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Suggested solution for Exam FY3403: Particle Physics

NOTE: The solutions below are meant as guidelines for how the problems may be solved and do not necessarily contain all the required details.

PROBLEM 1

(a) Neutrino: nothing lighter to decay into via weak interaction. Photon: nothing lighter to decay into. Electron: lightest charged particle. Proton: lightest baryon (conservation of baryon number saves it). The four-vectors for the electrons q and q' may be written as (neglecting the mass):

$$q = (E/c, E/c, 0, 0), q' = (E'/c, E'\cos\theta/c, E'\sin\theta/c, 0) \tag{1}$$

where we set the scattering to take place in the xy-plane without loss of generality. We may then directly compute:

$$-(q-q')^2 = 4EE'\sin(\theta/2). \tag{2}$$

(b) First: no (unmatched index). Second: yes (transforms like 4-vector). Third: yes (invariant scalar). For the scattering process, the momenta are equal and opposite in the CM frame. Since the initial particles have equal masses, they will have equal energies E. Same argument applies to the final particles which also will have equal energies E. The momenta are found from the standard relation $E^2 = \mathbf{p}^2 + m^2$ for each particle.

(c) Using conservation of 4-momentum we find for the electron $E = (m_W^2 - m_e^2)c^2/(2m_e)$. In numbers, this is 6 PeV, which is an extremely high energy (non-realistic in practice). NB! It was not clear from the problem text whether one should consider the electron-generation or muon- and taon-generations as well. Therefore, full score has been given to those that derived the analytical expression for the threshold energy E (without necessarily inserting numbers).

(d) The high mass of the vector bosons W, Z mediating the weak interaction strongly reduces the amplitude of the Feynman diagrams. However, for high-energy scattering near the pole of the propagator the weak interactions becomes very strong and overtakes the electromagnetic contribution. The gauge group is $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. Electrons, neutrinos, photons, and Z are colorless. The quarks come in 3 colors whereas the gluon has 8 different color combinations.

(e) Renormalization is the procedure of accounting for higher-order Feynman diagrams by obtaining effective coupling constants and/or masses for the theory. The contribution from higher-order diagrams to these renormalized (effective) quantities may contain both infinite and finite momentum-dependent terms. The renormalized coupling constants and/or masses are what is measured experimentally, which means that there evidently are cancelling infinities. The finite contributions make the coupling constants and/or masses "running" in the sense that they depend on the energy (or equivalently separation distance) of the involved particles. Asymptotic freedom refers to the distance-dependence of the strong coupling constant: for small distances, the strong coupling constant is small whereas it grows larger for larger distances. The interaction strength of quarks thus depends on their separation distance. A gauge theory is a field theory in which the Lagrangian has a continuous symmetry. The different choices exploiting this degree of freedom are called gauges.

(f) The cross-section has a contribution from all the hadrons made up of quarks that have sufficiently small mass to allow for production at a given energy of the initial pair of particles. As this energy is increased and exceeds additional quark masses, the ratio increases. Put differently: as the energy increases above certain thresholds, more and more types of hadrons are allowed for production and this is manifested as a increase (step-up) in the scattering cross section.

PROBLEM 2

(a) The fermion must have charge +1/3 due to conservation of charge, which means that it must be an anti-quark of the type $\bar{d}, \bar{s}$, or $\bar{b}$.

(b) The amplitude and diagram are given below:

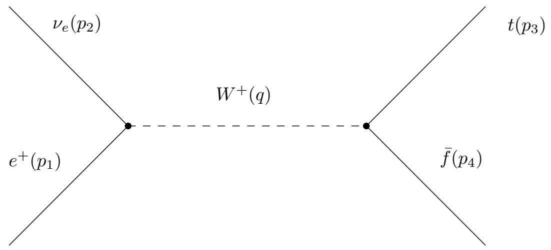

FIG. 1: Feynman diagram.

$$-\mathrm{i}\mathcal{M} = \frac{(-\mathrm{i}g_W)(\mathrm{i}g_c)}{2\sqrt{2}}[\bar{v}_1\gamma^\mu(1-\gamma_5)u_2]\frac{-\mathrm{i}(g_{\mu\nu} - q_\mu q_\nu/M_W^2)}{q^2 - M_W^2}[\bar{u}_3\gamma^\nu(1-\gamma_5)v_4]$$

The second term in the propagator makes no contribution when the positron and neutrino are massless since $\bar{\nu}_1 q'(1-\gamma_5)u_2 = 0$ as seen when anticommuting and using the massless Dirac equation. I used the notation $a' \equiv a^\mu\gamma_\mu$. The computation of the spin-averaged Feynman amplitude $\langle|\mathcal{M}|^2\rangle$ is shown in detail on page 250-252 in the textbook.

(c) See the treatment in the textbook for a detailed explanation. In particular, the gauge field may become massive when spontaneously breaking a local symmetry: the gauge field "eats" the Goldstone bosons.

(d) 1st term: "kinetic term" associated with time- and space-variations of the field. 2nd term: irrelevant constant. 3rd term: mass-term with $m \propto \alpha$. 4th term: interaction term for four ϕ-fields. If the mass-term had opposite sign, we would have to identify the ground-state of the Lagrangian (since $\phi = 0$ would not be the ground-state), reexpress the Lagrangian in terms of deviation from the ground-state, and then identify the mass of the field representing an excitation above the ground-state.