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Department of Physics

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Examination, course FY3403 Particle physics
Monday December 14, 2009
Time: 09.00–13.00

Grades made public: Wednesday December 30, 2009

Allowed to use: Particle physics booklet, calculator, mathematical tables.

A table of physical constants can be found in the Particle physics booklet.
All subproblems are given the same weight in the grading.

Problem 1:

The differential cross section for a two-body scattering process $1+2 \to 3+4$ in the centre of mass (CM) frame is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = S \left(\frac{\hbar c}{8\pi(E_1+E_2)}\right)^2 \frac{|\vec{p}_f|}{|\vec{p}_i|} |\mathcal{M}|^2 \, .$$

The energy and momentum of particle $j = 1,2,3,4$ are E_j and $\vec{p}_j$, and we have $\vec{p}_1 = -\vec{p}_2 = \vec{p}_i$ and $\vec{p}_3 = -\vec{p}_4 = \vec{p}_f$ in the CM frame. The statistical factor S is 1/2 if particles 3 and 4 are identical and is 1 otherwise. $\mathcal{M}$ is the scattering amplitude.

If the incoming particles 1 and 2 are prepared in a state of unpolarized spins, and if we do not measure the spins of the outgoing particles 3 and 4, then $|\mathcal{M}|^2$ should be replaced by $\langle|\mathcal{M}|^2\rangle$, which is $|\mathcal{M}|^2$ *averaged* over incoming spins and *summed* over outgoing spins.

Time reversal symmetry implies that $|\mathcal{M}|^2$ *summed* over both incoming and outgoing spins is the same for the process $1+2 \to 3+4$ and for the time reversed process $3+4 \to 1+2$.

The total cross section for the process $1+2 \to 3+4$ is

$$\sigma = \int \mathrm{d}\Omega \, \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \, .$$

The total cross section for the process $e^+ + e^- \to \gamma + \gamma$ (annihilation of a positron and an electron into two photons) at low energy (non-relativistic energies for the positron and electron) is

$$\sigma(e^+ + e^- \to \gamma + \gamma) = \frac{\pi {r_e}^2 c}{|\vec{v}|} \, , \tag{1}$$

where $r_e = 2.818 \times 10^{-15}$ m is the classical electron radius (see table 1.1, p. 4 in the Particle physics booklet), and where $\vec{v} = \vec{v}_1 - \vec{v}_2$ is the relative velocity of the positron and the electron.

a) What is the relation between the total cross section in the CM frame, where $\vec{p}_1 + \vec{p}_2 = 0$, and in the laboratory frame, where $\vec{p}_2 = 0$? Give a brief argument for your answer.

b) Use time reversal symmetry (which is known experimentally to hold for electromagnetic interactions) in order to find a formula for the total cross section, in the CM frame, of the pair creation process $\gamma + \gamma \to e^+ + e^-$ at an energy just above the threshold energy.

c) The cosmic microwave background radiation filling the Universe is electromagnetic black body radiation at a temperature of $T_{\text{CMB}} = 2.73$ K.

The average photon energy is of the order of kT_{CMB}, where k is Boltzmann's constant (table 1.1, p. 4 in the Particle physics booklet). What is kT_{CMB} in electronvolt?

A high energy photon, with energy above a certain threshold value, may collide with a photon of the cosmic microwave background and produce an electron positron pair.

What is the threshold energy of the high energy photon for this reaction?

d) The number density of cosmic microwave background photons is about 400 per cm^3.

Assume that the cross section for the reaction $\gamma + \gamma \to e^+ + e^-$ is πr_e^2, and estimate the average distance which a high energy photon can travel before it is destroyed in a collision with a photon of the cosmic microwave background.

Comment?

Problem 2:

a) A bound state of a spin 1/2 particle (a quark or an electron) and its antiparticle has parity $P = (-1)^{\ell+1}$ and charge conjugation symmetry $C = (-1)^{\ell+s}$, where ℓ is the relative orbital angular momentum and s is the total spin.

What are the possible values of ℓ and s?

Why do we expect that the ground state will have $\ell = 0$?

Based on this, what are the quark model predictions for the parity and charge conjugation symmetry of the spin zero mesons π^0 and η, and of the spin one mesons ρ^0 and ω? Explain your reasoning.

b) The ground state of positronium (a positron and an electron bound together) can have either $s = 0$ (parapositronium) or $s = 1$ (orthopositronium).

Parapositronium and orthopositronium have very different lifetimes (by a factor of 10^3), because they can not decay into the same number of photons. Why?

Which of them has the shortest lifetime, and why?

c) The decay rate, or inverse lifetime, for the two photon decay of positronium can be computed as

$$\Gamma = \frac{1}{\tau} = \mathcal{L}\sigma \ ,$$

where σ is the cross section for annihilation into two photons, and $\mathcal{L}$ is a luminosity,

$$\mathcal{L} = |\psi_{\rm rel}(0)|^2 \ |\vec{v}| \ .$$

Here $\vec{v} = \vec{v}_1 - \vec{v}_2$ is again the relative velocity of the positron and the electron, and $\psi_{\rm rel}$ is the relative wave function, found by solving the Schrödinger equation for the relative motion. The resulting probability density at the origin is

$$|\psi_{\rm rel}(0)|^2 = \frac{1}{\pi}\left(\frac{\alpha m_e c}{2\hbar}\right)^3 ,$$

where m_e is the electron mass.

The cross section given in equation (1) must be multiplied by 4 here, because the incoming spins are now fixed and there is no averaging. Thus,

$$\sigma = \frac{4\pi {r_e}^2 c}{|\vec{v}|} = \frac{4\pi c}{|\vec{v}|}\left(\frac{\alpha\hbar}{m_e c}\right)^2 .$$

Derive the formula for the positronium lifetime τ, and compute the numerical value.

Problem 3:

a) We observe three generations of leptons and quarks, for example with the electron, the electron neutrino, the u and d quarks, and their antiparticles, in the first generation.

How can the particles of one generation be transformed into particles of other generations?

b) What is meant by asymptotic freedom in quantum chromodynamics?

c) What is meant by a "Grand Unified Theory"?

Can you mention briefly one or two arguments in favour of grand unification, and one or two arguments against?