Oppgave 1

a) $R = \rho \cdot \frac{L}{S} = 0.424 \cdot 10^{-6} \cdot \frac{1{,}000}{0{,}200 \cdot 10^{-6}} \,\Omega = \underline{2{,}12\,\Omega}$

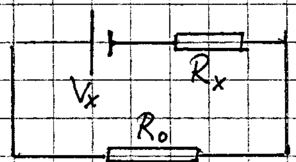
b) $\frac{V_a}{R+R_a} \cdot R \cdot \frac{AC}{AB} = V_s = \frac{V_a}{\frac{\rho L}{S} + R_a} \cdot \frac{\rho L}{S} \cdot \frac{AC}{L}$

$$AC = \frac{\frac{V_s}{V_a} \cdot (\rho L + S R_a)}{\rho} = \frac{1{,}02\,(0{,}424 \cdot 1{,}000 + 0{,}200 \cdot 0{,}030)}{4{,}00 \cdot 0{,}424}\, m = \underline{0{,}259\, m}$$

c) $V_s = I \cdot R \cdot \frac{AC_1}{AB} \qquad V_x = I \cdot R \cdot \frac{AC_2}{AB}$

$$V_x = V_s \cdot \frac{AC_2}{AC_1} = 1{,}02 \cdot \frac{0{,}763}{0{,}259}\, V = \underline{3{,}00\, V}$$

d)



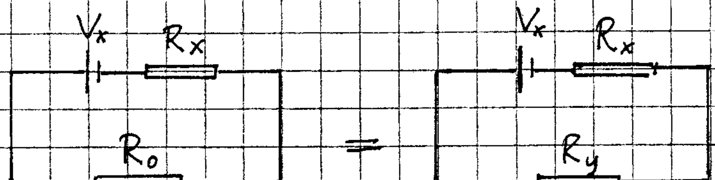
$I_o = \frac{V_o}{R_o}$

$V_x = I_o(R_o + R_x)$

$$R_x = \frac{V_x - I_o R_o}{I_o} = \frac{V_x - \frac{V_o}{R_o} \cdot R_o}{\frac{V_o}{R_o}} = \frac{(V_x - V_o) R_o}{V_o} = \frac{(3{,}00 - 2{,}95) \cdot 300}{2{,}95}\,\Omega$$

$$R_x = \underline{5{,}1\,\Omega}$$

e)



$$R_y = \frac{R_o R_e}{R_o + R_e}$$

$$P = R_y I^2 = R_y \left(\frac{V_x}{R_x + R_y}\right)^2 = \frac{R_o R_e}{R_o + R_e} \left(\frac{V_x}{R_x + \frac{R_o R_e}{R_o + R_e}}\right)^2$$

$$= \frac{R_o R_e V_x^2 (R_o + R_e)^2}{(R_o + R_e)(R_x R_o + R_x R_e + R_o R_e)^2} = \frac{R_o R_e (R_o + R_e) V_x^2}{(R_x R_o + R_x R_e + R_o R_e)^2}$$

Oppg 1 forts

$$\frac{dP}{dR_e} = 0 = \frac{(R_0^2V_x^2 + 2R_eR_0V_x^2)(R_xR_0 + R_xR_e + R_0R_e)^2 - 2(R_xR_0 + R_xR_e + R_0R_e)(R_x + R_0)(R_0 + R_e)R_0R_eV_x^2}{(R_xR_0 + R_xR_e + R_0R_e)^4}$$

$$R_0^3R_xV_x^2 + R_0^2R_xR_eV_x^2 + R_0^3R_eV_x^2 + 2R_0^2R_xR_eV_x^2 + 2R_0R_xR_e^2V_x^2 + 2R_0^2R_e^2V_x^2$$
$$-2R_0^2R_xR_eV_x^2 - 2R_0R_xR_e^2V_x^2 - 2R_0^3R_eV_x^2 - 2R_0^2R_e^2V_x^2 = 0$$

$$\underline{R_e = \frac{R_0R_x}{R_0 - R_x}} = \frac{300 \cdot 5.08}{300 - 5.08}\ \Omega = \underline{5.168\ \Omega}$$

$$R_y = \frac{R_0R_e}{R_0 + R_e} = \frac{300 \cdot 5.168}{300 + 5.168}\ \Omega = 5.08\ \Omega = R_x$$

$$P = R_yI^2 = R_y\left(\frac{V_x}{R_x + R_y}\right)^2 = \frac{5.08 \cdot 3.00^2}{(5.08 + 5.08)^2}\ W = \underline{0.443\ W}$$

$$P_x = R_x \cdot I^2 = R_x\left(\frac{V_x}{R_x + R_x}\right)^2 = \frac{V_x^2}{4R_x} = \frac{3.00^2}{4 \cdot 5.08}\ W = \underline{0.443\ W}$$

Enklere:

$$P = R_y \cdot I^2 = R_y \cdot \left(\frac{V_x}{R_x + R_y}\right)^2$$

$$\frac{dP}{dR_y} = 0 = \left(\frac{V_x}{R_x + R_y}\right)^2 - \frac{2R_yV_x^2}{(R_x + R_y)^3}$$

$$R_y = \underline{R_x} = \frac{R_0R_e}{R_0 + R_e}$$

$$R_e = \underline{\frac{R_0R_x}{R_0 - R_x}} = \underline{5.168\ \Omega}$$

Oppgave 2

a) Svingeligning:

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0$$

Udempet ($b=0$):

$$\omega=\omega_0=\sqrt{\frac{k}{m}}=\frac{2\pi}{T_0}=\frac{2\pi}{20}\,s^{-1}\Rightarrow k=m\cdot\left(\frac{2\pi}{T_0}\right)^2=5\left(\frac{2\pi}{20}\right)^2 N/m=\underline{0{,}493\,N/m}$$

Dempet ($b\neq0$):

$$\omega=\sqrt{\frac{k}{m}-\left(\frac{b}{2m}\right)^2}=\frac{2\pi}{T}=\frac{2\pi}{21}\,s^{-1}$$

$$b=2m\sqrt{\frac{k}{m}-\omega^2}=2m\sqrt{\frac{k}{m}-\left(\frac{2\pi}{T}\right)^2}=2\cdot5\cdot2\pi\sqrt{\frac{1}{20^2}-\frac{1}{21^2}}\,Ns/m=\underline{0{,}958\,Ns/m}$$

b) $$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=F_0\cos(\omega_f\cdot t)$$

$$x=A\sin(\omega_f\cdot t+\alpha)$$

$$\frac{dx}{dt}=\omega_f\cdot A\cdot\cos(\omega_f\cdot t+\alpha)$$

$$\frac{d^2x}{dt^2}=-\omega_f^2\cdot A\cdot\sin(\omega_f\cdot t+\alpha)$$

Innsatt:

$$-m\omega_f^2A\sin(\omega_f\cdot t+\alpha)+b\omega_f\cdot A\cos(\omega_f\cdot t+\alpha)+kA\sin(\omega_f\cdot t+\alpha)=F_0\cos\omega_f\cdot t$$

$$\sin(\omega_f\cdot t+\alpha)=\sin\omega_f t\cos\alpha+\cos\omega_f t\sin\alpha$$

$$\cos(\omega_f\cdot t+\alpha)=\cos\omega_f t\cos\alpha-\sin\omega_f t\sin\alpha$$

$$-m\omega_f^2A\sin\omega_f t\cos\alpha-m\omega_f^2A\cos\omega_f t\sin\alpha+b\omega_f A\cos\omega_f t\cos\alpha$$
$$-b\omega_f A\sin\omega_f t\sin\alpha+kA\sin\omega_f t\cos\alpha+kA\cos\omega_f t\sin\alpha=F_0\cos\omega_f t$$

$$A[(-m\omega_f^2+k)\cos\alpha-b\omega_f\sin\alpha]\sin\omega_f t$$
$$+A[(k-m\omega_f^2)\sin\alpha+b\omega_f\cos\alpha]\cos\omega_f t=F_0\cos\omega_f t$$

Dette er OK hvis

$$A[(k-m\omega_f^2)\sin\alpha+b\omega_f\cos\alpha]=F_0 \qquad (1)$$

$$A[(k-m\omega_f^2)\cos\alpha-b\omega_f\sin\alpha]=0 \qquad (2)$$

Oppg 2 forts

Vi antar $A \neq 0$

Av (2): $\tan\alpha = \frac{k - m\omega_f^2}{b\omega_f}$ eller $\underline{\alpha = \arctan\frac{k - m\omega_f^2}{b\omega_f}}$

Av (1): $A[(k - m\omega_f^2)\tan\alpha + b\omega_f] = F_0\sqrt{1+\tan^2\alpha}$

$$A = \frac{F_0\sqrt{b^2\omega_f^2 + (k - m\omega_f^2)^2}}{b\omega_f\left[\frac{(k - m\omega_f^2)(k - m\omega_f^2)}{b\omega_f} + b\omega_f\right]}$$

$$\underline{A = \frac{F_0}{\sqrt{(k - m\omega_f^2)^2 + b^2\omega_f^2}}} \qquad \text{q.e.d.}$$

c) A er størst når $N = b^2\omega_f^2 + (k - m\omega_f^2)^2$ er minst

$$\frac{dN}{d\omega_f} = 2b^2\omega_f + 2(k - m\omega_f^2)(-2m\omega_f) = 0$$

$$\Rightarrow \omega_f = \underline{\sqrt{\frac{k}{m} - \frac{b^2}{2m^2}}} = \sqrt{\frac{0.493}{5} - \frac{0.958^2}{2\cdot 5^2}}\ s^{-1} = \underline{0{,}283\ s^{-1}}$$

d) $$\overline{P} = \overline{b\cdot\frac{dx}{dt}\cdot\frac{dx}{dt}} = \overline{bv^2} = \overline{b\cdot A^2\cdot\omega_f^2\cdot\cos^2(\omega_f t + \alpha)}$$

$$\overline{\cos^2(\omega_f t + \alpha)} = 1/2$$

$$\underline{\overline{P} = \frac{bA^2\omega_f^2}{2}} = \frac{0.958\cdot 0.400^2}{2[(0.493 - 5\cdot 0.400^2)^2 + 0.958\cdot 0.400^2]}\ W = \underline{0{,}32\ W}$$

e) Amplituden dempes eksponentielt

$$A\cdot e^{-\frac{b}{2m}t} = \frac{A}{2} \Rightarrow \frac{b}{2m}t = \ln 2$$

$$t = \underline{\frac{2m}{b}\ln 2} = \frac{2\cdot 5{,}00}{0.958}\cdot\ln 2\ s = \underline{7{,}24\ s}$$

Opgave 3

a) Gauss: $\oint \vec{E}\, d\vec{A} = \frac{q}{\varepsilon}$

i) $r < a$: $q = 0 \Rightarrow E = 0$

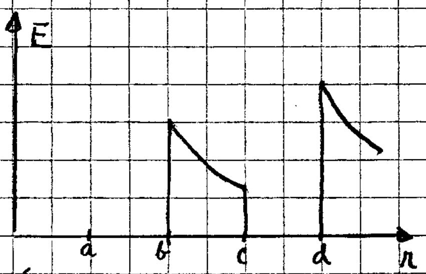
ii) $a < r < b$: Ladningen vil samle seg på ytterflaten av den indre sylinderen $\Rightarrow E = 0$

iii) $b < r < c$: Vi velger sylindrisk Gaussflate, lengde l

$$\oint \bar{E}\, d\bar{A} = E(r) \cdot 2\pi r \cdot l = \frac{\lambda_1 \cdot l}{\varepsilon_0}$$

$$E(r) = \frac{\lambda_1}{2\pi\varepsilon_0 r} \Rightarrow \vec{E}(\vec{r}) = \underline{\frac{\lambda_1}{2\pi\varepsilon_0 r} \cdot \hat{r}} = \underline{\frac{\lambda_1 \cdot \vec{r}}{2\pi\varepsilon_0 r^2}}$$

iv) $c < r < d$: $E = 0$ i metallet.



v) $r > d$:

$$\oint \bar{E}\, d\bar{A} = E(r) \cdot 2\pi r l = \frac{(\lambda_1 + \lambda_2) \cdot l}{\varepsilon_0}$$

$$E(r) = \frac{(\lambda_1 + \lambda_2)}{2\pi\varepsilon_0 r} \Rightarrow \bar{E}(\vec{r}) = \underline{\frac{(\lambda_1 + \lambda_2)\hat{r}}{2\pi\varepsilon_0 r}} = \underline{\frac{(\lambda_1 + \lambda_2)\vec{r}}{2\pi\varepsilon_0 r^2}}$$

b) Ladning pr lengdeenhet i indre sylindervegg av den ytre sylinderen er λ_{2i} mens ladning pr lengdeenhet i ytre sylindervegg av ytre sylinder er λ_{2y}.

$(\lambda_{2i} + \lambda_{2y}) = \lambda_2$

Vi legger en Gaussflate i ytre sylinder. Den ladningen som befinner seg innenfor, er $(\lambda_1 + \lambda_{2i}) \cdot l$

$E(r) = 0$ i metallet

$$\Rightarrow E(r) \cdot 2\pi r l = \frac{(\lambda_1 + \lambda_{2i}) \cdot l}{\varepsilon} = 0$$

$$\Rightarrow \lambda_{2i} = \underline{-\lambda_1}$$

$$\lambda_{2y} = \lambda_2 - \lambda_{2i} = \underline{\lambda_2 + \lambda_1}$$

Oppgave 3 forts.

c) Potensialforskjellen mellom indre og ytre sylinder, dvs mellom b og c:

$$\Delta V = V_b - V_c = -\int_c^b \vec{E}(\vec{r})\,d\vec{r} = \frac{-\lambda_1}{2\pi\varepsilon_0}\int_c^b \frac{1}{r}\,dr = \frac{\lambda_1}{2\pi\varepsilon_0}\ln\frac{c}{b}$$

Kapasitans pr. lengdeenhet:

$$C = \frac{\lambda_1}{V_b - V_c} = \underline{\frac{2\pi\varepsilon_0}{\ln\frac{c}{b}}}$$

d) $$\frac{\text{Energi}}{\text{lengde}} = \frac{1}{2}\,\frac{q\cdot\Delta V}{\text{lengde}} = \frac{1}{2}\lambda_1(V_b - V_c) = \underline{\frac{\lambda_1^2}{4\pi\varepsilon_0}\ln\frac{c}{b}}$$

Alternativt: Energitetthet/lengde $= u = \frac{1}{2}\varepsilon_0 E^2 \cdot \frac{1}{\ell}$

$$\frac{\text{Energi}}{\text{lengde}} = \int_b^c \frac{1}{2}\varepsilon_0 E^2 \cdot \frac{1}{\ell}\cdot 2\pi r\,dr\cdot \ell$$

$$= \int_b^c \frac{1}{2}\varepsilon_0\,\frac{\lambda_1^2}{4\pi^2\varepsilon_0^2\cdot r^2}\,\frac{2\pi r\,\ell\,dr}{\ell} = \underline{\frac{\lambda_1^2}{4\pi\varepsilon_0}\ln\frac{c}{b}}$$

e) Elektrostatisk energi i kondensatoren i lengde s:

$$U_1 = \frac{\lambda_1^2\cdot s}{4\pi\varepsilon_0}\cdot\ln\frac{c}{b}$$

Når pluggen er satt i, blir energien i lengde s:

$$U_2 = \frac{\lambda_1^2\cdot s}{4\pi\varepsilon}\ln\frac{c}{b} \qquad \text{der } \varepsilon = \varepsilon_0\cdot\kappa$$

Endring ved at pluggen settes inn:

$$\Delta U = U_2 - U_1 = \underline{\frac{\lambda_1^2\cdot s}{4\pi\varepsilon_0}\left(\frac{1}{\kappa} - 1\right)\ln\frac{c}{b}}$$

f) ΔU er negativ, dvs den potensielle energien minker.

$\Longrightarrow$ Pluggen dras innover

$$F_x = -\frac{dU}{ds} = -\frac{\lambda_1^2}{4\pi\varepsilon_0}\left(\frac{1}{\kappa} - 1\right)\ln\frac{c}{b} = \underline{\frac{\lambda_1^2}{4\pi\varepsilon_0}\left(1 - \frac{1}{\kappa}\right)\ln\frac{c}{b}}$$

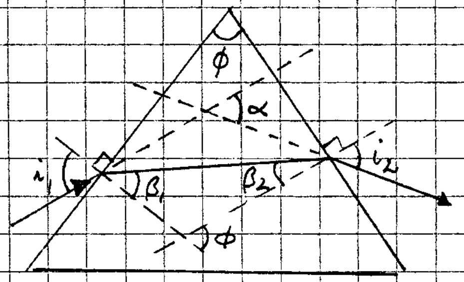
g) Intet E-felt i området $\Rightarrow U = 0 \Rightarrow \underline{\Delta U = 0}$

Oppgave 4.

a)

Snell: $\sin i_1 = n \sin \beta_1$

$\sin i_2 = n \sin \beta_2$



Vinkelen mellom innfallsloddene er ϕ. Vi ser også at ϕ er utvendig vinkel ved et hjørne i nederste trekant, slik at $\underline{\phi = \beta_1 + \beta_2}$.

Vi ser at α er utvendig vinkel ved et hjørne i øverste trekant, slik at $\alpha = (i_1 - \beta_1) + (i_2 - \beta_2) \Rightarrow \alpha = i_1 + i_2 - (\beta_1 + \beta_2) = \underline{i_1 + i_2 - \phi}$

b) Symmetrisk strålegang: $\beta_1 = \beta_2 = \beta$ og $i_1 = i_2 = i$

$\phi = 2\beta \Rightarrow \beta = \frac{\phi}{2}$ $\qquad \alpha = \alpha_{min} = 2i - \phi \Rightarrow i = \frac{1}{2}(\alpha_{min} + \phi)$

$n = \frac{\sin i}{\sin \beta} = \underline{\frac{\sin \frac{1}{2}(\alpha_{min} + \phi)}{\sin \frac{\phi}{2}}}$ q.e.d.

c) Prismet er en likesidet trekant $\Rightarrow \phi = 60°$

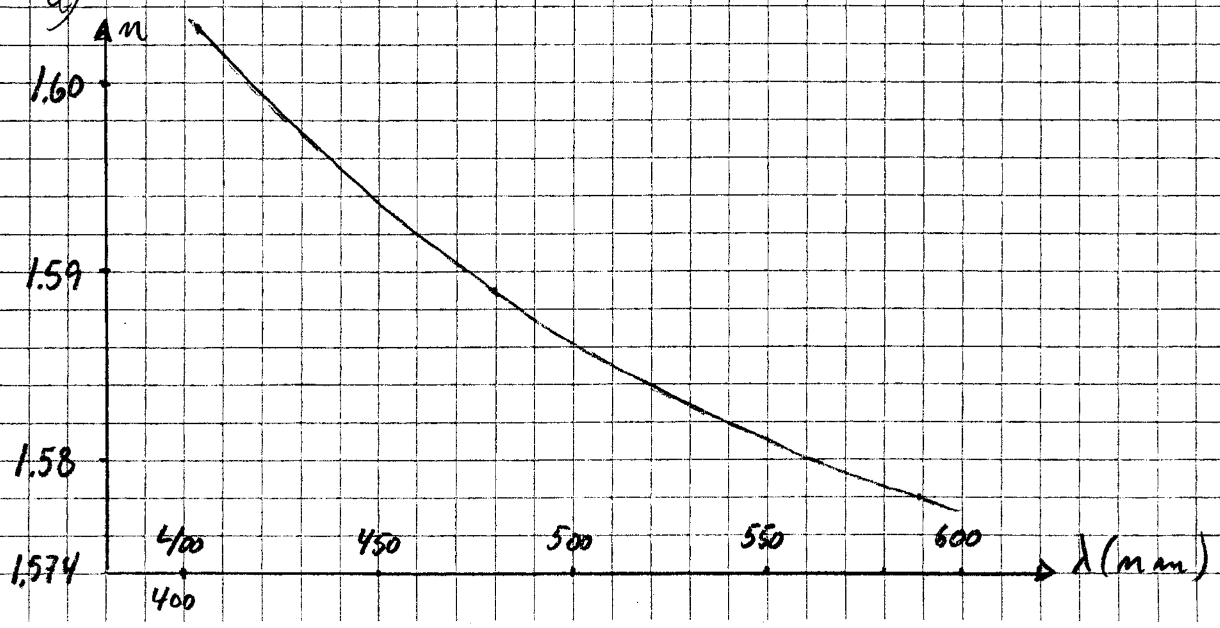
Ved innsetting i formelen for n finner vi

$\lambda_1 = 404.7$ nm $\qquad n_1 = \underline{1.603}$

$\lambda_2 = 480.0$ nm $\qquad n_2 = \underline{1.589}$

$\lambda_3 = 589.3$ nm $\qquad n_3 = \underline{1.578}$

d)

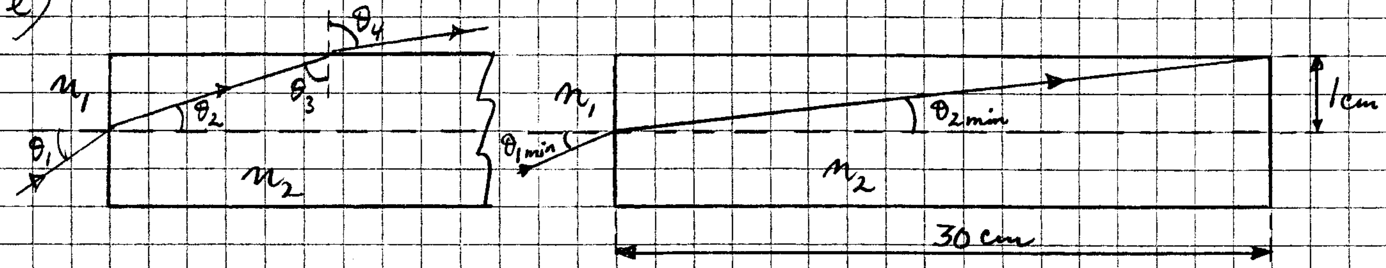


Oppgave 4 forts.

$$n = A + \frac{B}{\lambda^2}$$ Vi velger λ_1 og λ_3:

$$\left.\begin{aligned} 1{,}603 &= A + \frac{B}{404{,}7^2} \\ 1{,}578 &= A + \frac{B}{589{,}3^2} \end{aligned}\right\} \quad \underline{A = 1{,}556} \qquad \underline{B = 7{,}750 \cdot 10^3\ nm^2}$$

e)



For brytning på stavens sideflate: $n_1 \sin\theta_4 = n_2 \sin\theta_3$

Minste vinkel for totalrefleksjon når $\theta_4 = 90°$:

$n_1 \sin 90° = 1 \cdot \sin 90° = 1 = n_2 \sin\theta_{3min} = 1{,}53 \cdot \sin\theta_{3min} \Rightarrow \theta_{3min} = \underline{40{,}8°}$

Max vinkel θ_{2max} for brytning ved stavens inngangsflate:

$\theta_{2max} = 90° - \theta_{3min} = 90° - 40.8° = \underline{49{,}2°}$

$n_1 \sin\theta_{1max} = n_2 \sin\theta_{2max} \Rightarrow \sin\theta_{1max} = \frac{n_2}{n_1} \sin\theta_{2max}$

$\sin\theta_{1max} = \frac{1.53}{1.0} \sin 49{,}2° > 1 \Rightarrow \underline{\theta_{1max} = 90°}$

Skal lysstrålen treffe stavens sideflate, er $\theta_{2min} = \arctan\left(\frac{1}{30}\right)$

$\theta_{2min} = \underline{1{,}91°} \Rightarrow \theta_{1min} = \arcsin(1.53 \cdot \sin 1{,}91°) = \underline{2{,}92°}$

$\Rightarrow$ <u>Totalrefleksjon inne i staven for $2{,}92° < \theta_1 < 90°$. $\theta_{1max} = 90°$</u>

f) $n_1 \sin 90° = n_2 \sin\theta_{3min}$

$1{,}33 \cdot 1 = 1.53 \cdot \sin\theta_{3min} \Rightarrow \theta_{3min} = 60{,}38° \Rightarrow \theta_{2max} = 90° - 60{,}38° = \underline{29{,}62°}$

$n_1 \sin\theta_{1max} = n_2 \sin\theta_{2max} \Rightarrow 1.33 \sin\theta_{1max} = 1.53 \sin 29.62° \Rightarrow \theta_{1max} = \underline{34{,}7°}$

$n_1 \sin\theta_{1min} = n_2 \sin\theta_{2min}$

$\theta_{1min} = \arcsin\left(\frac{1.53}{1.33} \cdot \sin 1{,}91°\right) = \underline{2{,}20°}$

$\Rightarrow$ <u>Totalrefleksjon inne i staven for $2{,}20° < \theta_1 < 34{,}7°$. $\theta_{1max} = 34{,}7°$</u>