NTNU Trondheim, Institut for fysikk

Examination for FY2450 Astrophysics 1

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Acceptable languages for your answers: Bòkmal, English, Italian, Nynorsk.
Allowed tools: Pocket calculator, mathematical tables
Some formulas and numerical values of constants can be found at the end.
Grades: 29.6.2006

1. Stars.

a. A star has the twice the temperature of the Sun and a luminosity 64 times that of the Sun. Roughly, how large is the star compared to the Sun? (1 pt)

Approximating a star as blackbody radiator, $L = 4\pi R^2 \sigma T^4$, it follows

$$\frac{R}{R_\odot} = \left(\frac{L}{L_\odot}\right)^{1/2} \left(\frac{T_\odot}{T}\right)^2 = 8 \times 1/4 = 2\,.$$

b. Photons produced in the center of the Sun escapes from the Sun in roughly: (1 pt)

☐ 2.3 seconds
☐ 8.3 minutes
☐ a month
☐ a year
⊗ 200 000 years.

Since the interaction length $l_{\rm int}$ of photons is much smaller than $R_\odot$ and they are scattered isotropically, they perform a random walk and need much longer than $R_\odot/c \sim 2.3$ s. For an estimate see solutions to exercise sheet of week 3.

c. A star is stabilized by a pressure gradient from its center to its boundary. What is the main source of pressure in a

i) main-sequence star, (0.5 pt)
ii) white dwarf star, (0.5 pt)
iii) neutron star. (0.5 pt)

The E.o.S of an main-sequence star is close to the one of an ideal gas; main contribution to the pressure and its gradient are photons ("radiation pressure") traveling outward and the thermal motion of the gas.The energy source to both are in turn fusion reactions in the center.

The main contribution to the pressure of a white dwarf star comes from a degenerate electron gas, while a degenerate neutron gas dominates the pressure in a neutron star. (The Pauli principle forbids that fermions can occupy the same quantum state; thus fermions need to occupy higher momentum states even at $T = 0$, if pushed together, because of Heisenberg's uncertainty principle.)

2. The Sun and nuclear fusion.

The energy flux from the Sun above the Earth atmosphere is $\mathcal{F} = 1370$ W/m^2. The energy

emitted by the Sun is produced by fusion, $4p + 2e^- \rightarrow {}^4\text{He}+2\nu + 26.2\,\text{MeV}$.
a. Estimate the luminosity $L_\odot$ of the Sun. (2 pt)
b. How big is the neutrino flux at the Earth, i.e. how many neutrinos cross an area of unit size per unit time at the Earth distance? (1 pt)

a. Since $\mathcal{F}$ is given at the distance of the Earth, we have to use $d = 1\ AU$ in $L = 4\pi d^2\mathcal{F}$. Then

$$L_\odot = 4\pi(1.5 \times 10^{11}m)^2 \times 1370W/m^2 = 3.85 \times 10^{33}erg/s = 3.85 \times 10^{26}J/s.$$

b. The number of emitted neutrinos per time is

$$\dot{N}_\nu = 2L_\odot/E_b = \frac{2 \times 3.85 \times 10^{26}J/s \times 6.24 \times eV/J}{26.2 \times 10^6 eV} = 1.8 \times 10^{38}s^{-1},$$

the neutrino flux at the distance of the Earth

$$\phi_\nu = \frac{\dot{N}_\nu}{4\pi d^2} = \frac{1.8 \times 10^{38}s^{-1}}{4\pi(1.5 \times 10^{11}m)^2} = 6.6 \times 10^{14}m^{-2}s^{-1}$$

(Alternative: directly from $2\mathcal{F}/26.2MeV$).

3. Virial theorem.

The virial theorem, $-2U_{\rm kin} = U_{\rm pot}$, relates the total kinetic energy $U_{\rm kin}$ to the potential energy $U_{\rm pot} = \alpha GM^2/D$ of a gravitationally bound system. The mean radial velocity of stars in a cluster with size $D = 5$ pc is measured via Doppler-shift as $\langle v_{\rm r}^2\rangle \approx (16\text{km/s})^2$.
a. Find the total mass M of this cluster using $\alpha = 3/5$. (1.5 pts)
b. Does the mean velocity of the gravitationally bound stars remains constant, increases or decreases with time? Give a brief argument. (1 pt)

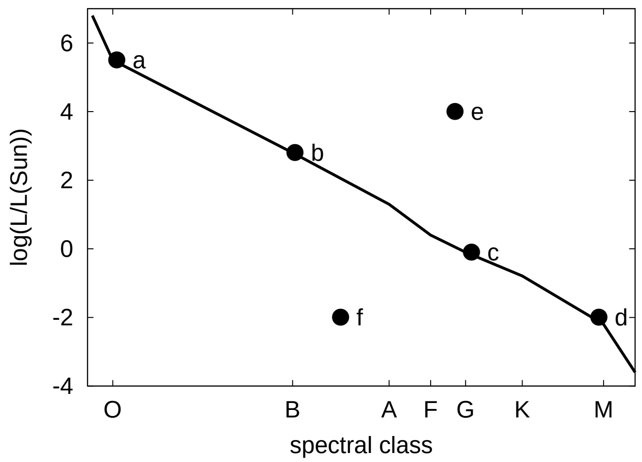
a. As told at the exam, a minus sign in $U_{\rm pot}$ or α is missing.
With $3\langle v_{\rm r}^2\rangle = \langle v^2\rangle$, it follows

$$M = \frac{5D\langle v_{\rm r}^2\rangle}{G} = \frac{5 \times 5 \times 3.07 \times 10^{16}m \times (16.000m/s)^2}{6.67 \times 10^{-11}m^3kg^{-1}s^{-2}} = 3 \times 10^{36}kg$$

b. A gravitationally bound system like a star cluster has no true equilibrium state: Stars with $v \geq v_{esc}$ escape from the cluster, and the cluster contracts to a new "quasi-equilibrium". Half of the released potential energy is used to heat-up the stars $\Rightarrow$ mean velocity of stars in the cluster increases slowly.
(An simple example is the space shuttle: to reach a lower orbit, it brakes and becomes thereby faster.)

3. Hertzspung-Russel diagram.

Use the schematic Hertzsprung-Russell shown below to answer the following questions:



a. Of the following choices, which best represents the Main Sequence? (1 pt)

☐ b and c

⊗ a, b, c, and d

☐ c and d

☐ e

☐ f

b. Of the following, the star with the largest radius is: (1 pt)

☐ b

☐ c

☐ d

⊗ e

☐ f

c. Of the following, the star with the largest mass is: (1 pt)

⊗ a

☐ b

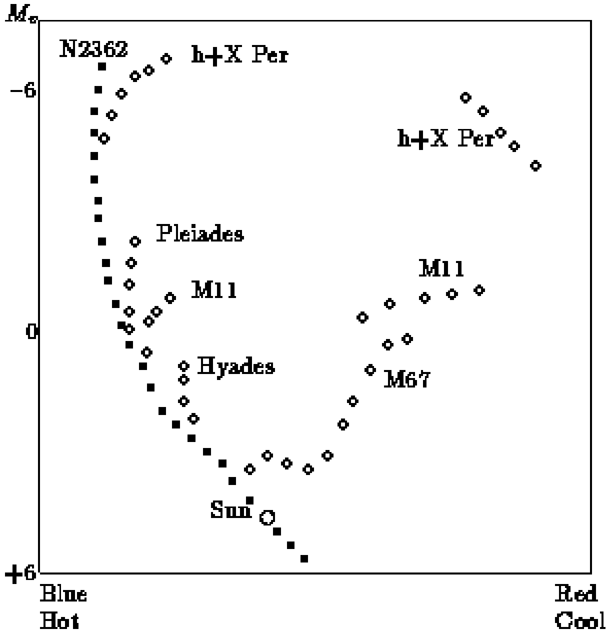
☐ c

☐ d

☐ f

4. Hertzspung-Russel diagram of star clusters.

The figure below shows a schematic Hertzsprung-Russell of several star clusters.



Of the following choices, which one is the youngest? (1 pt)

⊗ N2362
☐ h+X Per
☐ M11
☐ M67
☐ Hyades
☐ Pleiades

6. Cosmology.

The acceleration equation for a homogeneous, isotropic Universe is

$$\frac{\ddot{R}}{R} = \pm\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

a. Explain the meaning of $\dot{R}/R$, $\ddot{R}$ and Λ. (1.5 pts)

b. Give the correct sign in front of $4\pi G/3(\rho + 3p)$ and explain your choice. (1 pt)

b. The best estimate for the importance of different energy forms today is $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$ and $\Omega_{\rm rad} = 4.5 \times 10^{-5}$. Which energy form will dominate for $t \to \infty$? (1 pt)

c. Derive the time-dependence of the scale factor $R(t)$ for $p = \rho = 0$. (1 pt)

d. The temperature of the cosmic microwave background (CMB) is $T \approx 2.7$K today. (Equivalently, the mean energy of CMB photon is today $\langle E \rangle = 2.3 \times 10^{-4}$eV). What will be the temperature T (or the mean energy $\langle E \rangle$) of the CMB photons, when the expansion of the universe increased distances by a factor 10? (1 pt)

d. The current expansion velocity of the Universe is 70km/s/Mpc. How big is the energy density $\rho_{\rm m}$ of matter in the Universe (with $\Omega_m = \rho_{\rm m}/\rho_{\rm cr} = 0.3$, where $\rho_{\rm cr}$ is the energy density required for a flat universe)? (1.5 pts)

a. The Hubble parameter $H = \dot{R}/R$ determines via Hubble's law the expansion velocity of the (nearby) universe (and gives a first estimate for its age $\sim 1/H_0$, $\ddot{R}$ gives the (de-) acceleration of the expansion, Λ is the cosmological constant, i.e. a specific form of energy with the EoS $p = -\rho$.
b. The correct sign can be determined by a comparison with Newtonian gravity. There only ρ ("matter") enters, is attractive, i.e. decelerate the expansion $\Rightarrow$ negative sign is correct.
c. The universe expands forever for these parameters. Then $\rho_{rad} \propto 1/R^4 \to 0$, $\rho_m \propto 1/R^3 \to 0$, while $\rho_\Lambda = const.$ $\Rightarrow$ the cosmological constant dominates.
d. $\ddot{R}(t) = (\Lambda/3)R(t)$ and thus $R(t) = R_0 \exp(\sqrt{\Lambda/3}(t - t_0))$ $\Rightarrow$ exponential expansion (for $\Lambda > 0$).
e. The wave-length of photon is stretched with the expansion of the universe. $E \propto 1/\lambda \propto 1/R$ and thus $E = 2.3 \times 10^{-5} eV$. The same holds for the temperature of the Planck distribution, $\Rightarrow T \approx 0.27K$
f. In a Newtonian picture, a flat universe corresponds to $E = E_{kin} + E_{pot} = 0$ or $\rho_{cr} = 3H_0^2/(8\pi G)$. Then

$$\rho_m = 0.3\rho_{cr} = 0.3 \times \frac{3 \times (2.28 \times 10^{-18} s^{-1})^2}{8\pi \times 6.674 \times 10^{-11} m^3 kg^{-1} s^{-2}} = 2.8 \times 10^{-27} kg/m^3.$$

Some constants, parameters and formula.

Astronomical Unit	1 AU= 1.496×10^{13} cm
Parsec	1 pc = 3.086×10^{18} cm = 3.261 ly
Solar radius	$R_\odot = 6.960 \times 10^{10}$ cm

Gravitational constant	$G_N = 6.674 \times 10^{-11}\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2} = 6.674 \times 10^{-8}\text{cm}^3\,\text{g}^{-1}\,\text{s}^{-2}$
Stefan-Boltzmann constant	$\sigma = (2\pi^5 k^4)/(15c^2h^3) \approx 5.7 \times 10^{-8}$ W m^{-2} K^{-4}g
	$= 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4}

1 J = 10^7 erg = 6.242×10^{18}eV

Stefan-Boltzmann law—*Stefan–Boltzmanns lov*: $\mathcal{F} = \sigma T^4$