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Home examination in the course 74 355 Nuclear Physics
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Problem 1:

a) If one wants to measure the age of biological material by measuring the radioactivity in 10 g of carbon from this material, and if one can spend 10 days (240 hours) on the task, what is the maximum age that one can measure with 10 % accuracy or better?
The ideal way to measure would be to count exactly the number of atoms of ^{14}C in the 10 g of carbon. If this was possible, what would then be the maximum age that one could measure with 10 % accuracy or better?

b) The potassium isotope ^{40}K is radioactive with a half life of 1,28 billion years. It decays with 89 % probability into ^{40}Ca and with 11 % probability into ^{40}Ar.
What kinds of radioactivity are involved?
Why is the lifetime so long?

c) How old is a rock where the ratio between the numbers of atoms of ^{40}Ar and ^{40}K is found to be 0.2? What are the conditions for this method of measuring the age of a rock to be valid?
Describe a possible method for measuring age by means of ^{40}K, ^{40}Ca and one more stable isotope of Ca, assuming one has many rocks of the same age and with the same original ratio between the isotopes of Ca.

d) Heavy elements must have been produced in two essentially different kinds of processes, called "s processes" and "r processes".
What charcterizes the two types of processes, and what is it that shows that both of them must have taken place?

e) If the isotopes ^{235}U and ^{238}U are produced all the time with a constant rate, in the ratio 1.64 to 1, how old is then the Universe?

Problem 2:

a) Which elementary particles exist, according to the "standard model" of particle physics? Which are present in ordinary matter?
In 1970 the existence of a fourth type of quark was predicted, in addition to the three already known. What was the argument?

b) How can the parity of the π meson be measured?
What general relation is there between the parity of a particle and its antiparticle?
The parity of the π meson is an important test of the validity of the quark model. Why?

c) What was the original motivation for the hypothesis that quarks exist in three variants, having three different "coulours": "red", "green" and "blue"? (The three coulour variants have different values of two new charge quantum numbers.)
Describe briefly an experiment where one meusures rather directly the square sum of the electric charges of the quarks, and thereby indirectly the number of coulour variants of the quarks.

Problem 3:

a) In the shell model the spherically symmetric one particle potential

$$V(r) = \frac{-V_0}{1 + \exp\left(\frac{r-R}{a}\right)} \tag{1}$$

is used, with $V_0 = 50\,\text{MeV}$, $R = 1.25A^{\frac{1}{3}}\,\text{fm}$ where A is the mass number, and $a = 0.524\,\text{fm}$. We write $r = |\boldsymbol{r}|$, where $\boldsymbol{r}$ is the position of the particle.
A nucleon in this potential with $A = 33$ has the following bound energy levels:

Level	Energy in MeV
1s	−37.52
1p	−26.51
1d	−14.20
2s	−11.90
1f	− 1.46
2p	− 0.91

Count the total number of bound states, when you take into account that the nucleon has spin, but neglect the spin-orbit interaction.
From this (over-)simplified model, what is the separation energy of a proton and of a neutron in the nucleus ${}^{33}_{16}\text{S}$?
Which effects have we neglected, in addition to the spin-orbit interaction?

b) The spin-orbit interaction in the shell model is described by a velocity dependent potential

$$V_1(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{S}) = W(r)\, \boldsymbol{L} \cdot \boldsymbol{S}\ , \tag{2}$$

where $\boldsymbol{p}$ is the momentum, $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$ the orbital angular momentum, and $\boldsymbol{S}$ is the spin of the nucleon.
Take for simplicity $W(r)$ to be constant, for example $\hbar^2\, W(r) = -2\,\text{MeV}$, and use first order perturbation theory to calculate corrected energy levels for $A = 33$, using the above table.
Use the corrected levels to predict spin and parity of the ground state and the lowest excited states in ${}^{33}_{16}\text{S}$, together with the excitation energies.

c) The origin of the spin-orbit interaction in the shell model is the spin-orbit interaction between two nucleons, which has the same form as in equation (2), with the modification that $\boldsymbol{r}$ is the relative position and $\boldsymbol{L}$ the relative orbital angular momentum.
In the interaction between two protons there is also an *electromagnetic* spin-orbit interaction, which has

$$W(r) = \frac{2}{m_p^2\, c^2}\, \frac{1}{r}\, \frac{\mathrm{d}V_C}{\mathrm{d}r} \; .$$

Here m_p is the proton mass, c is the speed of light, and V_C is the Coulomb potential between the protons. Compare the electromagnetic spin-orbit interaction with that which is due to the nuclear interaction (with respect to sign, strength, range).

d) Explain briefly how the spin-orbit interaction between two particles may lead to a polarization of the spin of the scattered particles in the scattering of unpolarized particles.

e) The spin-orbit potential $W(r)$ for a nucleon in the shell model is not constant, but is nonzero mostly when the nucleon is close to the surface of the nucleus. There is a simple physical explanation of this, from the fact that the spin-orbit interaction in the nucleus comes from a two-particle interaction. Can you suggest an explanation?

Problem 4:

Use the potential (1) to calculate numerically, with an accuracy of 0.1 MeV or better, all the bound energy levels for a nucleon when $A = 15$. Neglect the spin-orbit interaction.
Give the degeneracies of the energy eigenvalues.
Here are some hints for numerical solution of the egenvalue problem:
Write the three dimensional, time independent wave function in spherical coordinates as

$$\psi(r, \theta, \varphi) = \frac{1}{r}\, u(r)\, Y_{\ell m}(\theta, \varphi) \; ,$$

where $Y_{\ell m}$ is a spherically harmonic function. The radial Schrödinger equation determining the energy eigenvalue E and the radial wave function $u = u(r)$ is then, with $u'' = \mathrm{d}^2 u/\mathrm{d}r^2$,

$$u''(r) = \left[\frac{2m}{\hbar^2}\,(V(r) - E) + \frac{\ell(\ell+1)}{r^2}\right] u(r) \; .$$

For bound states $E < 0$.
The boundary condition as $r \to \infty$ is that $u(r)$ decreases exponentially, i.e. that $u'(r)/u(r) = -\sqrt{-2mE}/\hbar$ when r is “large enough”.
The boundary condition as $r \to 0$ is that $u(r) \to 0$. More precisely: $u(r)$ goes to 0 as $r^{\ell+1}$, so that $u(r) \to r\, u'(r)/(\ell + 1)$.
Note that the principal quantum number n is the number of zeroes of the radial wave function $u(r)$, including the zero at the origin.

A suitable Runge–Kutta method for numerical integration of the second order differential equation $u'' = f(r, u)$ says that

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\begin{aligned}
u(r+h) &= u(r) + h\,u'(r) + \frac{h^2}{6}\,(k_1 + 2k_2) + \mathcal{O}(h^5)\ , \\
u'(r+h) &= u'(r) + \frac{h}{6}\,(k_1 + 4k_2 + k_3) + \mathcal{O}(h^5)\ , \\
k_1 &= f(r, u(r))\ , \\
k_2 &= f\left(r + \frac{h}{2}\,,\ u(r) + \frac{h}{2}\,u'(r) + \frac{h^2}{8}\,k_1\right), \\
k_3 &= f\left(r + h\,,\ u(r) + h\,u'(r) + \frac{h^2}{2}\,k_2\right).
\end{aligned}
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It is advantageous to integrate numerically from a "large" value $r = r_1$ (e.g. $r_1 = 10$ or $r_1 = 20$ in units of fm) and in towards $r = 0$ (we can integrate numerically all the way to $r = 0$ only if $\ell = 0$). This because the function $u(r)$ then must *increase* exponentially to begin with (if we integrate outwards, we have to calculate an exponentially decreasing fuction, and that problem is numerically unstable). Hence we start at $r = r_1$, with $u(r_1) = 1$, since the normalization of the wave function is uninteresting, and with $u'(r_1) = -\sqrt{-2mE}/\hbar$. Then we adjust the energy E until we obtain that $u(r) \to 0$ for $r \to 0$.