NORGES TEKNISK-
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EKSAMEN I EMNE FY8901 Målesensorer/transdusere

EKSAMEN I EMNE FY3006 Målesensorer/transdusere

Mandag 20. desember 2010
Tid: kl 09.00–13.00

Hjelpemidler: B1 – Typegodkjent kalkulator, med tomt minne, i henhold til liste utarbeidet av NTNU tillatt

Ingen trykte eller håndskrevne hjelpemidler tillatt

Sensuren faller i uke 3.

Answer all three problems.

Some useful formulae are listed on the last page.

Problem 1 (25 marks)

Figure 1 depicts a photomultiplier tube (PMT) and the circuit elements involved in converting the anode current to a voltage signal. The connections between a high voltage power supply, the PMT bleeder chain and the dynodes have been omitted in order to avoid unnecessary clutter, and you are not required to draw these either. We will assume that a PMT may be regarded as an ideal current source. We will ignore the dark current, the spread in the electron transit time, and the influence of the instrument used for measuring the output of the PMT.

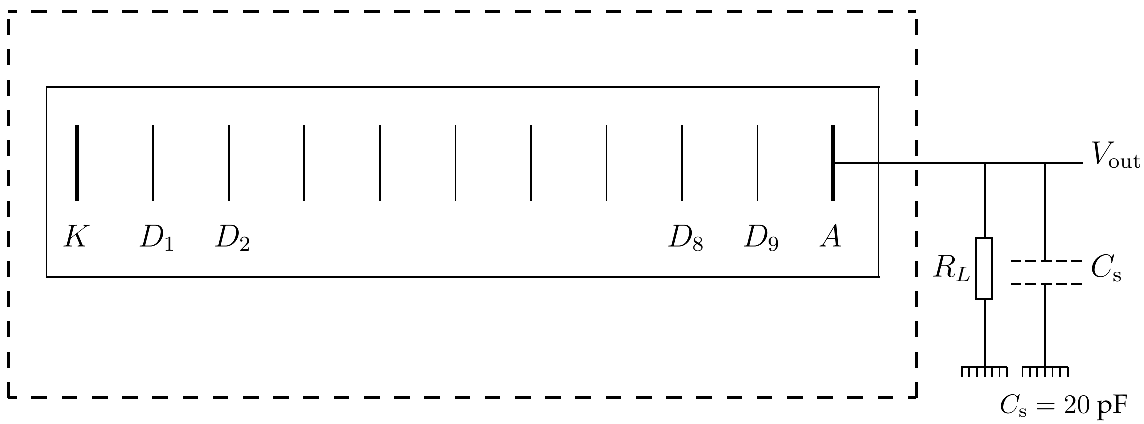

Figure 1: A photomultiplier tube with some ancillary components

a) Use elementary circuit theory to derive the differential equation satisfied by the the output voltage V_{out}. Express this equation in the form shown below:

$$\frac{\mathrm{d}V_{\mathrm{out}}}{\mathrm{d}t} + \alpha V_{\mathrm{out}} = \beta I_A, \qquad (\alpha, \beta = \text{constants}). \tag{1}$$

b) Assume that a triangular current pulse of duration T arrives at the anode at $t = 0$; its

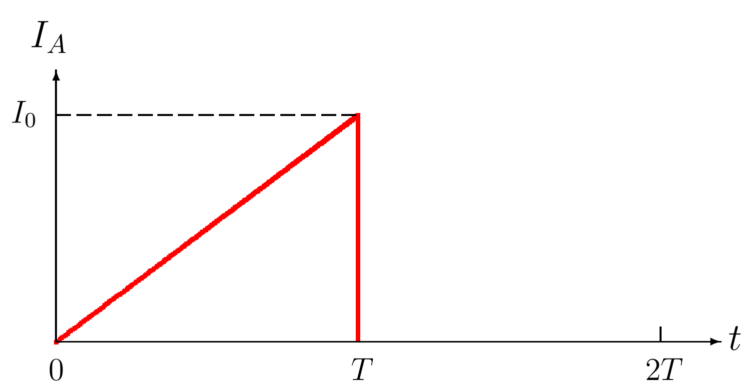

Figure 2: A triangular current pulse arriving at the anode of the PMT in Fig. 1.

shape is shown in the Fig. 2. Draw three sketches, showing the shapes of V_{out} for the following cases:

1. $R_L = 50\ \Omega$ and $T = 10$ ns,
2. $R_L = 5\ \text{K}\Omega$ and $T = 10$ ns,
3. $R_L = 5\ \text{K}\Omega$ and $T = 1\ \mu$s.

The horizontal scale should cover the interval $[0, 2T]$. Each sketch should be based on replacing Eq. (1) with an *approximate* equation for $0 \le t \le T$, and finding the solution of the approximate equation. (You will not get any credit for solving Eq. (1) and replacing the solution by approximations; you may, of course, use this approach for verifying your answers.)

Problem 2 (25 marks)

Figure 3 is a schematic representation of the mass and spring part of an accelerometer. A metallic beam is clamped at the centre to form two cantilevers, with concentrated end-loads. Two strain gauges are used on each cantilever, one on top and the other underneath.

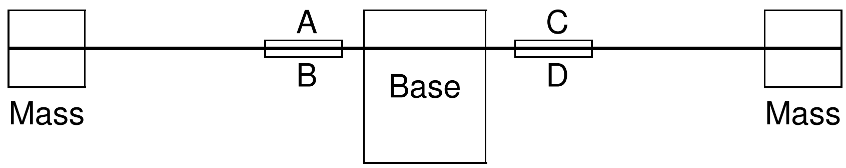

Figure 3: A mass-spring system used as an accelerometer.

The cantilevers are mounted in an aluminium box with an oil-tight cover plate, and filled with a viscous fluid; the clearance between the case and the cantilevers is designed to provide the necessary viscous damping. Figure 4 provides an overview with the cover plate removed. (*This problem can be solved even without Fig. 4; ignore the figure if it is reproduced poorly.*)

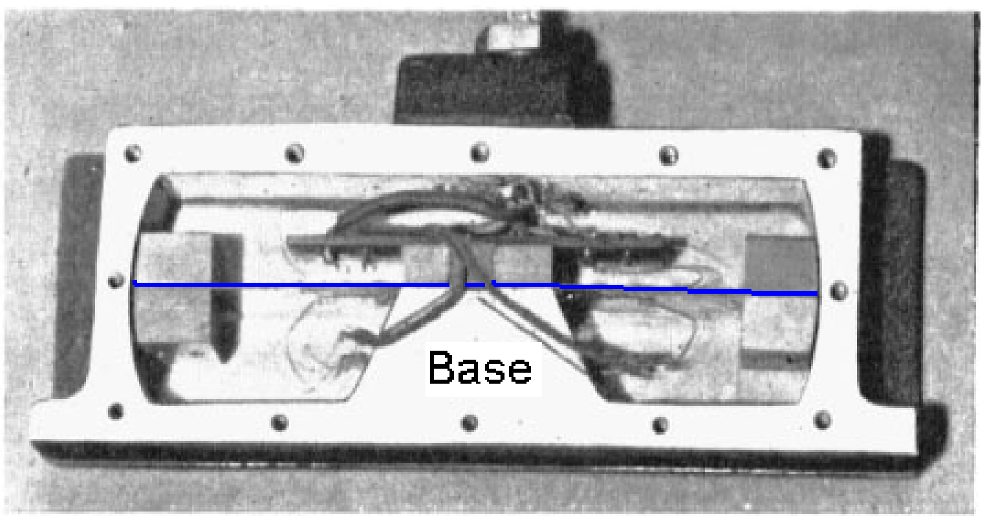

Figure 4: Internal view of the accelerometer.

Answer the following questions:–

1. Draw a diagram showing all four gauges (with their labels A, B, C, and D) forming a Wheatstone bridge with a sensitivity four times that which could be obtained from a single gauge. Present your analysis of the bridge circuit to support your claim.Your answer should start with an equation of the form shown below:

$$V_{\text{out}} = V_{\text{in}} \left[\frac{R_U}{R_U + R_V} - \frac{R_W}{R_W + R_Y} \right], \tag{2}$$

where U, V, W, and Y stand for one of the four labels (A, B, C, and D) used in Fig. 3. *There is no need for you to derive or justify this equation.*

2. With a properly connected bridge, the instrument depicted in Figure 3 would give automatic temperature compensation. Is this true for your bridge circuit? Justify your answer.

3. With a properly connected bridge, the instrument depicted in Fig. 3 would respond to acceleration in one direction only. If the acceleration is along the line of the cantilevers, the bridge will remain balanced. Is this statement valid for your bridge circuit? (N.B. Accelerations across the width of the beams do tend to twist them, but the effect is small, and you should neglect it.)

4. *For the rest of this problem*, base your answers on Eq. (3), which governs the time dependence of the amplitude of vibration of the sensor under consideration:

$$m\ddot{y} + b\dot{y} + Ky = ? \tag{3}$$

   The equation is left incomplete because you are supposed to be familiar with the dynamic behaviour of second-order sensors. If $b^2/(mK)$ is very close to 2, the response of the instrument to a frequency $\omega = \omega_{\mathrm{n}}/2$ will be within 3% of the response at a much lower frequency (say, $\omega = \omega_{\mathrm{n}}/100$). True or false? Justify your answer on the basis of Eq. (10) on the last page. (Here ω_{n} stands for the natural frequency of vibration of the accelerometer.)

5. In order to produce an instrument with as high a sensitivity as possible for a particular application, it is necessary to design the accelerometer to have as low a natural frequency as possible. True or false? Justify your answer. Can you think of a disadvantage associated with an instrument that has a low natural frequency?

Comment

Your answers to questions 1–3 should be based on the changes (if any) in the resistances of the different gauges, and Eq. (2). You may find it convenient to use some or all of the text given below:–

1. When all gauges are unstressed, each will have a resistance equal to R, say. If only one gauge, say ..., is bonded to the accelerometer, its resistance will change from R to .... Eq. (2) now becomes

$$V_{\mathrm{out}} = \ldots.$$

   When all four gauges are used, the resistances of .... If the base moves towards the top of the page, ... In this case, Eq. (2) leads to the following expression for the output

$$V_{\mathrm{out}} = \ldots$$

2. A change in temperature will .... When this information is inserted into Eq. (2), one gets

$$V_{\text{out}} = \ldots$$

3. If the acceleration is to the right, ...

Problem 3 (Part a: 15 marks; Part b: 10 marks)

Part a: Consider the arrangement shown in Fig. 5. We will assume that the wavelength indicator of the monochromator is calibrated in nm, and the bandwidth of the light leaving the slit is independent of the wavelength setting. Introduce the following notation: $E_\lambda \, \mathrm{d}\lambda$ is the relative energy output per unit wavelength interval, and $Q_\lambda \, \mathrm{d}\lambda$ is the relative number of quanta per unit wavelength interval. Answer the following questions. Provide enough

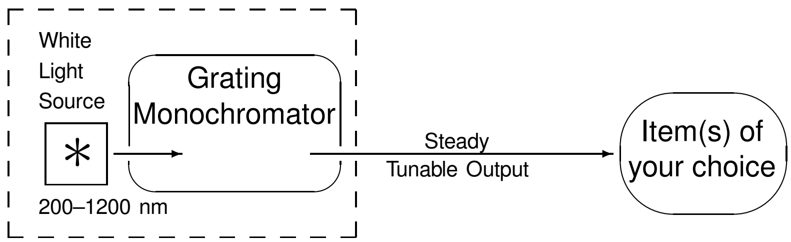

Figure 5: An arrangement for recording the spectral output of a tunable light source.

practical details to show that you know more than just the name of the device(s); mention, in particular, the underlying principle(s) and any restrictions on the wavelength region that can be sensed by your selected item(s).

1. What item(s) will you choose for measuring E_λ? If you can think of more than one solution, mention them all.

2. Assume that whatever sensor you chose above has a low sensitivity; what measures will you take for improving the signal-to-noise ratio. If you can think of more than one solution, mention them all.

3. What item(s) will you choose for measuring Q_λ? If you can think of more than one solution, mention them all.

4. What is the relation between E_λ and Q_λ?

5. What is the relation between Q_λ and Q_σ? Here $\sigma = 1/\lambda$ is the wavenumber, and $Q_\sigma \, \mathrm{d}\sigma$ denotes the relative number of quanta emitted in the wavenumber interval between σ and $\sigma + \mathrm{d}\sigma$.

Part b: Consider the following reaction scheme, which refers to a binary organic solution scintillator designed for detecting β-particles; here X and Y denote the solvent and the fluorescent solute, respectively. We will assume that no quenching species are present.

Process			Rate constant	Units
0.	X^*	$\longleftarrow X^0 + \beta$		
1.	X^*	$\longrightarrow X^0 + h\nu'$	k_1	s^{-1}
2.	X^*	$\longrightarrow X^0 + \text{heat}$	k_2	s^{-1}
3.	$X^* + Y^0$	$\longrightarrow X^0 + Y^*$	k_3	$M^{-1}\ s^{-1}$
4.	Y^*	$\longrightarrow Y^0 + h\nu''$	k_4	s^{-1}
5.	Y^*	$\longrightarrow Y^0 + \text{heat}$	k_5	s^{-1}

Let ϕ_X and ϕ_Y denote the intrinsic fluorescence quantum yields of X (in the absence of Y) and of Y, respectively. Let N_0 be the number of X^* molecules generated by the absorption of a single β-particle, and define ϕ, the fluorescence efficiency of the binary solution, as

$$\phi = \frac{\text{Number of photons emitted by } Y^*}{N_0}.$$

Use the data given in Table 1 to find the value of k_3.

Table 1: Values of parameters

Parameter	Value	Parameter	Value
ϕ_X	5%	k_2	$9.5 \times 10^7 s^{-1}$
ϕ_Y	80%	N_0	not needed
ϕ	40%	k_3	to be determined

Appendix I

Taylor series:

$$f(x+a) = f(x) + \sum_{n=1}^{\infty} \frac{a^n}{n!} \frac{\mathrm{d}f^n}{\mathrm{d}x^n}. \tag{4}$$

MacLaurin series:

$$f(x) = f(0) + \sum_{n=1}^{\infty} \frac{x^n}{n!} \left. \frac{\mathrm{d}f^n}{\mathrm{d}x^n} \right|_{x=0}. \tag{5}$$

Two special cases are:

$$\sin x \quad = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots (\text{all real values of } x), \tag{6}$$

$$\cos x \quad = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots (\text{all real values of } x). \tag{7}$$

Binomial series:

$$(a+x)^n = a^n + na^{n-1}x + \frac{n(n-1)}{2!}a^{n-2}x^2 + \cdots . \tag{8}$$

Two special cases are:

$$(1+x)^{1/2} \quad = 1 + \frac{1}{2}x - \frac{1}{2 \cdot 4}x^2 + \cdots \qquad -1 < x \leq 1, \tag{9}$$

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 + \cdots \qquad -1 < x \leq 1. \tag{10}$$

Geometric series:

$$1/(1-x) = 1 + x + x^2 + x^3 \cdots . \tag{11}$$

Euler's formula:

$$e^{\mathrm{i}n\phi} = \cos n\phi + \mathrm{i} \sin n\phi. \tag{12}$$

Three corollaries of particular interest are:

$$\sin x = \frac{e^{\mathrm{i}x} - e^{-\mathrm{i}x}}{2\mathrm{i}} = \mathrm{i} \sinh(\mathrm{i}x) \qquad \left[\sinh y \equiv \frac{e^y - e^{-y}}{2}\right], \tag{13}$$

$$\cos x = \frac{e^{\mathrm{i}x} + e^{-\mathrm{i}x}}{2} = \cosh(\mathrm{i}x) \qquad \left[\cosh y \equiv \frac{e^y + e^{-y}}{2}\right]. \tag{14}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x \tag{15}$$

A useful integral:

$$\int_0^{\infty} \mathrm{e}^{-kx} \, \mathrm{d}x = 1/k, \qquad (k > 0). \tag{16}$$