NTNU **Institutt for fysikk**

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Eksamen i FY8306 KVANTEFELTTEORI

Fredag 9. juni 2006
09:00–13:00

Tillatte hjelpemidler: Alternativ **C**

Typegodkjent kalkulator, med tomt minne (i henhold til liste utarbeidet av NTNU).
K. Rottman: *Matematisk formelsamling* (alle språkutgaver).
Schaum's Outline Series: *Mathematical Handbook of Formulas and Tables.*

Sensur legges ut på fagets webside, `http://web.phys.ntnu.no/~kolausen/FY8306/`, såsnart den er klar

Dette oppgavesettet er på 3 sider, pluss et vedlegg på 2 sider.

Oppgave 1. Henfall av Z_0 vektormesonet i Standardmodellen

I denne oppgaven skal du studere henfall av Z^0 vektormesonet i Standardmodellen for partikkelfysikk. De nødvendige Feynmanreglene og annet relevant formelverk er vedlagt oppgavesettet. Anta et Lorentz system der Z^0 partikkelen er i ro før henfallet.

a) Tegn Feynmandigrammet for den generiske henfallsprosessen

$$Z^0 \to f\bar{f},$$

der f står for ett av de mulige fermionene i Standardmodellen.

b) Skriv ned den tilhørende henfallsamplituden $\mathcal{M}_{fi}$.

c) Finn amplitudekvadratet $\overline{|\mathcal{M}_{fi}|^2}$, midlet over spinnet til Z^0-partikkelen, og summert over spinnene til f- og $\bar{f}$-partiklene. Du kan neglisjere alle ledd som er proporsjonale med fermionmassen m_f. (Dette er ekvivalent med å anta at $m_f^2 \ll M_Z^2$.)

d) Bruk dette resultatet, og ligning (11) i vedlegget, til å finne den integrerte henfallsraten $\Gamma_{Z\to f\bar{f}}$.

e) Den totale henfallsraten er gitt som

$$\Gamma_{\text{tot}} = \sum_f \Gamma_{Z\to f\bar{f}}, \tag{1}$$

der summen løper over alle kjente typer leptoner, nøytrinoer, og kvarker som er lette nok til at henfallsprosessen kan gå.

Hva er sannsynligheten $\Gamma_{Z\to e\bar{e}}/\Gamma_{\text{tot}}$ for at Z^0 skal henfalle til et elektron–positron par?

f) Hva er sannsynligheten for at Z^0 skal henfalle til et nøytrino–antinøytrino par?

g) Hva blir den numeriske verdien til den totale henfallsraten Γ_{tot}? Sammenlign dette svaret med den eksperimentelle verdien $\Gamma_{\text{tot}} \approx 2.4952$ GeV.

Oppgitt:

$M_Z = 91.19$ GeV

$\sin^2\theta_W = 0.231$

$\alpha = 1/137.036$

Oppgave 2. SU(2) gauge modeller

I denne oppgaven skal du se på noen aspekter av kvantefelt modeller der gaugegruppen er ren SU(2) (isospinn). Dvs. at de kovariant deriverte kan skrives på formen

$$D_{\mu ab} = \partial_\mu \delta_{ab} + \mathrm{i} g T^k_{ab} A^k_\mu, \tag{2}$$

der μ er en rom-tids indeks, a og b er isospinn indekser, og k skal summeres over de tre generatorene til SU(2). Matrisene T^k avhenger av isospinnet til feltet som den kovariant deriverte virker på, men oppfyller alltid kommuteringsregelen

$$\left[T^k, T^\ell\right] = \mathrm{i}\,\varepsilon^{k\ell m} T^m. \tag{3}$$

For et isospinn-$\frac{1}{2}$ felt har vi

$$T^k_{ab} = \frac{1}{2}\sigma^k_{ab}, \tag{4}$$

der σ^k er en Pauli-matrise (altså en kompleks 2×2 matrise). For et isospinn-1 felt har vi

$$T^k_{ab} = \mathrm{i}\,\varepsilon^{kab} \tag{5}$$

(disse er altså rent imaginære 3×3 matriser).

Merk at mange av de spørsmålene som følger er uavhengig av hverandre, slik at du ikke trenger å få til hvert enkelt delpunkt for å gå videre.

a) Skriv ned, på matriseform, sammenhengen mellom den kovariant deriverte D_μ (som en matrise med isospinn indekser) og felttensoren $F_{\mu\nu}$ (som en matrise med isospinn indekser).

b) Matrisen $F_{\mu\nu}$ kan skrives som en sum over generatorene T^k,

$$F_{\mu\nu ab} = T^k_{ab} F^k_{\mu\nu}. \tag{6}$$

Vis at feltene $F^k_{\mu\nu}$ ikke avhenger av representasjonsmatrisene T^k, dvs om de f.eks. er definert ved ligning (4) eller (5) sålenge $T^k \neq 0$, men bare av feltene A^k_μ. Skriv ned den eksplisitte sammenhengen mellom A^k_μ og $F^k_{\mu\nu}$.

c) Langrangetettheten for modellene skal ha et bidrag som avhenger av felttensoren $F^k_{\mu\nu}$ ("Maxwell"-bidraget $\mathcal{L}_A$). Hvordan ser dette leddet ut?

d) Vi antar nå først en modell der vi også har et isospinn-$\frac{1}{2}$ skalarfelt φ, med bidrag til Lagrangetettheten

$$\mathcal{L}_\varphi = (D_\mu\varphi)^\dagger D^\mu\varphi + m^2\varphi^\dagger\varphi - \frac{1}{4}\lambda(\varphi^\dagger\varphi)^2, \tag{7}$$

med m^2 og λ positive. (Den totale Langrangetettheten er altså $\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\varphi$).
Hvilken verdi av $\varphi^\dagger\varphi$ minimaliserer potensialet $V = -m^2\varphi^\dagger\varphi + \frac{1}{4}\lambda(\varphi^\dagger\varphi)^2$?

e) Vakuumtilstanden for systemet vil være karakterisert av den verdien av φ-feltet som minimaliserer potensialet V. Ved passende valg av gauge kan vi anta at dette har formen

$$\langle\,\Omega\,|\,\varphi\,|\,\Omega\,\rangle = \varphi_0 = \begin{pmatrix} 0 \\ \phi_0 \end{pmatrix}, \tag{8}$$

med ϕ_0 reell. Dette gir opphav til masseledd for gaugefeltene A^k_μ (Higg's mekanismen). Hva blir massene til de tre gaugefeltene i dette tilfellet?

f) Vi vil nå i stedet studere tilfellet der skalarfeltet har isospinn-1.
Vis først at representasjonsmatrisene T^k definert av ligning (5) oppfyller kommuteringsregelen (3).

Tips: Bruk relasjonen $\varepsilon^{abc}\varepsilon^{dec} = \delta^{ad}\delta^{be} - \delta^{ae}\delta^{bd}$, og at ε-symbolet er totalt antisymmetrisk.

g) Vi antar nå en modell med et reellt isospinn-1 skalarfelt $\boldsymbol{\chi}$, med bidrag til Lagrangetettheten

$$\mathcal{L}_\chi = \frac{1}{2}D_\mu\boldsymbol{\chi}\cdot D^\mu\boldsymbol{\chi} + \frac{1}{2}m^2\boldsymbol{\chi}\cdot\boldsymbol{\chi} - \frac{1}{4!}\lambda(\boldsymbol{\chi}\cdot\boldsymbol{\chi})^2 \tag{9}$$

med m^2 og λ positive, og der $\boldsymbol{\chi}$-feltet har tre reelle komponenter (derfor vektor-notatasjonen). (Den totale Langrangetettheten er altså $\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\chi$).
Hvilken verdi av $\boldsymbol{\chi}\cdot\boldsymbol{\chi}$ minimaliserer potensialet $V = -\frac{1}{2}m^2\boldsymbol{\chi}\cdot\boldsymbol{\chi} + \frac{1}{4!}\lambda(\boldsymbol{\chi}\cdot\boldsymbol{\chi})^2$?

h) Vakuumtilstanden for systemet vil nå være karakterisert av den verdien av $\boldsymbol{\chi}$-feltet som minimaliserer potensialet V. Ved passende valg av gauge kan vi anta at dette har formen

$$\langle\,\Omega\,|\,\boldsymbol{\chi}\,|\,\Omega\,\rangle = \varphi_0 = \begin{pmatrix} 0 \\ 0 \\ \chi_0 \end{pmatrix}, \tag{10}$$

med χ_0 reell. Dette gir opphav til masseledd for gaugefeltene A^k_μ (Higg's mekanismen). Hva blir massene til de tre gaugefeltene i dette tilfellet?

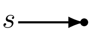
1 Sammenheng mellom amplitude $\mathcal{M}_{fi}$ og henfallsrate Γ

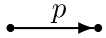
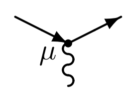
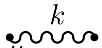
Sammenhengen mellom Feynman amplitude $\mathcal{M}_{fi}$ og henfallsrate $\mathrm{d}\Gamma$ er gitt som

$$\mathrm{d}\Gamma = (2\pi)^4 \delta^{(4)}(p - \Sigma p'_f)\, \frac{1}{2E}\, |\mathcal{M}_{fi}|^2 \prod_f \frac{\mathrm{d}^3 \boldsymbol{p}'_f}{(2\pi)^2 2E'_f}, \tag{11}$$

der p, E er henholdsvis firerimpuls og energi til partikkelen som henfaller; de øvrige størrelsene refererer til partiklene i sluttilstanden.

2 Noen Feynmanregler for $-\mathrm{i}\mathcal{M}_{fi}$:

1. Utgående partikler			2. Innkommende partikler		
Type partikler	Grafisk symbol	Algebraisk uttrykk	Type partikler	Grafisk symbol	Algebraisk uttrykk
$e^-, \mu^-, \cdots$	p, s	$\overline{u}(p,s)$	$e^-, \mu^-, \cdots$	p, s 	$u(p,s)$
$e^+, \mu^+, \cdots$	p, s	$v(p,s)$	$e^+, \mu^+, \cdots$	p, s 	$\overline{v}(p,s)$
Z^0	k, r	$\varepsilon_\mu(k,r)^*$	Z^0	k, r 	$\varepsilon_\mu(k,r)$

3. Propagatorer			4. Vekselvirkningsknuter		
Type partikler	Grafisk symbol	Algebraisk uttrykk	V.virkning $\mathcal{L}_{\text{int}}$	Grafisk symbol	Algebraisk uttrykk
$e^\pm, \mu^\pm, \cdots$	p 	$\frac{\mathrm{i}(\not{p}+m)}{p^2 - m^2 + \mathrm{i}\epsilon}$	$\frac{-e}{\sin 2\theta_W}\overline{\psi}\gamma^\mu(g_V - g_A\gamma^5)\psi Z_\mu$	 μ	$\frac{-\mathrm{i}e}{\sin 2\theta_W}\gamma^\mu(g_V - g_A\gamma^5)$
Z^0	k  μ ν	$\frac{-\mathrm{i}(\eta_{\mu\nu} - k_\mu k_\nu/M_Z^2)}{k^2 - M_Z^2 + \mathrm{i}\epsilon}$			

Her er

$$g_V = \begin{cases} \frac{1}{2} & \text{for } \nu_e,\ \nu_\mu,\ \nu_\tau \\ \frac{1}{2}\left(-1 + 4\sin^2\theta_W\right) & \text{for } e,\ \mu,\ \tau \\ \frac{1}{2}\left(1 - \frac{8}{3}\sin^2\theta_W\right) & \text{for } u,\ c,\ t \\ \frac{1}{2}\left(-1 + \frac{4}{3}\sin^2\theta_W\right) & \text{for } d,\ s,\ b \end{cases} \qquad g_A = \begin{cases} \frac{1}{2} & \text{for } \nu_e,\ \nu_\mu,\ \nu_\tau,\ u,\ c,\ t \\ -\frac{1}{2} & \text{for } e,\ \mu,\ \tau,\ d,\ s,\ b \end{cases}$$

3 Noen fullstendighetsrelasjoner

Dirac partikler, Dirac antipartikler, og Z^0 vektorbosoner

$$\sum_{s=1}^{2} u(p,s)\,\overline{u}(p,s) = \not{p} + m, \qquad \sum_{s=1}^{2} v(p,s)\,\overline{v}(p,s) = \not{p} - m \tag{12}$$

$$\sum_{r=1}^{3} \varepsilon_\mu(k,r)\,\varepsilon^*_\nu(k,r) = -\eta_{\mu\nu} + k_\mu k_\nu / M_Z^2 \tag{13}$$

4 Dirac's γ-matriser

4.1 Standardrepresentasjonen

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \qquad \vec{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \tag{14}$$

der I er en 2×2 enhetsmatrise, og $\boldsymbol{\sigma}$ er Pauli-matrisene,

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{15}$$

som oppfyller den algebraiske relasjonen

$$\sigma^i\,\sigma^j = \delta^{ij} + \mathrm{i}\varepsilon^{ijk}\,\sigma^k, \text{ dvs. at } (\boldsymbol{\sigma}\cdot\mathbf{a})\,(\boldsymbol{\sigma}\cdot\mathbf{b}) = \mathbf{a}\cdot\mathbf{b} + \mathrm{i}\,\boldsymbol{\sigma}\cdot(\mathbf{a}\times\mathbf{b}) \tag{16}$$

4.2 Algebraiske relasjoner

$$\{\gamma^5, \gamma^\nu\} = 0, \tag{17}$$

$$(\gamma^5)^2 = 1, \tag{18}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2\,\eta^{\mu\nu} \quad \Longrightarrow \not{p}\,\not{p} = p^2 \tag{19}$$

$$\gamma^\mu\,\gamma^\nu\,\gamma_\mu = -2\,\gamma^\nu \quad \Longrightarrow \gamma_\mu\,\not{p}\,\gamma^\mu = -2\,\not{p} \tag{20}$$

$$\gamma^\mu\,\gamma^\nu\,\gamma^\lambda\,\gamma_\mu = 4\,\eta^{\nu\lambda} \quad \Longrightarrow \gamma_\mu\,\not{p}\,\not{q}\,\gamma^\mu = 4\,(pq) \tag{21}$$

$$\gamma^\mu\,\gamma^\nu\,\gamma^\lambda\,\gamma^\sigma\,\gamma_\mu = -2\,\gamma^\sigma\,\gamma^\lambda\,\gamma^\nu \quad \Longrightarrow \gamma_\mu\,\not{p}\,\not{q}\,\not{r}\,\gamma^\mu = -2\not{r}\,\not{q}\,\not{p} \tag{22}$$

4.3 Noen spor-uttrykk

$$\mathrm{Tr}\,1 = 4 \tag{23}$$

$$\mathrm{Tr}\,\gamma^5 = 0 \tag{24}$$

$$\mathrm{Tr}\,\gamma^\mu = 0 \tag{25}$$

$$\mathrm{Tr}\,\gamma^\mu\gamma^5 = 0 \tag{26}$$

$$\mathrm{Tr}\,\gamma^\mu\,\gamma^\nu = 4\,\eta^{\mu\nu} \quad \Longrightarrow \mathrm{Tr}\,\not{p}\,\not{q} = 4\,(pq) \tag{27}$$

$$\mathrm{Tr}\,\gamma^\mu\,\gamma^\nu\gamma^5 = 0 \tag{28}$$

$$\mathrm{Tr}\,\gamma^\mu\,\gamma^\nu\,\gamma^\lambda = 0 \tag{29}$$

$$\mathrm{Tr}\,\gamma^\mu\,\gamma^\nu\,\gamma^\lambda\gamma^5 = 0 \tag{30}$$

$$\mathrm{Tr}\,\gamma^\mu\,\gamma^\nu\,\gamma^\lambda\,\gamma^\sigma = 4\left(\eta^{\mu\nu}\,\eta^{\lambda\sigma} - \eta^{\mu\lambda}\,\eta^{\nu\sigma} + \eta^{\mu\sigma}\,\eta^{\nu\lambda}\right) \tag{31}$$

$$\Longrightarrow \mathrm{Tr}\,\not{p}\,\not{q}\,\not{r}\,\not{s} = 4(pq)(rs) - 4(pr)(qs) + 4(ps)(qr)$$

$$\mathrm{Tr}\,\gamma^\mu\,\gamma^\nu\,\gamma^\lambda\,\gamma^\sigma\,\gamma^5 = -4\mathrm{i}\,\varepsilon^{\mu\nu\lambda\sigma} \tag{32}$$

NTNU **Institutt for fysikk**

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Exam in FY8306 QUANTUM FIELD THEORY

Friday June 9, 2006
09:00–13:00

Allowed help: Alternativ **C**

Standard pocket calculator (according to list made by NTNU).
K. Rottman: *Matematisk formelsamling* (any language).
Schaum's Outline Series: *Mathematical Handbook of Formulas and Tables.*

Grades are posted on the web page of the course,
`http://web.phys.ntnu.no/~kolausen/FY8307/`,
as soon as they are ready

This problem set consists of 3 pages, plus an Appendix of 2 pages.

Problem 1. Decay of the Z_0 vector meson in the Standard Model
In this problem we shall study the decay of the Z^0 vector meson according to the Standard Model of Particle Physics. The Feynman rules needed, together with other relevant formulas, can be found in the Appendix. Assume a Lorentz frame where the Z^0 particle is at rest before it decays.

a) Draw the Feynman diagram for the decay process

$$Z^0 \to f\bar{f},$$

where f denotes one of the posssible fermions in the Standard Model.

b) Write down the corresponding decay amplitude $\mathcal{M}_{fi}$.

c) Find the squared amplitude $\overline{|\mathcal{M}_{fi}|^2}$, averaged over the spin of the Z^0 particle, and summed over the spins of the f and $\bar{f}$ particles.

d) Use the result above, and equation (11) in the appendix, to find the integrated decay rate $\Gamma_{Z\to f\bar{f}}$.

e) The total decay rate is given as

$$\Gamma_{\text{tot}} = \sum_f \Gamma_{Z\to f\bar{f}}, \tag{1}$$

with the sum running over all known types of leptons, neutrinos and quarks which are light enough to make the decay possible.
What is the probability $\Gamma_{Z\to e\bar{e}}/\Gamma_{\text{tot}}$ that Z^0 decays into a electron–positron pair?

f) What is the probability for Z^0 to decay into a neutrino–antineutrino pair?

g) What is the numerical value of the total decay rate Γ_{tot}? Compare your answer with the experimental value $\Gamma_{\text{tot}} \approx 2.4952$ GeV.

Some constants of nature:

$M_Z = 91.19$ GeV

$\sin^2\theta_W = 0.231$

$\alpha = 1/137.036$

Problem 2. SU(2) gauge models

In this problem we shall consider some aspects of Quantum Field models where the gauge group in pure SU(2) (i.e. isospin). This means that the covariant derivatives can be written as

$$D_{\mu ab} = \partial_\mu \delta_{ab} + \mathrm{i} g T^k_{ab} A^k_\mu, \tag{2}$$

where μ is a space-time index, a and b are isospin indices, and k is to be summed over the three generators of SU(2). The matrices T^k depend on the isospin representation of the field on which the covariant derivative acts, but always fulfills the commutation relation

$$\left[T^k, T^\ell\right] = \mathrm{i}\,\varepsilon^{k\ell m} T^m. \tag{3}$$

We have for a isospin $\frac{1}{2}$ field,

$$T^k_{ab} = \frac{1}{2}\sigma^k_{ab}, \tag{4}$$

where σ^k is a Pauli matrix (e.g. a complex 2×2 matrix). We have for a isospin 1 field,

$$T^k_{ab} = \mathrm{i}\,\varepsilon^{kab} \tag{5}$$

(e.g. purely imaginary 3×3 matrices).

Not that many of the questions which follows are independent of each other, so that you don't have to solve every point in order to proceed.

a) Write down, in matrix form, the connection between the covariant derivative D_μ (as a matrix with isospin indices) and the field tensor $F_{\mu\nu}$ (as a matrix with isospin indices).

b) The matrix $F_{\mu\nu}$ can be written as a som over the generators T^k,

$$F_{\mu\nu ab} = T^k_{ab} F^k_{\mu\nu}. \tag{6}$$

Show that the fields $F^k_{\mu\nu}$ don't depend on the representation matrices T^k, i.e. whether they are defined by e.g. equation (4) or (5) as long as $T^k \neq 0$,but only upon the fields A^k_μ. Write down the explicit connection between A^k_μ and $F^k_{\mu\nu}$.

c) The Lagrangian density for these modeles have a contribution which depends on the field tensor $F^k_{\mu\nu}$ (the "Maxwell" term $\mathcal{L}_A$). How does this term look like?

d) We now assume a model which also includes a isospin $\frac{1}{2}$ scalar field φ, whose contribution to the Lagrangian density is

$$\mathcal{L}_\varphi = (D_\mu\varphi)^\dagger D^\mu\varphi + m^2\varphi^\dagger\varphi - \frac{1}{4}\lambda(\varphi^\dagger\varphi)^2, \tag{7}$$

with m^2 and λ positive. (The total Lagrangian density thus is $\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\varphi$).
Which value of $\varphi^\dagger\varphi$ minimizes the potential $V = -m^2\varphi^\dagger\varphi + \frac{1}{4}\lambda(\varphi^\dagger\varphi)^2$?

e) The vacuum state for this system is characterized by the value of the φ-field which minimizes the potential V. By a suitable choice of gauge this may be assumed to have the form

$$\langle\,\Omega\,|\,\varphi\,|\,\Omega\,\rangle = \varphi_0 = \begin{pmatrix} 0 \\ \phi_0 \end{pmatrix}, \tag{8}$$

where ϕ_0 is real. This gives rise to a mass term for the gauge fields A^k_μ (the Higg's mechanism).
What are the masses of the three gauge fields in this case?

f) We now switch to study the case when the scalar field has isospin 1.
First show that the representation matrices T^k defined by equation (5) satisfy the commutation relation (3).

Hint: Use the relation $\varepsilon^{abc}\varepsilon^{dec} = \delta^{ad}\delta^{be} - \delta^{ae}\delta^{bd}$, and the fact that the ε-symbol is totally anti-symmetric.

g) We now assume a model with a real isospin 1 scalar field $\boldsymbol{\chi}$, contributing to the Lagrangian density with the term

$$\mathcal{L}_\chi = \frac{1}{2}D_\mu\boldsymbol{\chi}\cdot D^\mu\boldsymbol{\chi} + \frac{1}{2}m^2\boldsymbol{\chi}\cdot\boldsymbol{\chi} - \frac{1}{4!}\lambda(\boldsymbol{\chi}\cdot\boldsymbol{\chi})^2 \tag{9}$$

med m^2 og λ positive, og der $\boldsymbol{\chi}$-feltet har with m^2 and λ positive, and where the der $\boldsymbol{\chi}$-field has three real components (hence the vector notation). (The total Lagrangian density is thus $\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\chi$).
Which value of $\boldsymbol{\chi}\cdot\boldsymbol{\chi}$ minimizes the potential $V = -\frac{1}{2}m^2\boldsymbol{\chi}\cdot\boldsymbol{\chi} + \frac{1}{4!}\lambda(\boldsymbol{\chi}\cdot\boldsymbol{\chi})^2$?

h) The vacuum state for this system is now characterized by the value of the $\boldsymbol{\chi}$-field which minimizes the potential V. By a suitable choice of gauge this may be assumed to have the form

$$\langle\,\Omega\,|\,\boldsymbol{\chi}\,|\,\Omega\,\rangle = \varphi_0 = \begin{pmatrix} 0 \\ 0 \\ \chi_0 \end{pmatrix}, \tag{10}$$

with χ_0 real. This gives rise to mass terms for the gauge fields A^k_μ (The Higgs mechanism).

What are the masses of the three gauge fields in this case?

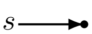
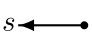
1 Connection between amplitude $\mathcal{M}_{fi}$ and decay rate Γ

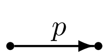
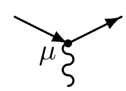
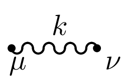
The connection between the Feynman amplitude $\mathcal{M}_{fi}$ and the decay rate $\mathrm{d}\Gamma$ is given by

$$\mathrm{d}\Gamma = (2\pi)^4\delta^{(4)}(p - \Sigma p'_f)\,\frac{1}{2E}\,|\mathcal{M}_{fi}|^2 \prod_f \frac{\mathrm{d}^3\boldsymbol{p}'_f}{(2\pi)^2 2E'_f}, \tag{11}$$

where p, E are respectively the four-momentum and the energy of the decaying particle; the other quantities refer to the particles in the final state.

2 Some Feynman rules for $-\mathrm{i}\mathcal{M}_{fi}$:

1. Outgoing particles			2. Incoming particles		
Type of particles	Graphical symbol	Algebraic expression	Type of particles	Graphical symbol	Algebraic expression
$e^-, \mu^-, \cdots$	p, s	$\overline{u}(p,s)$	$e^-, \mu^-, \cdots$	p, s 	$u(p,s)$
$e^+, \mu^+, \cdots$	p, s	$v(p,s)$	$e^+, \mu^+, \cdots$	p, s 	$\overline{v}(p,s)$
Z^0	k, r	$\varepsilon_\mu(k,r)^*$	Z^0	k, r 	$\varepsilon_\mu(k,r)$

3. Propagators			4. Vertices		
Type of particles	Graphical symbol	Algebraic expression	Interaction $\mathcal{L}_{\text{int}}$	Graphical symbol	Algebraic expression
$e^\pm, \mu^\pm, \cdots$		$\frac{\mathrm{i}(\not{p}+m)}{p^2-m^2+\mathrm{i}\epsilon}$	$\frac{-e}{\sin 2\theta_W}\overline{\psi}\gamma^\mu(g_V - g_A\gamma^5)\psi Z_\mu$		$\frac{-\mathrm{i}e}{\sin 2\theta_W}\gamma^\mu(g_V - g_A\gamma^5)$
Z^0		$\frac{-\mathrm{i}(\eta_{\mu\nu}-k_\mu k_\nu/M_Z^2)}{k^2-M_Z^2+\mathrm{i}\epsilon}$			

Here we have

$$g_V = \begin{cases} \frac{1}{2} & \text{for } \nu_e,\ \nu_\mu,\ \nu_\tau \\ \frac{1}{2}\left(-1+4\sin^2\theta_W\right) & \text{for } e,\ \mu,\ \tau \\ \frac{1}{2}\left(1-\frac{8}{3}\sin^2\theta_W\right) & \text{for } u,\ c,\ t \\ \frac{1}{2}\left(-1+\frac{4}{3}\sin^2\theta_W\right) & \text{for } d,\ s,\ b \end{cases} \qquad g_A = \begin{cases} \frac{1}{2} & \text{for } \nu_e,\ \nu_\mu,\ \nu_\tau,\ u,\ c,\ t \\ -\frac{1}{2} & \text{for } e,\ \mu,\ \tau,\ d,\ s,\ b \end{cases}$$

3 Some completeness relations

Dirac particles, Dirac antiparticles, and Z^0 vector bosons

$$\sum_{s=1}^{2} u(p,s)\,\overline{u}(p,s) = \not{p} + m, \qquad \sum_{s=1}^{2} v(p,s)\,\overline{v}(p,s) = \not{p} - m \tag{12}$$

$$\sum_{r=1}^{3} \varepsilon_\mu(k,r)\,\varepsilon_\nu^*(k,r) = -\eta_{\mu\nu} + k_\mu k_\nu / M_Z^2 \tag{13}$$

4 Dirac γ-matrices

4.1 Standard representation

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \qquad \vec{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \tag{14}$$

where I is a 2×2 unit matrix, and $\boldsymbol{\sigma}$ are the Pauli matrices,

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{15}$$

obeying the algebraic relation

$$\sigma^i\,\sigma^j = \delta^{ij} + \mathrm{i}\varepsilon^{ijk}\,\sigma^k, \text{ dvs. at } (\boldsymbol{\sigma}\cdot\mathbf{a})\,(\boldsymbol{\sigma}\cdot\mathbf{b}) = \mathbf{a}\cdot\mathbf{b} + \mathrm{i}\,\boldsymbol{\sigma}\cdot(\mathbf{a}\times\mathbf{b}) \tag{16}$$

4.2 Algebraic relations

$$\{\gamma^5, \gamma^\nu\} = 0, \tag{17}$$

$$(\gamma^5)^2 = 1, \tag{18}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2\,\eta^{\mu\nu} \quad \Longrightarrow \not{p}\not{p} = p^2 \tag{19}$$

$$\gamma^\mu\,\gamma^\nu\,\gamma_\mu = -2\,\gamma^\nu \quad \Longrightarrow \gamma_\mu \not{p}\,\gamma^\mu = -2\not{p} \tag{20}$$

$$\gamma^\mu\,\gamma^\nu\,\gamma^\lambda\,\gamma_\mu = 4\,\eta^{\nu\lambda} \quad \Longrightarrow \gamma_\mu \not{p}\not{q}\,\gamma^\mu = 4\,(pq) \tag{21}$$

$$\gamma^\mu\,\gamma^\nu\,\gamma^\lambda\,\gamma^\sigma\,\gamma_\mu = -2\,\gamma^\sigma\,\gamma^\lambda\,\gamma^\nu \quad \Longrightarrow \gamma_\mu \not{p}\not{q}\not{r}\,\gamma^\mu = -2\not{r}\not{q}\not{p} \tag{22}$$

4.3 Some traces of γ-matrices

$$\mathrm{Tr}\,1 = 4 \tag{23}$$

$$\mathrm{Tr}\,\gamma^5 = 0 \tag{24}$$

$$\mathrm{Tr}\,\gamma^\mu = 0 \tag{25}$$

$$\mathrm{Tr}\,\gamma^\mu\gamma^5 = 0 \tag{26}$$

$$\mathrm{Tr}\,\gamma^\mu\,\gamma^\nu = 4\,\eta^{\mu\nu} \quad \Longrightarrow \mathrm{Tr}\,\not{p}\not{q} = 4\,(pq) \tag{27}$$

$$\mathrm{Tr}\,\gamma^\mu\,\gamma^\nu\gamma^5 = 0 \tag{28}$$

$$\mathrm{Tr}\,\gamma^\mu\,\gamma^\nu\,\gamma^\lambda = 0 \tag{29}$$

$$\mathrm{Tr}\,\gamma^\mu\,\gamma^\nu\,\gamma^\lambda\gamma^5 = 0 \tag{30}$$

$$\mathrm{Tr}\,\gamma^\mu\,\gamma^\nu\,\gamma^\lambda\,\gamma^\sigma = 4\left(\eta^{\mu\nu}\,\eta^{\lambda\sigma} - \eta^{\mu\lambda}\,\eta^{\nu\sigma} + \eta^{\mu\sigma}\,\eta^{\nu\lambda}\right) \tag{31}$$

$$\Longrightarrow \mathrm{Tr}\,\not{p}\not{q}\not{r}\not{s} = 4(pq)(rs) - 4(pr)(qs) + 4(ps)(qr)$$

$$\mathrm{Tr}\,\gamma^\mu\,\gamma^\nu\,\gamma^\lambda\,\gamma^\sigma\,\gamma^5 = -4\mathrm{i}\,\varepsilon^{\mu\nu\lambda\sigma} \tag{32}$$