Norwegian University of Science and Technology
Department of Physics

samme oppg gjelder også FY 8902

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**EXAMINATION IN FY3201 ATMOSPHERIC PHYSICS AND CLIMATE CHANGE**

Faculty for Natural Sciences and Technology

27 May 2009

Time: 09:00-13:00

Permitted help sources: 1 sheet A5 with printed or handwritten formulas permitted
English dictionary permitted
Approved calculators are permitted

You may take:

Mass of water vapour ~18 kg/kmole

Mass of dry air ~29 kg/kmole

1 hPa = $10^2$ Pa = $10^2$ N $m^{-2}$

Scale Height, H=R·T/g

g=9.8 m $s^{-2}$ and constant in z

$R_d$=287 J $K^{-1}$ $kg^{-1}$

273 K = 0 °C

# SOLUTIONS

**1) Atmospheric structure (20 %):**

a) A hurricane at sea has a central sea-level pressure of 940 hPa and is surrounded by air with a sea-level pressure of 1014 hPa. At 200 hPa, the depression in the pressure field vanishes (that is, the 200 hPa surface is perfectly horizontal as one goes from the storm centre to the open ocean). In the region outside of the hurricane, the average temperature of the layer between 1014 and 200 hPa is -3°C. Assuming dry air both inside and outside the hurricane, what is the average temperature of the layer between 940 and 200 hPa in the hurricane's centre? (15 %)

*They can use the Hypsometric equation directly:*

$$z_2 - z_1 = \frac{RT}{g}\ln\left(\frac{p_1}{p_2}\right)$$

*Or if they forget that one, they can easily derive it from the Hydrostatic Equation:* $\frac{dp}{dz} = -g\rho$, *and the Perfect Gas Law:* $\rho = \frac{p}{RT}$, *to get:* $\frac{dp}{p} = -\frac{g}{RT}dz$, *which integrates directly into the Hypsometric equation for constant T. Something we have done countless times in class!*

*Then outside the hurricane:* $z_{200}^{out} - z_{1014}^{out} = \frac{RT_{out}}{g}\ln\left(\frac{p_{1014}}{p_{200}}\right)$,

*and inside:* $z_{200}^{in} - z_{940}^{in} = \frac{RT_{in}}{g}\ln\left(\frac{p_{940}}{p_{200}}\right)$.

*The stated boundary conditions are:* $z_{200}^{in} = z_{200}^{out}$, $z_{940}^{in} = z_{1015}^{out} = 0$, *and* $T_{out} = -3°C = 270K$, *they get:*

$$T_{in} = T_{out}\frac{\ln\left(\frac{p_{1014}}{p_{200}}\right)}{\ln\left(\frac{p_{940}}{p_{200}}\right)} = 270K\frac{1.62}{1.55} = 283.2K = 10.2°C$$

b) Describe briefly the air flow and winds in the vicinity of the hurricane. (5 %)

*Winds flow from high pressure to low, and at a given altitude below the 200 hPa level the pressure outside the hurricane is greater than inside at any altitude. Flow will be into the centre of the hurricane would give full marks. Spinning anti-clockwise with the Coriolis force is great, but beyond what is needed.*

**2) Radiation and atmospheric structure (20 %)**

A planet has an atmosphere composed of a gas with a constant absorption coefficient, $k_\nu$=0.01 $m^2\ kg^{-1}$. The atmospheric pressure at the surface is 1000 hPa, the temperature lapse rate is isothermal, the scale height is 10 km and the gravitational acceleration is 10 m $s^{-2}$. From the definition of optical depth and the basic atmospheric structure equations, determine the height and pressure of the point where the optical depth is one.

*From the definition of optical depth, the point at which it is one is:*

$$\tau_\nu(z) = k_\nu \int_z^\infty \rho \cdot dz = 1$$

*Given the hydrostatic equation,* $\frac{dp}{dz} = -g\rho$, *we can say:* $\rho \cdot dz = -\frac{dp}{g}$.

*This means that the pressure at which the optical depth is one may be obtained:*

$\tau_\nu(z) = 1 = k_\nu \int_z^\infty \rho \cdot dz = -\frac{k_\nu}{g}\int_z^\infty dp = \frac{k_\nu p_z}{g}$, *since* $p(\infty)=0$

*Solving for p gives* $10^3$ *Pa (using g=10).*
*They should get ½ credit for reaching this point.*

*If we go farther with the hydrostatic equation and substitute the Perfect gas law:* $\rho = \frac{p}{RT}$, *we get* $\frac{dp}{p} = -\frac{g}{RT}dz$, *which we can integrate for an isothermal atmosphere to be:*

$$\ln(p_z) = \ln(p_o) - \frac{g}{RT}z = \ln(p_o) - \frac{1}{H}z$$

*So*

$$z = H \cdot \frac{\ln(p_o)}{\ln(p_z)} = 10 \cdot \ln(100) = 46\ km$$

*Of course the derivation of scale height is not needed. We have had that in the case of an isothermal atmosphere, the pressure is given by:*

$p_z = p_o \exp\left(-\frac{z}{H}\right)$, *which many may remember and use directly to convert* $p_z$ *to z.*

**3) Radiation transfer (20 %)**

The Schwarzschild radiation transfer equation may be integrated to give the radiance at any altitude, z, as follows:

$$L_\nu^\uparrow(z) = L_{\nu s}^\uparrow \cdot e^{-(\tau_s-\tau_z)/\mu} - \int_{\tau_s}^{\tau_z} J_\nu(\tau') e^{(\tau'-\tau_z)/\mu} \cdot \frac{d\tau'}{\mu}$$

$$L_\nu^\downarrow(z) = L_{\nu\infty}^\downarrow \cdot e^{\tau_z/\mu} - \int_{\tau_z}^{\tau_z} J_\nu(\tau') e^{-(\tau'-\tau_z)/\mu} \cdot \frac{d\tau'}{\mu}$$

Explain each of the terms in the equations. If we ignore scatter, which terms are dominant for short wavelength light and which terms are important for far-infrared light?

*A* ***complete*** *explanation of each of the terms would be:*

$L_\nu^\uparrow(z)$ *is the upward radiance at altitude z*

$L_{\nu s}^\uparrow$ *is the upward radiance at the surface*

$L_\nu^\downarrow(z)$ *is the downward radiance at altitude z*

$L_{\nu\infty}^\downarrow$ *is the downward radiance at the top of the atmosphere*

$\tau_s = \int_0^\infty k_\nu \cdot \rho_a \cdot dz$, *is the optical depth at the surface (integral not needed)*

$\tau_z = \int_z^\infty k_\nu \cdot \rho_a \cdot dz$, *is the optical depth at altitude z (integral not needed)*

$\mu = \cos(\theta)$, *where $\theta$ is the angle the ray makes to the vertical*

$e^{-(\tau_s-\tau_z)/\mu}$ *is the attenuation the upward going surface radiance experiences between the surface and z along the path inclined at angle $\theta$ to the vertical*

$e^{\tau_z/\mu}$ *is the attenuation the downward going radiance from the top of the atmosphere experiences coming to altitude z along the path inclined at angle $\theta$ to the vertical*

$J_\nu(\tau')$ *is the source term, or how much radiance the atmosphere emits at some intermediate level $\tau'$.*

$e^{-(\tau'-\tau_z)/\mu}$ *is the attenuation source radiance experiences between the intermediate level and z along the path inclined at angle $\theta$ to the vertical*

***HOWEVER I would accept a more compact description such as:***

*First terms = boundary radiance attenuated over the path to z inclined at angle $\theta$ to the vertical, where $\mu=\cos(\theta)$*

*Second terms = atmospheric source radiance at an intermediate level, $\tau'$, attenuated over the path to z inclined at angle $\theta$ to the vertical*

*That carries all the essential understanding: Bounday radiance, Atmospheric source radiance, attenuation and path angle .*

***For the second part,***

*Short wavelengths: only* $L^{\downarrow}_{\nu\infty} \cdot e^{\tau_z/\mu}$ *is important*

*Long wavelengths, only* $L^{\downarrow}_{\nu\infty} \cdot e^{\tau_z/\mu}$ *can be ignored*

***Weighting:*** ***1st part is 15%***
***2nd part is 5%***

4) **Climate (20 %)**

a) What are the most important optical properties of a gas that allow it to create a greenhouse effect? (5 %)
b) With the addition of a greenhouse gas to the atmosphere, briefly describe the process by which the lower atmosphere warms? (5 %)
c) What is meant by a radiative equilibrium temperature? (5 %)
d) In a real atmosphere, where the temperature of the lower atmosphere is determined by convective equilibrium, why does a greenhouse gas like $CO_2$ have a radiative cooling rate of ~2 K/day? (5 %)

*a) Must transmit at short wavelengths and absorb at long*

*b) The Earth's long-wavelength radiation is absorbed and ½ of it is returned back to the surface to create additional heating in the lower atmosphere*

*c) When the radiative energy absorbed = radiative energy emitted. If we assume that the atmosphere radiates as a grey body, the Planck formula may be used to determine the temperature at which one must be to radiate that much energy. This temperature is the radiative equilibrium temperature.*

*d) In an atmosphere in radiative equilibrium, the distribution of radiation leads to a temperature profile that falls off faster than the dry or moist adiabatic rate in the lower atmosphere. Convective equilibrium redistributes the air so as to lessen that temperature fall off, which warms the lower atmosphere. Hence it gives additional energy to the atmosphere. A greenhouse gas like $CO_2$ now is at a temperature higher than its radiative equilibrium point, and thus tries to radiate away that extra energy to return back down to radiative equilibrium (ie, cool back to radiative equilibrium).*

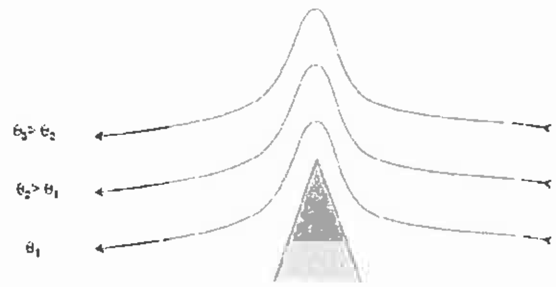
**5) Atmospheric Structure and Thermodynamics (20 %)**

$T^{c_p} \cdot p^{-R} = constant$ for an adiabatic (isentropic) process ($\delta q=0$). Use this to show that the potential temperature of a parcel of air when it goes adiabatically from one state to another is: $\theta = T(p_o / p)^{\kappa}$, where $\kappa=R/C_p$. How will the potential temperature of an air parcel change as it undergoes adiabatic expansion or compression? Use this to sketch the flow of air over a mountain range and compare this to the potential temperature structure of the atmosphere. What would happen to the potential temperature of a parcel of air in this flow if some of its water vapour condensed as the parcel went over the top of the mountain?

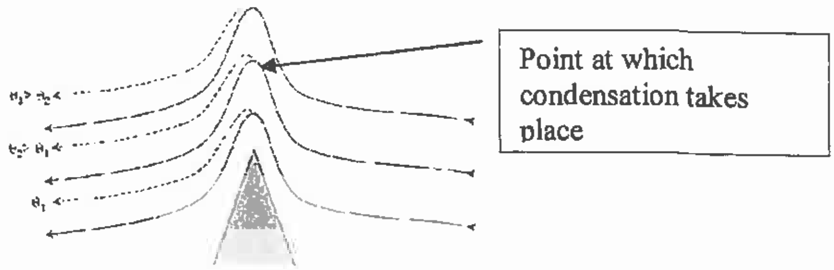
*a) At state 1, temperature =T and pressure = p. At state 2, temperature = $\theta$ and pressure = $p_o$. According to the relation, $\theta^{c_p} \cdot p_o^{-R} = T^{c_p} \cdot p^{-R}$. And this may be written $\theta = T(p_o / p)^{\kappa}$*

*b) The potential temperature of the air parcel is constant as long as it undergoes only adiabatic changes*

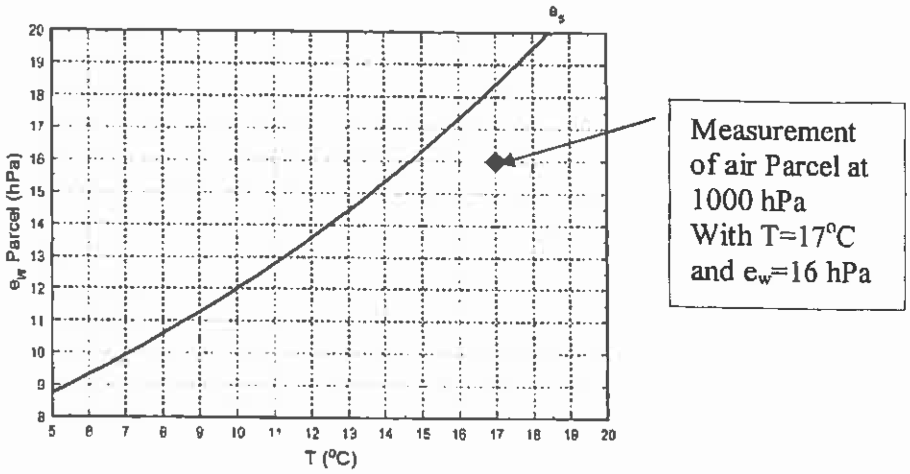
*c) As it flows over a mountain, the lines of potential temperature are the same as the air flow:*




*d) As condensation takes place, the latent heat of water is given off by the condensing vapour and heats the surrounding air. We no longer have an adiabatic process because $\delta q \neq 0$, and the parcel warms, increasing its potential temperature:*



**6) Moisture in the atmosphere (20 %)**



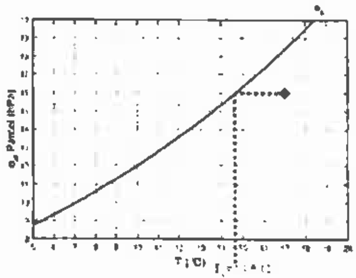

A water vapour pressure, $e_w$, of 16 hPa is measured in a parcel of air that is at a pressure of 1000 hPa and a temperature of 17 °C. This is given by the data point on the above graph, while the saturation vapour pressure curve is given by the line.

Use the graph above to determine:

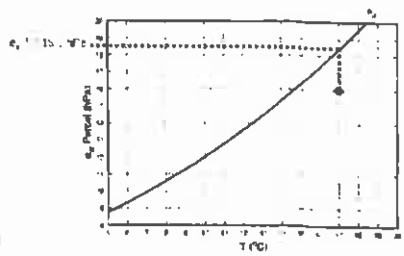
a) The dew point temperature ($T_d$) of the parcel
b) The saturation vapour pressure of the parcel

From these, calculate:

c) The relative humidity of the parcel
d) The saturation mixing ratio of the parcel, $\mu_s$, expressed in g/kg



*a)*



*b)*

*c) RH=$e_w(T)/e_s(T)$ = 16 hPa/18.5 hPa = 86% relative humidity*

*d)* $\mu_s(T,p) = \frac{m_{H_2O}}{m_d}\frac{e_s(T)}{p}$, *and here, $e_s(T=17C)$=18.5 hPa, p=1000 hPa*

*and from our table on page 1, Mass of water vapour ~18 kg/kmole, Mass of dry air ~29 kg/kmole, so $\mu_s=(18/29)(18.5/1000)x10^3$ g/kg=11.4g/kg*