Noen løsningsforslag lastes ned – de fleste løsninger kan ellers leses ut av kompendiene som vi delte ut.

Fra høsten 2006:

Oppgavene 3 og 4.

Løsningsforslag   Oppgave 3

a) v = banehastigheten

$\mu = G \cdot M$ i Newtons formel $F = G\frac{M \cdot m}{r^2} = \mu \frac{m}{r^2}$

r = avstand fra massepunktet til jorda

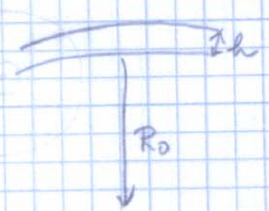
a = den elliptiske banens halve storakse

For sirkulær bane er $a = r$

$$\therefore \quad v = \sqrt{\mu \cdot \left(\frac{2}{r} - \frac{1}{r}\right)} = \sqrt{\frac{\mu}{r}}$$

b)

For ISS gjelder at banhøyden er $h = 400\,km$



$$R = R_0 + h$$
$$= 6400 + 400\ km = 6800\ km$$

$$F = \mu \frac{m}{R^2} = \frac{m v_{iss}^2}{R}$$

$$\Rightarrow v_{iss}^2 = \frac{\mu}{R} \Rightarrow v_{iss} = 7626{,}7 \left[\frac{m}{s}\right]$$

$v_{astr} = v_{iss} + \Delta v$ etter 'utblåsing'

$$v_{iss} + \Delta v = \sqrt{\mu\left(\frac{2}{R} - \frac{1}{a}\right)}$$

$$v_{iss}^2 + 2\Delta v \cdot v_{iss} = \mu\left(\frac{2}{R}\right) - \frac{\mu}{a}$$

$$\Rightarrow \quad 2\Delta v \cdot v_{iss} = \mu\left(\frac{1}{R} - \frac{1}{R+\Delta a}\right)$$

$$\frac{\Delta a}{R^2}$$

$$\Delta a = \frac{2\, v_{iss} \cdot R^2 \cdot \Delta v}{\mu} = \frac{2\, v_{iss} \cdot R^2 \cdot \Delta v}{R \cdot v_{iss}^2} = \frac{2R \cdot \Delta v}{v_{iss}}$$

$$= \frac{2 \cdot 6800 \cdot 10^3 \cdot 10}{\left(\frac{398603 \cdot 10^3}{6800 \cdot}\right)^{1/2}} \quad \left[\frac{m \cdot m/s}{\left(\frac{m^3}{s^2} \cdot \frac{1}{m}\right)^{1/2}}\right]$$

$$= \frac{13{,}6 \cdot 10^7}{7626{,}7} = 17832 \ m$$

$$= 17{,}832 \ km$$

$$a = 6800 + 17{,}832 = 6818 \ km$$

c) $T = \text{omloppstiden} = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{\frac{\mu}{R}}} = \frac{2\pi R^{3/2}}{\mu^{1/2}}$

d) $$T = 2\pi \sqrt{\frac{(R+\Delta a)^3}{\mu}}$$

$$= 6.28 \sqrt{\frac{(6800+18)^3}{\mu}} = 6.28 \sqrt{795715.} = 5599.8 \ [s]$$

$$T_{iss} = 2\pi \sqrt{\frac{R^3}{\mu}} = 6.28\sqrt{788781{,}6} = 5577.5 \ [s]$$

$$\Delta T = 22{,}33 \ s$$

med en hastighet $\Delta v = 10 \, m/s$ relativt ISS
vil avstanden da bli

$$10 \times 22{,}33 \text{ s} \sim 220 \text{ m}$$

Det er tvilsomt om astronauten kan 'fanges inn' -
- men kanskje en spesiell EVU kan komme
til hjelp!

Høst 2006

Løsningsforslag Oppgave 4

Spesifikke impulsen er gitt av

$$I_{sp} = \frac{F_{thrust}}{\frac{\Delta m}{\Delta t} \cdot g_0} =$$

Enheter

$$[I_{sp}] = s$$
$$[F] = N$$
$$[\Delta m] = kg$$
$$[\Delta t] = s$$
$$[g_0] = m/s^2$$

Altså $F = I_{sp} \cdot g_0 \cdot \Delta m / \Delta t$

$$= \frac{300 \cdot 9.81 \cdot \Delta m}{5 \cdot 60}$$

Δm søkes

$$\Delta m = m_0 \left(1 - e^{-\frac{\Delta v}{I_{sp} \cdot g_0}}\right)$$

$$\frac{\Delta v}{I_{sp} \cdot g_0} \sim \frac{5}{10 \cdot 300} \sim 0.001667$$

$$e^{-x} = \left(1 - x + \frac{x^2}{2!} \cdots \right) \approx 1 - x$$

$$\Delta m = 400 \cdot 0.001667 = 0.67 \text{ kg}$$

Og altså skyvekraften $F = 9.81 \cdot 0.67 = 6.7 \text{ N}$

- Fra høst 2006 oppg 5
- og fra høst 2005 oppg 2

høst 2005 Oppg 2

høst 2006 oppg 5

The station broadcasts a *carrier signal* at a specified frequency, regulated and licensed by the Federal Communications Commission (FCC) in the U.S. The transmitter then super-imposes the message being sent—music, news, or mission data—on top of the carrier signal, using some type of modulation scheme. The schemes we're most familiar with are amplitude or frequency modulation (AM and FM, see Figure 13-9). Spacecraft applications use other schemes as well. This signal travels outward from the station's antenna and hits your radio antenna. There, more charges accelerate. Your receiver detects this charge movement in the antenna and re-translates it to the original signal. The receiver demodulates the AM or FM signal to separate the message from the carrier signal and, suddenly, you're listening to tunes while cruising down the road.

Now we want to look at more details of communication systems to understand some of the basic principles and limitations. Let's use a light bulb to demonstrate some of these key principles. Similar to a radio transmitter, a light bulb emits EM radiation, but at a different frequency—visible light. If we put a light bulb in the center of a room, as shown in Figure 15-14, light radiates outward in all directions (assuming it's a perfect bulb with no light blockage). The intensity or brightness of the light at some distance from the bulb is called the *power-flux density, F.* Of course, the farther we get from the light bulb, the dimmer it appears. In other words, the power-flux density, which we perceive as brightness, decreases as we move farther away. From test measurements, we know the brightness actually decreases with the square of the distance, because all the output is distributed over the surface of a sphere surrounding the source. We express this as

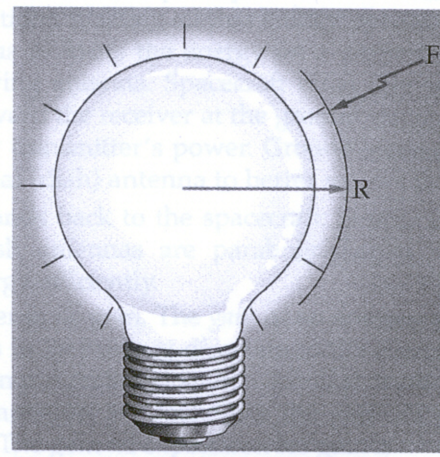


Figure 15-14. Power-flux Density. An ideal light bulb radiates equally in all directions. The brightness, or power-flux density, F, at any given distance, R, depends on the bulb's output, P.

$$F = \frac{\text{Power}}{\text{Surface area of a sphere}} = \frac{P}{4\pi R^2} \tag{15-1}$$

where

F = power-flux density (W/m^2)

P = power rating of the light bulb (W)

R = distance from the bulb (m)

We know that visible light, like that of a light bulb, is simply electromagnetic radiation. Radiation, moving equally in all directions, similar to our light bulb example from Figure 15-14, is called *omnidirectional* or *isotropic*. Now, what if we wanted to increase the brightness or power-flux density in only one direction using the same bulb? As Figure 15-15 shows, that's just what a flashlight does. This time we're still using our ideal light bulb, but we've put a parabolic-shaped mirror on one side of it. Thus, most of the light in one direction reflects off the mirror and heads in the opposite direction, and we have a directed beam of light—a spotlight—rather than an omnidirectional source. Doing this, we effectively concentrated most of the light energy into a smaller area. As a result, we get a brightness in that one direction that is much, much greater than it was when the bulb emitted light isotropically. We've "gained" extra power density by using the parabolic mirror.

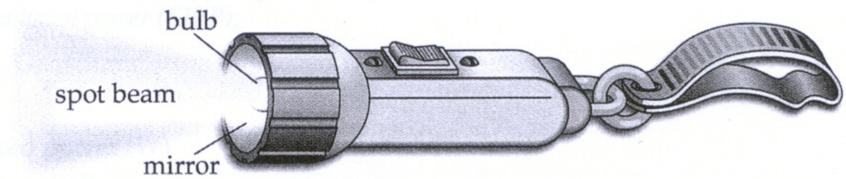


Figure 15-15. Directed Output from a Light Bulb. A parabolic mirror can direct the bulb output to give us an effective spot light. The mirror allows us to focus the bulb's energy in one direction, thus increasing the gain.

The flashlight example illustrates the basic principle of an antenna. Instead of broadcasting in all directions, wasting all that energy, specially designed "dish" antennas allow us to focus the energy on a particular point of interest, such as a receiving antenna. Spacecraft often rely on directional antennas that point toward the receiver at the ground station, making more efficient use of their transmitter's power. Ground stations usually employ another directional (dish) antenna to better receive the signal, as well as transmit commands back to the spacecraft. Similar to our flashlight's mirror, these dish antennas are parabolic-shaped to transmit and receive the radio energy efficiently.

An important antenna parameter is its gain. The *gain* of an antenna is the ratio of the energy it transmits in its primary direction to the energy that would be available from an omnidirectional source. In other words, the gain for an omnidirectional antenna is 1, whereas the gain for a directed antenna is greater than 1. The general expression for gain is

$$G = \frac{\text{Energy on target with a directed antenna}}{\text{Energy on target with an omnidirectional antenna}}$$

We can relate these two values for energy to an antenna's area, its efficiency, and the wavelength of the energy it's transmitting by

$$\boxed{G = \frac{4\pi A\eta}{\lambda^2} = \frac{4\pi A_e}{\lambda^2}} \tag{15-2}$$

where

G = gain (unitless)

A = physical area of the antenna (m^2)

η = antenna's efficiency (0.55–0.75 for parabolic antennas)

A_e = antenna's effective area (= $A\eta$, m^2)

λ = signal's wavelength (m)

This relationship tells us that if we want to increase the gain of an antenna (and transmit our message more efficiently), we can either increase its effective area or decrease our signal's wavelength. We use the same expression for the gain of transmitting and receiving antennas.

If we multiply the transmitter's power output by its antenna gain, we get an expression that represents the amount of power an isotropic transmitter would have to emit to get the same amount of power on a target. We call this the *effective isotropic radiated power (EIRP)*.

$$EIRP = P_t G_t \tag{15-3}$$

where

EIRP = effective isotropic radiated power (W)

P_t = transmitter's power output (W)

G_t = transmitter's antenna gain (unitless)

How much of the transmitter's power does the receiver collect? Think about collecting rainfall in a bucket. The amount of rain water collected depends on how hard it's raining—the rain's density—and the bucket's size or cross-sectional area. Similarly, the signal strength at a receiver is a function of the power-flux density at the receiver and the area of the receiver's antenna. The resulting expression for the signal gathered by the receiving antenna is then

$$S = \left(\frac{P_t G_t}{4\pi R^2}\right) A_{e_{receiver}} \tag{15-4}$$

where

S = received signal strength (W)

R = range to receiver. Worst case is $R_{max} = \sqrt{(R_\oplus + h)^2 - R_\oplus^2}$ (m), where $R_\oplus$ is Earth's sea level radius, and h is the spacecraft's height above sea level (m)

$\left(\frac{P_t G_t}{4\pi R^2}\right)$ = transmitter's effective power spread over a sphere of radius, R (W/m^2)

$A_{e_{receiver}}$ = receiving antenna's effective area (m^2)

Solving the right-hand expression in Equation (15-2) for $A_{e_{receiver}}$ and substituting into Equation (15-4) results in

$$\boxed{S = P_t G_t \left(\frac{\lambda}{4\pi R}\right)^2 G_r} \tag{15-5}$$

where

S = received signal strength (W)

$\left(\frac{\lambda}{4\pi R}\right)^2$ = space loss term (0 < space loss < 1.0) (unitless)

G_r = receiver's antenna gain (computed the same way as the transmitter's antenna gain) (unitless)

Notice we have a term representing space loss. *Space loss* is not a loss in the sense of power being absorbed in the atmosphere; rather, it accounts for the way energy spreads out as an electromagnetic wave travels away from a transmitting source. As distance increases, this term becomes smaller, which means space losses get worse. This situation makes sense. The greater the distance between a transmitter and receiver, the greater the total space losses (smaller space loss term). When this term is multiplied by the transmitter's power, and the receiver's and transmitter's antenna gains, the total signal strength, S, gets smaller for longer distances.

So we now have several ways to increase the received signal

- Increase the transmitter's power—P_t
- Increase the transmitter's antenna gain, concentrating the focus of the energy—G_t
- Increase the receiver's gain so it collects more of the signal—G_r
- Decrease the distance between the transmitter and receiver—R

A few pages back we discussed the concept of signal-to-noise (S/N) ratio in communication systems. So far in this discussion we've talked about the received signal, S. Earlier, when we discussed communicating across a room, noise came from some rambunctious kids. But where does noise come from for a radio signal? One important source of radio noise is heat. Recall from our discussion of black-body radiation in Chapter 11 that any object having a temperature greater than absolute 0 K emits EM radiation. While a receiver is running, just like your TV set, it gets hot and produces EM radiation as noise. The amount of noise power is given by

$$N = kTB \tag{15-6}$$

where

N = noise power (W)

k = Boltzmann's constant = 1.381×10^{-23} joules/K

T = receiver system's temperature (K)

B = receiving system's bandwidth (Hz)

Bandwidth is the range of frequencies the receiver is designed to receive. For example, the range of human eyesight, or the bandwidth of our eyes, is about 3.90×10^{14} Hz to 8.13×10^{14} Hz, which is a bandwidth of 4.23×10^{14} Hz. This represents the small portion of the EM spectrum we can see—visible light. Note that the noise in the receiver increases as the bandwidth increases. This should make sense, because the more information a receiver attempts to receive, the more likely it'll pick up noise. Ideally, we try to reduce the receiver temperature as much as possible and restrict the bandwidth of interest to minimize the noise.

Combining Equation (15-5) and Equation (15-6), we get the signal-to-noise ratio for a radio signal

$$\boxed{\frac{S}{N} = \left(\frac{P_t G_t}{kB}\right)\left(\frac{\lambda}{4\pi R}\right)^2\left(\frac{G_r}{T}\right)} \tag{15-7}$$

where

S/N = signal-to-noise ratio (unitless)

P_t = transmitter's power (W)

G_t = transmitter's gain (unitless)

k = Boltzmann's constant = 1.381×10^{-23} joules/K

B = receiving system's bandwidth (Hz)

λ = signal's wavelength (m)

R = range to receiver. Worst case is $R_{max} = \sqrt{(R_\oplus + h)^2 - R_\oplus^2}$ (m), where $R_\oplus$ is Earth's sea level radius, and h is the spacecraft's height above sea level (m)

G_r = receiver's gain (unitless)

T = receiver system's temperature (K)

Remember, for effective communication, the signal-to-noise ratio must be greater than or equal to 1.0. (The voice you hear must be louder than the background noise in the room.) To improve the S/N we can

- Increase the strength of the signal using the methods outlined above
- Reduce the signal's bandwidth—B
- Reduce the receiver's temperature—T

So far we haven't said much about changing the signal's frequency or wavelength. What effect does this have? Looking at Equation (15-7), we'd expect that increasing the wavelength would improve the S/N ratio, but remember the relationship for gain, given in Equation (15-2). The transmitter and receiver gains are inversely related to wavelength. That is, as wavelength increases (lower frequency), gain decreases. This means the net effect of increasing wavelength (decreasing frequency) is to decrease the antenna gains and thus reduce the S/N ratio. In other words, all other system parameters being equal, higher frequency gives us improved S/N. We show all these relationships in action in Example 15-2 applied to our FireSat scenario.

We must also plan what frequencies to use to avoid losses from the atmosphere and heavy precipitation.

Figure 15-16. Tracking and Data Relay Satellite's (TDRS) Second Terminal. This ground station controls NASA's TDRS constellation and receives telemetry and mission data from many satellites, including the Space Shuttle. *(Courtesy of NASA/White Sands)*

Satellite Control Networks

Now that we've looked at the theoretical aspects of communication networks, let's look at some examples of control networks in place to support NASA and the DoD space missions. NASA has two networks for tracking and receiving data from space. The Spaceflight Tracking and Data Network (STDN) mostly tracks and relays data for the Space Shuttle and other near-Earth missions. The STDN includes ground-based antennas at White Sands, New Mexico (Figure 15-16), as well as space-based portions using the Tracking and Data Relay Satellites (TDRS) in geostationary orbits. The deep-space network (DSN) includes very large antennas (70 m in diameter), used for tracking and receiving data from interplanetary space missions. These antennas are located in Madrid, Spain; Canberra, Australia (Figure 15-17); and Goldstone, California.

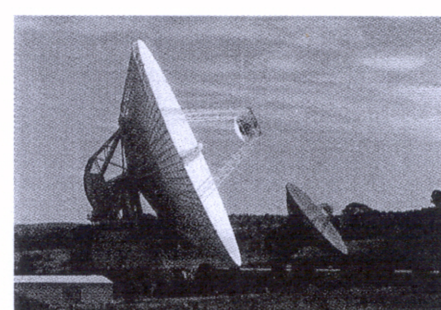
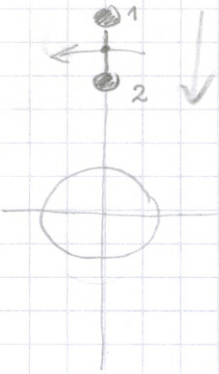


Figure 15-17. Deep Space Network (DSN). This complex of giant antennas at Canberra, Australia, keeps a constant watch for radio signals from NASA's interplanetary satellites, such as Stardust, Mars Exploration Rovers Spirit and Opportunity, and Cassini. *(Courtesy of NASA/Goddard Space Flight Center)*

Fra høst 2005 oppgave 5:1

(Ingen var interessert i oppgave 5:2!):

høst 2005 oppg 5:1



$$F_1 = GMm\frac{1}{r_1^2}$$

$$F_2 = GMm\frac{1}{r_2^2}$$

$$F_{cg} = GMm\frac{1}{r_0^2} = m\cdot\omega^2\cdot r_0 = F_{centr.}$$

$$\begin{cases} F_{centr\,1} = m\cdot\omega^2\cdot r_1 \\ F_{centr\,2} = m\omega^2\cdot r_2 \end{cases}$$

$$\begin{cases} F_{tot\,1} = GM\cdot m\frac{1}{r_1^2} - m\omega^2\cdot r_1 = GM\cdot m\frac{1}{(r_0+\Delta r)^2} - m\omega^2\cdot(r_0+\Delta r) \\ F_{tot\,2} = GM\cdot m\frac{1}{r_2^2} - m\omega^2\cdot r_2 = GMm\left(\frac{1}{(r_0-\Delta r)^2}\right) - m\omega^2(r_0-\Delta r) \end{cases}$$

$$\omega^2 = \frac{GM}{r_0^3}$$

$$\Rightarrow F_{tot\,1} = GMm\left(\frac{1}{(r_0+\Delta r)^2} - \frac{(r_0+\Delta r)}{r_0^3}\right) \approx GMm\left(\frac{(r_0-\Delta r)^2}{r_0^4} - \frac{(r_0+\Delta r)}{r_0^3}\right)$$

$$\approx GMm\left(\frac{(r_0^2-2r_0\Delta r) - (r_0^2+\Delta r\cdot r_0)}{r_0^4}\right)$$

$$\approx GMm\left(\frac{-2r_0\Delta r - \Delta r\cdot r_0}{r_0^4}\right)$$

$$\approx GMm\left(\frac{-3\Delta r}{r_0^3}\right)$$

$$F_{tot\,2} = GMm\left(\frac{r_0^2+2r_0\Delta r}{r_0^4} - \frac{r_0(r_0-\Delta r)}{r_0^4}\right) = GMm\left(\frac{2r_0\Delta r + \Delta r\cdot r_0}{r_0^4}\right)$$

$$\approx GMm\frac{3\Delta r}{r_0^3}$$

Forskjellen blir $\quad GMm \cdot \frac{6\Delta r}{r_0^3}$

$g = GM \frac{1}{r_0^2}$

$F = m \cdot g$

$$\therefore \frac{\cancel{GMm} \cdot 6\Delta r \cdot \cancel{r_0^2}}{r_0^{\cancel{3}} \cdot \cancel{GMm}}$$

$$= \frac{6\Delta r}{r_0}$$

med $\Delta r \approx 25$ km $\quad \Rightarrow \sim \frac{\cancel{6} \times 25}{\cancel{6000}\ 1000} = 0.025\ g$

LEO $\quad r_0 \simeq 6500 + 1000 = 7500$

$$\Rightarrow \frac{6 \times \cancel{25}}{\cancel{7500}\ 300} \approx 0.02\ g$$

eller $\underline{0.01\ g}$ i tvilling satellittene