattering. η_ℓ is the elasticity, which satisfies the inequality $0 \le \eta_\ell \le 1$.
The total elastic scattering cross section is

$$\sigma_{\mathrm{el}} = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1) \left| \eta_\ell \,\mathrm{e}^{2\mathrm{i}\delta_\ell} - 1 \right|^2 \; ,$$

whereas the total inelastic scattering cross section is

$$\sigma_{\mathrm{inel}} = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1) \left(1 - {\eta_\ell}^2 \right) \; .$$

a) Sketch (briefly!) how the expressions for $\sigma_{\rm el}$ and $\sigma_{\rm inel}$ may be derived.
What is the meaning of each of the limiting cases $\eta_\ell = 1$ and $\eta_\ell = 0$?

b) Deduce an upper limit to the cross section for absorption of a thermal neutron by a heavy nucleus. A typical temperature in a fission reactor might be 600 K.
Compare with the measured cross section of 2.7×10^6 b for neutron absorption in ^{135}Xe.

c) Deduce both an upper and a lower limit for the elastic scattering cross section of a thermal neutron on ^{135}Xe.
What does the angular distribution look like for elastic scattering at low energy?

d) What is meant by a *fissile*, and a *fertile*, isotope?
There is (at least) one other characteristic property distinguishing these two classes of heavy isotopes. Which property, and what is the theoretical explanation?

e) The fission products in a reactor are important for the functioning of the reactor.
Some fission products are sources of *delayed* neutrons.
What is the reason for the delay? And why are the delayed neutrons important?
Other fission products *poison* the reactor.
How? Give an example of a fission product poison!

Problem 2:

a) There exist four separate decay series of heavy, radioactive isotopes.
Why exactly four?

b) The most long lived isotope in one of the four series is $^{238}_{92}$U, with a half life of 4.47×10^9 years. This series contains another uranium isotope, $^{234}_{92}$U, with a half life of 245 000 years.
Natural uranium contains the isotopes 238 (99.275%), 235 (0.720%) and 234 (0.0057%).
The half life of ^{235}U is 7.13×10^8 years.
How much do each of the three isotopes contribute to the specific activity (activity per mass) of natural uranium? Any comment?

c) The ^{238}U series ends in the stable lead isotope $^{206}_{82}$Pb.
How many α and β decays are needed for transforming a ^{238}U nucleus into a ^{206}Pb nucleus? How much energy is liberated in total?
Atomic masses:
$M_A(^{238}\text{U}) = 238.050785\,\text{u}$, $M_A(^{206}\text{Pb}) = 205.974440\,\text{u}$, $M_A(^{4}\text{He}) = 4.002603\,\text{u}$.

d) One kg of sea water contains 3×10^{-6} g of uranium.
What is the activity of ^{238}U in one kg of sea water?
What is the total contribution to the specific activity in sea water if all the radioactive isotopes in the ^{238}U series are included?
Compare to the activity from ^{40}K, given that one kg of sea water contains 0.38 g of potassium, which in turn contains 0.012% of ^{40}K. The half life of ^{40}K is 1.28×10^9 years.

Problem 3:

a) Use the level diagram of one-particle energies in the shell model (see below) to predict the spin and parity of the ground state of each of the following nuclei:
$^{2}_{1}$H, $^{4}_{2}$He, $^{6}_{3}$Li, $^{13}_{6}$C, $^{14}_{6}$C, $^{38}_{17}$Cl, $^{71}_{33}$As, $^{72}_{33}$As, $^{73}_{33}$As.