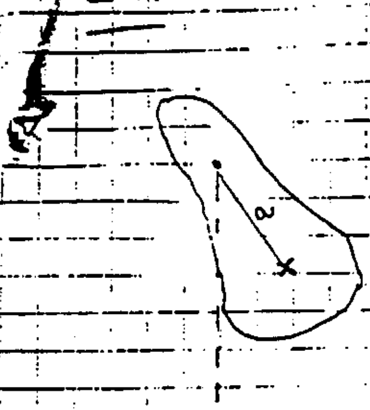
L.1.




$$I = I_0 + Ml^2 = M(l^2 + a^2) \quad \text{(Steiners sats)}$$

$$I\alpha_z = N_z = -Mga\sin\varphi \approx -Mga\varphi$$

$$\frac{d^2\varphi}{dt^2} = -\frac{Mga}{I}\varphi = -\omega_0^2\varphi$$

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{I}{Mga}} = 2\pi\sqrt{\frac{l^2+a^2}{ga}}$$

b) $T_A = 2\pi\sqrt{\dfrac{l^2+a^2}{ga}}$ $\quad T_B = 2\pi\sqrt{\dfrac{l^2+b^2}{gb}}$

$$T_A = T_B \Rightarrow (l^2+a^2)b = (l^2+b^2)a$$

$$l^2(b-a) = ab(b-a) \qquad \underline{l^2 = ab}$$

$$T_0 = T_A = T_B = 2\pi\sqrt{\frac{ab+a^2}{ga}} = 2\pi\sqrt{\frac{a+b}{g}} = 2\pi\sqrt{\frac{x_0}{g}}$$

$$\underline{g = 4\pi^2\frac{x_0}{T_0^2}}$$

L.2

a) $M\frac{d^2Z}{dt^2} = M\frac{Z}{L}g \qquad \underline{\frac{d^2Z}{dt^2} = \frac{g}{L}Z = \alpha^2 Z}$

$Z = Ae^{\alpha t} + Be^{-\alpha t}$

Grensebet.: $\left.\begin{array}{l} Z_0 = Z(0) = A+B \\ 0 = \left.\frac{dZ}{dt}\right|_{t=0} = \alpha(A-B) \end{array}\right] \Rightarrow A = B = Z_0/2$

$\underline{Z(t) = Z_0 \cosh \alpha t} \qquad \underline{v(t) = Z_0\alpha \sinh \alpha t} \qquad \alpha = \sqrt{\frac{g}{L}}$

b) $(Z/Z_0)^2 - (v/Z_0\alpha)^2 = \cosh^2\alpha t - \sinh^2\alpha t = 1 \Rightarrow$

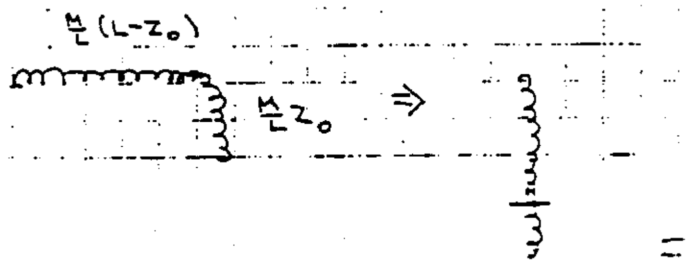
$Z^2 - (v/\alpha)^2 = Z_0^2 \qquad \underline{v = \alpha\sqrt{Z^2 - Z_0^2}}$

Kontroll: $Z = Z_0 \Rightarrow v = 0$.

Når kjeden forlater bordet er $Z = L$:

$\underline{v = \sqrt{\frac{g}{L}(L^2 - L_0^2)}}$ (Fritt fall videre)

c)




Energibevarelse: $K = -\Delta U \Rightarrow$

$\frac{1}{2}Mv^2 = \frac{M}{L}(L-Z_0)\cdot\frac{1}{2}(L-Z_0)\cdot g + \frac{M}{L}Z_0\cdot(L-Z_0)\cdot g$

$= \frac{Mg}{L}(L-Z_0)\left[\underbrace{\frac{1}{2}(L-Z_0)+Z_0}_{\frac{1}{2}(L+Z_0)}\right] = \frac{Mg}{2L}(L^2 - Z_0^2)$

$\underline{v = \sqrt{\frac{g}{L}(L^2 - L_0^2)}}$ som over