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**EKSAMEN I EMNE FY3006 Målesensorer/transdusere**

Mandag 25. mai 2009

Tid: kl 09.00-13.00

*Hjelpemidler:* B1 – Typegodkjent kalkulator, med tomt minne, i henhold til liste utarbeidet av  
NTNU tillatt

Ingen trykte eller håndskrevne hjelpemidler tillatt

Sensuren faller i uke 25.

*You are required to answer all three problems in this paper.*

*Some useful formulae are listed on the last page.*

### Problem 1

You are supposed to be familiar with the construction and operation of a photomultiplier tube (PMT). Let us denote the potential difference between successive dynodes as  $V_{DD}$ , that between  $K$  and the first dynode ( $D1$ ) as  $V_{K1}$ , and that between the last dynode ( $DL$ ) and  $A$  as  $V_{LA}$ . We will suppose, for the purpose of this exercise, that the distribution of the energies of the secondary electrons ejected by a particular dynode can be expressed as follows:

$$N(w) = \frac{N_t}{w_0} e^{-w/w_0}, \quad (1)$$

where  $w$  denotes the energy of an electron after it has been ejected,  $w_0$  is the work function of the material and  $N_t$  is the total number of ejected electrons. Note that this distribution is properly normalized, so that

$$\int_0^\infty N(w) dw = N_t. \quad (2)$$

We will now make the following simplifying assumptions:–

1. A primary photoelectron leaves  $K$  with zero kinetic energy.
2.  $V_{K1} = V_{DD} = V_{LA}$ .

Answer the questions listed below:–

1. Assume that a photoelectron leaves  $K$  with zero kinetic energy. Apply conservation of energy to show that the number of secondary electrons emitted by  $D1$  is

$$N_t = \frac{eV_{K1}}{2w_0}. \quad (3)$$

2. Find an expression for  $\bar{E}$ , the average energy acquired by an electron after it is ejected.
3. For this part of the problem, we will introduce a few more simplifying assumptions:
  - A secondary electron leaves a dynode with zero kinetic energy.
  - $V_{K1} = V_{DD} = V_{LA}$ .
  - The electron trajectory from  $K$  to  $A$  may be divided into  $N = L + 1$  segments, each of length  $s$ .

What is meant by the term *electron transit time*? We will use the symbol  $\Delta$  to denote this quantity. Show that  $\Delta = GU^p$ , where  $U$  is the potential difference between  $A$  and  $K$ . Find the value of  $p$ , and express the proportionality constant  $G$  in terms of the electronic charge-to-mass ratio and some (or all) of the quantities introduced above.

4. Take a look now at Fig. 1. Make a reasonable guess for the value of  $s$ , and use the data given below to estimate  $\Delta$ :

Electronic charge:  $e = 1.6 \times 10^{-19} \text{ C}$

Electronic mass:  $m = 9.1 \times 10^{-31} \text{ kg}$

Cathode-to-Anode

potential difference:  $U = 1.5 \text{ kV}$

Explain how you arrived at your estimate of  $s$ .

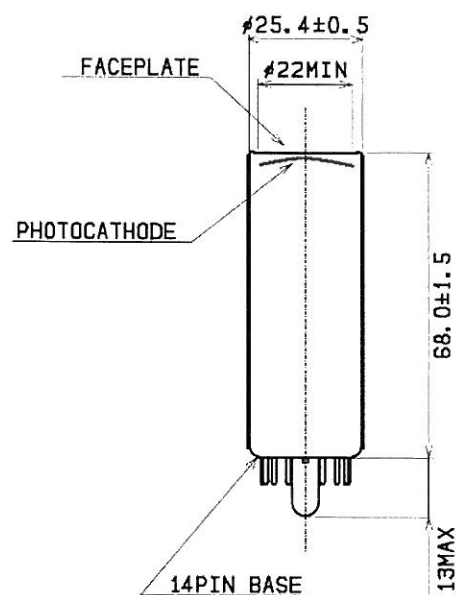
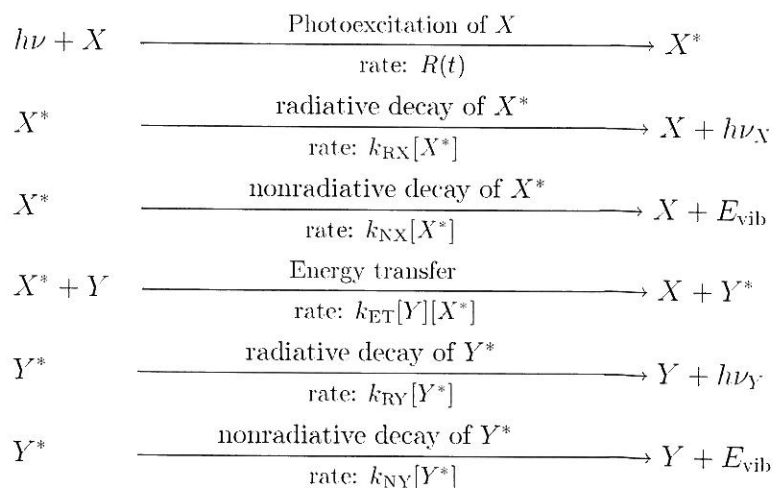


Figure 1: Physical dimensions (in mm) of a commercial PMT with ten dynodes

## Problem 2

We begin by listing the processes occurring in an oxygen-free solution comprising a primary solute  $X$  (the donor) and a secondary solute  $Y$  (the acceptor) dissolved in a transparent inert solvent. The system is initially excited by monochromatic radiation that is absorbed by only  $X$ . Asterisks denote molecules in the first excited singlet state, square brackets denote molar concentrations, and  $E_{\text{vib}}$  signifies vibrational energy.



You will notice that the triplet states ( $X^\dagger$  and  $Y^\dagger$ ) are not included in the above scheme. We will return to this point; until then you may assume that the quantum yields of triplet formation are negligible for the molecules under consideration.

*Your tasks are listed below:–*

1. Let us write down the rate equations governing the time-dependence of  $[X^*]$  and  $[Y^*]$  in order to reserve numbers for these equations:

$$\frac{d[X^*]}{dt} = R(t) - \alpha[X^*] \quad (4)$$

$$\frac{d[Y^*]}{dt} = S(t) - \beta[Y^*] \quad (5)$$

Express  $S(t)$ ,  $\alpha$  and  $\beta$  in terms of the symbols appearing in the above reaction scheme by completing the following equations:

$$S = ? \quad (6)$$

$$\alpha = ? \quad (7)$$

$$\beta = ? \quad (8)$$

2. For the rest of this problem, we will consider sinusoidal excitation in which  $I$ , the intensity of the excitation source, is modulated in a sinusoid manner described by the equation:

$$I = I_0(1 + m_0 \cos \omega t), \quad (9)$$

where  $m_0$  is the depth of modulation,  $\omega$  is the modulation frequency, and  $I_0$  is the average value of  $I$ . You may assume, as we have done throughout this course, that the rate of formation of  $X^*$  is given by  $R = AI$ , where  $A$  is a constant which you need not worry about. We will introduce some additional notation for the sake of making subsequent equations much tidier:

$$\phi_X = \arctan(\omega/\alpha), \quad \lambda_X = \frac{\alpha}{(\omega^2 + \alpha^2)^{1/2}}; \quad (10)$$

$$\phi_Y = \arctan(\omega/\beta), \quad \lambda_Y = \frac{\beta}{(\omega^2 + \beta^2)^{1/2}}. \quad (11)$$

Show that  $F_Y$ , intensity of the acceptor fluorescence, can be expressed as

$$F_Y = B\{1 + m \cos(\omega t + \phi)\}. \quad (12)$$

3. Assume that  $\omega$  can be made as large or as small as you desire, and that  $\omega$  can be measured with negligible error. You may also take it for granted that a lock-in amplifier and a reference signal (a square wave of period  $T = 2\pi/\omega$ ) are available. What steps would you take for determining  $\beta$ ? List the additional major items of equipment you would use, and explain your reasoning fully. Identify the sources of error in your determination. (A complete error analysis is not needed.)
4. Consider the case  $\beta \gg \alpha$ . Would the value of  $\beta$  determined by you be as reliable as in the case  $\beta \approx \alpha$ ? Notice that this question can be answered more easily on the basis of the differential equations, Eqs. (4) and (5), rather than by examining their solutions.
5. We have neglected the formation of  $X^\dagger$  and  $Y^\dagger$  so far. Argue that, even if triplet formation yields are far from negligible, the above analysis would remain valid despite the fact that the solutions are oxygen-free.

### Problem 3

**Part a:** The gauge factor of strain gauges is determined by observing the fractional change in resistance of a gauge when subjected to strain. We will now investigate how this can be done by using a Wheatstone bridge. Examine the circuit shown in Fig. (2a) below. Here  $P$  and  $Q$  are "reference" strain gauges with a known gauge factor  $g$ , and are attached to opposite surfaces of the beam of a calibrating apparatus.  $R$  is the "test" strain gauge, attached to the same surface of the beam as  $P$ , and having a gauge factor  $f$ ;  $S$  is a balancing resistance. The resistors  $M$  and  $N$  (called ganged resistors) are such that their values, though variable, are always equal to each other; let their initial values be  $M_0$  and  $N_0$ .

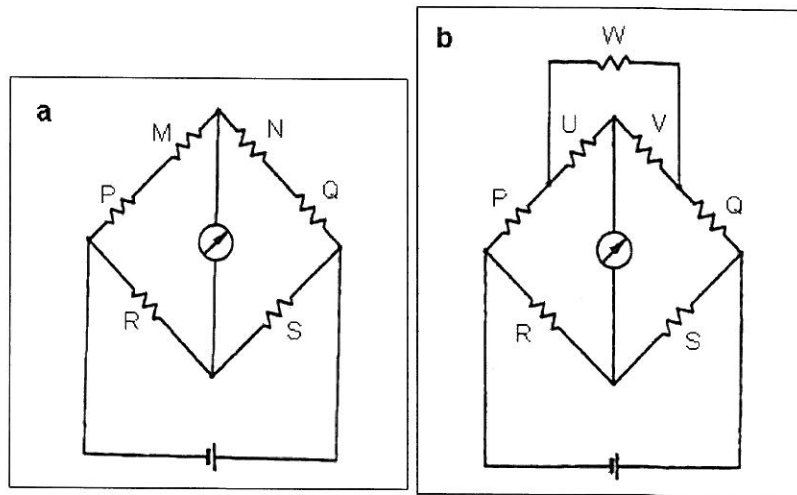


Figure 2: Wheatstone bridge configurations for Problem 3

Now proceed as follows:

1. Assume that the bridge is balanced initially when  $M_0 = N_0$ ,  $P = Q$ , and  $R = S$ ; write down the condition of balance. Call it Eq. (13). Assume next that when the beam is bent,  $P$  and  $R$  undergo strain  $\varepsilon$  (say) whilst  $Q$  undergoes strain  $-\varepsilon$ . The bridge is rebalanced by changing the ganged resistors (keeping  $M = N$ ); write down the new condition of balance, and call it Eq. (14).
2. Show that Eqs. (13) and (14) lead to the following result:

$$f = \frac{2Qg}{N + Q(1 - \varepsilon g)}. \quad (15)$$

Notice that if one could neglect the term  $\varepsilon g$  in the denominator, one would arrive at the very useful relation shown below:

$$f = \frac{2Qg}{N + Q}. \quad (16)$$

3. Justify the approximation made in obtaining Eq. (16) by showing that Eq. (15) leads, when  $\epsilon g \ll 1$ , to the following conclusion:

$$f \approx \frac{2Qg}{N+Q} + \text{correction term.} \quad (17)$$

Find the relation between the correction term and the parameters  $\varepsilon$  and  $f$ . Estimate the magnitude of the correction term for a metal wire strain gauge by using your knowledge of the typical magnitudes of  $\varepsilon$  and  $f$ . Why is Eq. (16) very useful?

*Part b:* In practice it is not easy to get suitable ganged resistors, and the circuit needs to be modified. Consider the modification shown in Fig. 2. Here  $U$  and  $V$  are equal resistors and  $W$  is a variable resistance. The new network can be regarded as equivalent to the old network if we manage to find suitable expressions for  $M$  and  $N$  in terms of  $U$ ,  $V$  and  $W$ .

1. Find the suitable expressions mentioned in the previous sentence and number your equations as Eq. (17).
2. Show that the unknown gauge factor can now be found by means of the relation

$$f = \frac{2Qg(2V+W)}{2QV+W(V+Q)}. \quad (18)$$

## Appendix I

*Taylor series:*

$$f(x+a) = f(x) + \sum_{n=1}^{\infty} \frac{a^n}{n!} \frac{df^n}{dx^n}. \quad (19)$$

*MacLaurin series:*

$$f(x) = f(0) + \sum_{n=1}^{\infty} \frac{x^n}{n!} \left. \frac{df^n}{dx^n} \right|_{x=0}. \quad (20)$$

*Two special cases are:*

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \quad (\text{all real values of } x), \quad (21)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \quad (\text{all real values of } x). \quad (22)$$

*Binomial series:*

$$(a+x)^n = a^n + na^{n-1}x + \frac{n(n-1)}{2!}a^{n-2}x^2 + \dots. \quad (23)$$

*Two special cases are:*

$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{2 \cdot 4}x^2 + \dots \quad -1 < x \leq 1, \quad (24)$$

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 + \dots \quad -1 < x \leq 1. \quad (25)$$

*Geometric series:*

$$1/(1-x) = 1 + x + x^2 + x^3 \dots. \quad (26)$$

*Euler's formula:*

$$e^{in\phi} = \cos n\phi + i \sin n\phi. \quad (27)$$

*Three corollaries of particular interest are:*

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} = i \sinh(ix) \quad \left[ \sinh y \equiv \frac{e^y - e^{-y}}{2} \right], \quad (28)$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} = \cosh(ix) \quad \left[ \cosh y \equiv \frac{e^y + e^{-y}}{2} \right]. \quad (29)$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x \quad (30)$$

*A useful integral:*

$$\int_0^{\infty} e^{-kx} dx = 1/k, \quad (k > 0). \quad (31)$$