Renorm Ex 15.12.89
Problem 1 Løsningsforslag

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a & b Som boka, avsnitt 12.3 & 12.2

c

$\left(\nabla^2 s(\vec{x})\right)^2$ Invariant overfor $s \to -s$ og for $x \to -x$ } OK

$(s\nabla s)^2 = \left(\frac{1}{2}\nabla s^2\right)^2$ —— " —— OK

$\left((\nabla s)^2\right)^2$ —— " —— OK

(i) $\int d^dx \left(\nabla^2 s\right)^2 \to \int d^dx_L\, L^d \left(\nabla_L^2 s_L\right)^2 \cdot L^{-4} c^{-2}$

$= L^{d-4} c^{-2} \int d^dx_L \left(\nabla_L^2 s_L\right)^2$

Krav: $\int d^dx (\nabla s)^2 = L^{d-2} c^{-2} \int d^dx_L (\nabla_L s_L)^2 = \int d^dx_L (\nabla_L s_L)^2$

ↄ: $\boxed{c^{-2} = L^{2-d}}$

$\int d^dx (\nabla^2 s)^2 \to L^{-2} \int d^dx_L \left(\nabla_L^2 s_L\right)^2$ irrelevant!

(ii) $\int d^dx (s\nabla s)^2 \to \int d^dx_L (s_L \nabla_L s_L)^2 \cdot L^{d-2} c^{-4}$

$=$ —— " —— L^{2-d} relevant for $d \leq 2$.
irrelevant for $d>$

(iii) $\int d^dx \left((\nabla s)^2\right)^2 \to \int d^dx_L \left((\nabla_L s_L)^2\right)^2 \cdot L^{d-4} c^{-4}$

$=$ —— " —— L^{-d} irrelevant!

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Problem 2 Løsningsforslag, etc.

Vi tar de oppgitte diffligningene for god fisk, med $4K_4 \equiv C$

$$\frac{dr}{dL} = 2r + \frac{C}{1+r}\Big((n+2)u + 3v\Big)$$

$$\frac{du}{dL} = \varepsilon u - \frac{C}{(1+r)^2}\Big((n+8)u^2 + 6uv\Big)$$

$$\frac{dv}{dL} = \varepsilon v - \frac{C}{(1+r)^2}\Big(9v^2 + 12uv\Big)$$

Fikspunkter

① Gauss: $r_1 = u_1 = v_1 = 0$ er åpenbart et fikspunkt siden det gir $dr/dL = \frac{du}{dL} = \frac{dv}{dL} = 0$.

② $v_2 = 0$

$$\frac{du_2}{dL} = \varepsilon u_2 - \frac{C}{(1+r_2)^2}(n+8)u_2^2 = 0 \qquad \Rightarrow u_2 = O(\varepsilon)$$

$$\Downarrow$$

$$\frac{dr_2}{dL} = 2r_2 + \frac{C}{1+r_2}(n+2)u_2 = 0 \qquad \Rightarrow r_2 = O(\varepsilon)$$

Dervad kan vi, til ledende orden i ε, la $(1+r_2) \to 1$ i forbindelse med bestemmelse av fixpkt.

$$\frac{du_2}{dL}=0 \Rightarrow \quad u_2=\frac{\varepsilon}{(n+8)C} \quad ; \quad r_2=-\frac{C(n+2)\,\varepsilon}{2\,(n+8)C}=-$$

$$=-\frac{n+2}{2(n+8)}\,\varepsilon$$

(3) $u_3=0$

Tilsvarende

$$\frac{dv_3}{dL}=0 \Rightarrow \quad v_3=\frac{\varepsilon}{9C} \quad ; \quad r_3=-\frac{C\cdot 3\cdot \varepsilon}{2\,9C} \;=-\frac{1}{6}\varepsilon$$

(4) $u_4\neq 0 \quad v_4\neq 0$

$$\left.\begin{aligned} 0=\frac{du_4}{dL} &\Rightarrow \quad \varepsilon = C\left((n+8)u_4+6v_4\right) \\ 0=\frac{dv_4}{dL} &\Rightarrow \quad \varepsilon = C\left(9v_4+12u_4\right)\end{aligned}\right\}(4-n)u_4+3v_4=0$$

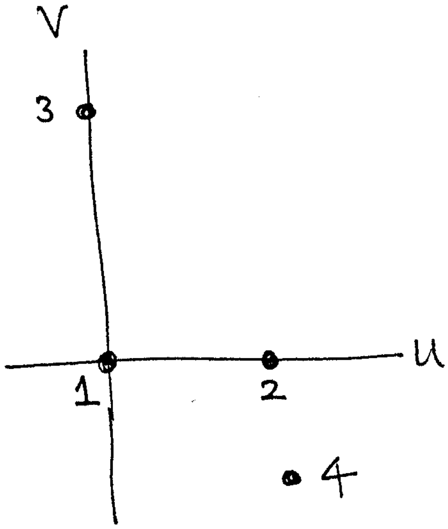
$$\varepsilon = C\left((n+8)u_4-2(4-n)u_4\right)=3nCu_4$$

$$u_4=\frac{\varepsilon}{3nC} \quad ; \quad v_4=-\frac{4-n}{9nC}\,\varepsilon$$

$$r_4=-\frac{C}{2}\left((n+2)\frac{\varepsilon}{3nC}-\frac{4-n}{3nC}\,\varepsilon\right)$$

$$=-\frac{\varepsilon}{6n}\left(n+2-4+n\right)$$

$$=-\frac{\varepsilon}{3n}(n-1)$$



Dette er svar på oppgaven, projisert på (u,v)-planet. Men hvordan er flytdiagrammet mellom disse fikspunktene?

For å finne ut dette må vi sjekke egenverdiene til diff lign. nær de fire fikspunktene

(1)

$$\frac{d\delta r_1}{dL} = 2\delta r_1 + C\left((n+2)\delta u_1 + 3\delta v_1\right) - \cancel{\frac{C}{(1+r)^2}\delta r_1}(\times 0)$$

$$\frac{d\delta u_1}{dL} = \varepsilon \delta u_1 - C\cdot 0$$

$$\frac{d\delta v_1}{dL} = \varepsilon \delta v_1 - C\cdot 0$$

$$\Rightarrow \begin{pmatrix} \frac{d\delta r_1}{dL} \\ \frac{d\delta u_1}{dL} \\ \frac{d\delta v_1}{dL} \end{pmatrix} = \begin{pmatrix} 2 & C(n+2) & 3C \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}$$

Egenverdier $y_1 = \begin{cases} 2 = y_{1r} \\ \varepsilon = y_{1u} \\ \varepsilon = y_{1v} \end{cases}$ alle > 0

I (u,v)-planet To: $\begin{cases} \varepsilon \\ \varepsilon \end{cases}$ begge positive

(2)

$$\frac{d\delta r_2}{dL} = 2\delta r_2 + C\Big((n+2)\delta u_2 + 3\delta v_2\Big) - \frac{C\delta r_2}{(1+\underset{=0}{r_2})^2}\Big((n+2)u_2 + 3v_2\Big)$$

$$= \delta r_2\left\{2 - C(n+2)\frac{\varepsilon}{(n+8)C}\right\} + C(n+2)\delta u_2 + 3C\,\delta v_2$$

$$= \left(2 - \frac{n+2}{n+8}\varepsilon\right)\delta r_2 + (n+2)C\,\delta u_2 + 3C\,\delta v_2$$

$$\frac{d\delta u_2}{dL} = \varepsilon\,\delta u_2 - C\Big(2(n+8)u_2\delta u_2 + 6\overset{0}{v_2}\delta u_2 + 6u_2\delta v_2\Big)$$
$$+ \frac{2C}{(1+\underset{=0}{r_2})^2}\delta r_2\Big((n+8)\underset{\uparrow}{u_2^2} + 6u_2\underset{=0}{v_2}\Big)$$

$\mathcal{O}(\varepsilon^2) \leftarrow$ kan droppes her

$$= \left(\varepsilon - 2(n+8)C\frac{\varepsilon}{(n+8)C}\right)\delta u_2 - 6C\frac{\varepsilon}{(n+8)C}\delta v_2$$

$$= -\varepsilon\,\delta u_2 - \frac{6\varepsilon}{n+8}\delta v_2$$

$$\frac{d\delta v_2}{dL} = \varepsilon\,\delta v_2 - C\Big(18\underset{=0}{v_2}\delta v_2 + 12u_2\delta v_2\Big)$$

$$= \left(\varepsilon - 12C\frac{\varepsilon}{(n+8)C}\right)\delta v_2 = \frac{n+8-12}{n+8}\varepsilon\,\delta v_2 = -\frac{4-n}{n+8}\varepsilon\delta$$

$$\begin{pmatrix} \frac{d\delta r_2}{dL} \\ \frac{d\delta u_2}{dL} \\ \frac{d\delta v_2}{dL} \end{pmatrix} = \begin{pmatrix} 2-\frac{n+2}{n+8}\varepsilon & (n+2)C & 3C \\ 0 & -\varepsilon & -\frac{6\varepsilon}{n+8} \\ 0 & 0 & -\frac{4-n}{n+8}\varepsilon \end{pmatrix}\begin{pmatrix} \delta r_2 \\ \delta u_2 \\ \delta v_2 \end{pmatrix}$$

Egenverdiene er

$$y_{2t} = 2 - \frac{n+2}{n+8}\varepsilon \quad ; \quad y_{2u} = -\varepsilon \quad ; \quad y_{2v} = -\frac{4-n}{n+8}\varepsilon$$

③ Tilsvarende ②:

$$\frac{d\delta r_3}{dL} = 2\delta r_3 + C\left((n+2)\delta u_3 + 3\delta v_3\right) - C\delta r_3 \cdot 3v_3$$

$$= \left(2 - \frac{\varepsilon}{3}\right)\delta r_2 + (n+2)C\delta u_3 + 3C\delta v_3$$

$$\frac{d\delta u_3}{dL} = \varepsilon\delta u_3 - C\left(6v_3\delta u_3\right)$$

$$= \left(\varepsilon - \frac{2}{3}\varepsilon\right)\delta u_3 = \frac{\varepsilon}{3}\delta u_3$$

$$\frac{d\delta v_3}{dL} = \varepsilon\delta v_3 - C\left(18v_3\delta v_3 + 12v_3\delta u_2\right)$$

$$= -\frac{4}{3}\varepsilon\,\delta u_2 + (\varepsilon - 2\varepsilon)\delta v_3 = -\frac{4}{3}\varepsilon\,\delta u_2 - \varepsilon\,\delta v_3$$

$$\begin{pmatrix} \frac{d\delta r_3}{dL} \\ \frac{d\delta u_3}{dL} \\ \frac{d\delta v_3}{dL} \end{pmatrix} = \begin{pmatrix} 2-\frac{\varepsilon}{3} & (n+2)C & 3C \\ 0 & \frac{\varepsilon}{3} & 0 \\ 0 & -\frac{4}{3}\varepsilon & -\varepsilon \end{pmatrix} \begin{pmatrix} \delta r_3 \\ \delta u_2 \\ \delta v_3 \end{pmatrix}$$

$$\Rightarrow y_{3t} = 2 - \frac{\varepsilon}{3} \quad ; \quad y_{3u} = \frac{1}{3}\varepsilon \quad ; \quad y_{3v} = -\varepsilon$$

(4) $$u_4 = \frac{\varepsilon}{3nC} \; ; \; v_4 = -\frac{4-n}{9nC}\varepsilon \; ; \; r_4 = -\frac{n-1}{3n}\varepsilon$$

$$\frac{d\delta r_4}{dL} = 2\delta r_4 - C\,\delta r_4\left((n+2)\frac{\varepsilon}{3nC} - 3\,\frac{4-n}{9nC}\varepsilon\right)$$
$$+ C\left((n+2)\delta u_4 + 3\delta v_4\right.$$

$$= \left[2 - \frac{\varepsilon}{3n}\left(n+2-4+n\right)\right]\delta r_4 + (n+2)\,C\,\delta u_4 + 3C\delta v_4$$

$$= \left(2 - \frac{2}{3n}(n-1)\varepsilon\right)\delta r_4 + (n+2)C\,\delta u_4 + 3C\delta v_4$$

$$\frac{d\delta u_4}{dL} = \varepsilon\,\delta u_4 - C\left(2(n+8)\frac{\varepsilon}{3nC}\,\delta u_4 + 6\left(-\frac{4-n}{9nC}\right)\varepsilon\,\delta u_4 + \frac{2\varepsilon}{nC}\,\delta v_4\right)$$

$$= \left[\varepsilon - \frac{2(n+8)}{3n}\varepsilon + \frac{2(4-n)}{3n}\varepsilon\right]\delta u_4 - \frac{2\varepsilon}{n}\delta v_4$$

$$= \frac{\varepsilon}{3n}\left(3n - 2n - 16 + 8 - 2n\right)\delta u_4 - \frac{2\varepsilon}{n}\delta v_4$$

$$= -\frac{n+8}{3n}\varepsilon\,\delta u_4 - \frac{2\varepsilon}{n}\delta v_4$$

$$\frac{d\delta v_4}{dL} = \varepsilon\,\delta v_4 - C\left(18\left(-\frac{4-n}{9nC}\right)\varepsilon\,\delta v_4 + 12\frac{\varepsilon}{3nC}\delta v_4 + 12\left(-\frac{4-n}{9nC}\right)\delta u\right)$$

$$= +\frac{4}{3}\,\frac{4-n}{n}\varepsilon\,\delta u_4 + \varepsilon\left[1 + \frac{2(4-n)}{n} - \frac{4}{n}\right]\delta v_4$$

$$\frac{n+8-2n-4}{n} = -\frac{n-4}{n}$$

$$= \frac{4}{3}\,\frac{4-n}{n}\varepsilon\,\delta u_4 + \frac{4-n}{n}\varepsilon\,\delta v_4$$

$$\begin{pmatrix} \dfrac{d\delta r_4}{dL} \\[2ex] \dfrac{d\delta u_4}{dL} \\[2ex] \dfrac{d\delta v_4}{dL} \end{pmatrix} = \begin{pmatrix} 2-\frac{2}{3n}(n-1)\varepsilon & (n+2)C & 3C \\[1ex] 0 & -\frac{n+8}{3n}\varepsilon & -\frac{2\varepsilon}{n} \\[1ex] 0 & \frac{4}{3}\frac{4-n}{n}\varepsilon & \frac{4-n}{n}\varepsilon \end{pmatrix} \begin{pmatrix} \delta r_4 \\[2ex] \delta u_4 \\[2ex] \delta v_4 \end{pmatrix}$$

Termisk egenverdi $y_{4t} = 2 - \frac{2}{3n}(n-1)\varepsilon$

De to andre:

$$\begin{vmatrix} -\frac{n+8}{3n}\varepsilon - y & -\frac{2\varepsilon}{n} \\[1ex] \frac{4}{3}\frac{4-n}{n}\varepsilon & \frac{4-n}{n}\varepsilon - y \end{vmatrix} = 0 \qquad y = \frac{\varepsilon}{3n} Y$$

$$\begin{vmatrix} -(n+8) - Y & -6 \\[1ex] 4(4-n) & 3(4-n) - Y \end{vmatrix} = 0$$

$$\big(-(n+8) - Y\big)\big(3(4-n) - Y\big) + 24(4-n) = 0$$

$$Y^2 + (n+8-12+3n)Y + 3[32 - 8n - 4n \mp 32 + n^2 + 8n] =$$

$$Y^2 + 4(n-1)Y + 3(n^2 - 4n) = 0$$

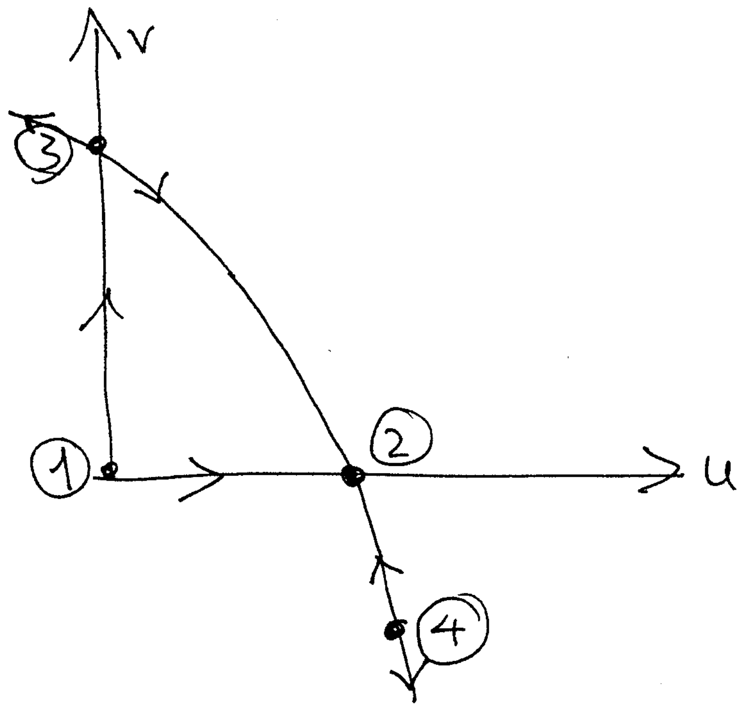
$$Y = -2(n-1) \pm \sqrt{4(n-1)^2 - 3n^2 + 12n} = -2(n-1) \pm (n+2)$$

$$4n^2 - 8n + 4 - 3n^2 + 12n = n^2 + 4n + 4$$

$$Y = \begin{cases} -2n+2+n+2 = -n+4 \\ -2n+2-n-2 = -3n \end{cases}$$

$$y = \frac{\varepsilon}{3n} Y = \begin{cases} \frac{4-n}{3n}\varepsilon \\ -\varepsilon \end{cases}$$

$\boxed{\text{Derved:}}$



Gauss fikspunktet er (for $\varepsilon > 0$, dvs $d < 4$) frastøtende i alle retn.

Det <u>isotrope</u> fikspunktet ② er <u>tiltrekkende</u> i (u,v)-planet (frastøtende i temperatur-retn.) innen en <u>sektor</u> gitt av ③ & ④

③ & ④ er begge tiltrekkende i én, frastøtende i én retning. <u>Utenfor</u> sektoren som tiltrekkes til ② gir dynamikken drift ut av området for (r,u,v) kont. faseovergang $\Rightarrow$ 6. ordens ledd nødvendig $\Rightarrow$ 1. ordens overgang.

$\boxed{\text{For mye anisotropi gir 1. ordens overgang!}}$