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Contact during the exam:
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Exam in SIF4052 SOLID STATE PHYSICS 1

Saturday, December 16, 2000
09:00–14:00

Allowed help: Alternativ B

Godkjent lommekalkulator.
K. Rottman: *Matematisk formelsamling* (alle språkutgaver).
O.H. Jahren og K.J. Knudsen: *Formelsamling i matematikk*.

This problem set consists of 2 pages.

Problem 1

For long wave lengths, the equation of motion for longitudinal waves along the [100] direction of a simple cubic crystal is

$$\frac{d^2u}{dt^2} = \left(\frac{c}{\rho}\right)\frac{d^2u}{dx^2}\,, \tag{1.1}$$

where c is the elastic modulus and ρ is the volume mass density of the crystal. In the same direction, the LA dispersion relation is

$$\omega^2 = \left(\frac{4K}{M}\right)\sin^2\left(\frac{ka}{2}\right)\,, \tag{1.2}$$

where a is the cubic lattice constant, K is the harmonic force constant between atoms and M is the mass of each atom.

a) Express c in terms of a and K.

Problem 2

Consider vibrations of a planar square lattice of rows and columns of N identical atoms, mass M spaced a distance a apart and constrained to only move in the 2D plane. $N \gg 1$.

a) Using periodic boundary conditions, show that the number of allowed values of $\vec{k}$ per unit area in $\vec{k}$ space is $(Na^2)/(2\pi)^2$.

b) What is the radius k_D of the circle in $\vec{k}$-space containing N allowed $\vec{k}$ values?

c) Assume that all waves in the plane have the same velocity v. The density of states is $g(\omega)$. Show that $g(\omega) = B\omega$ and express the constant B in terms of the given information.

d) Using the Debye model, prove that for $T \ll \Theta_D$ (where Θ_D is the Debye temperature), the heat capacity of this lattice is proportional to T^2. Express Θ_D in terms of the given information.

Problem 3

A linear monoatomic lattice has lattice constant a, and a basis of two identical atoms with mass m at 0 and $a/4$. The potential energy of an electron can be expressed as

$$V(x) = \sum_{j=-\infty}^{+\infty} V_0(x - x_j) \ , \tag{4.1}$$

where x_j is the x coordinate of the jth atom.

a) Assuming the potential is strong, sketch the electron wave function in light of the Bloch theorem.

b) Show that the Fourier coefficient of the potential energy function $\tilde{V}_G = 0$, where $G = l(2\pi/a)$, for $l = 2$. We define

$$\tilde{V}_G = \frac{2\pi}{a} \int_{-a/2}^{+a/2} V(x)\ e^{iGx}\ dx \ . \tag{4.2}$$

c) Assume that the potential is not strong enough to bind the electrons. Let the first and the third Fourier coefficients be given as $\tilde{V}_1$ and $\tilde{V}_3$ such that $\tilde{V}_1/\tilde{V}_3 = 3/2$. Make a good sketch of $E(k)$ for the extended zone scheme for $-(3.5\pi)/a < k < +(3.5\pi)/a$. Repeat showing the same bands in the reduced zone scheme on the same $E(k)$ graph.

Problem 4

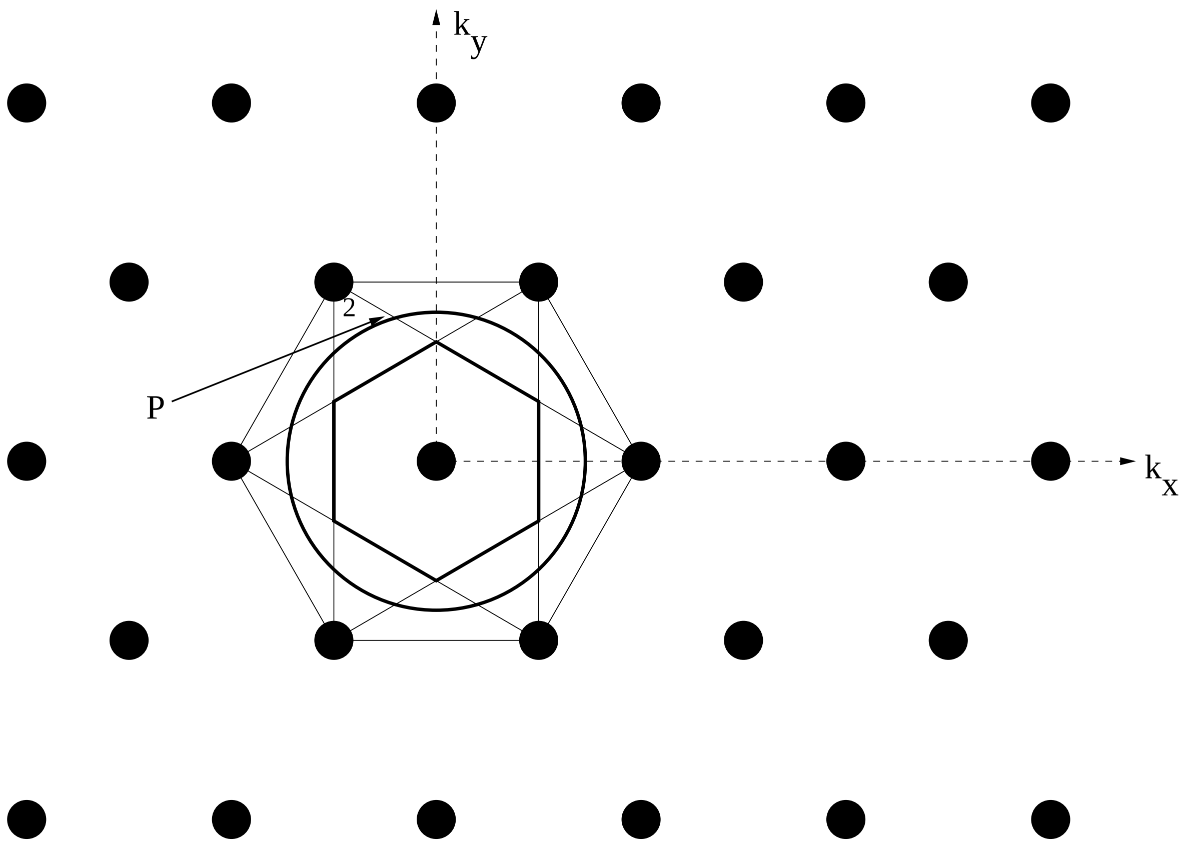
The figure shows the (k_x, k_y) plane for a two-dimensional metal having a hexagonal lattice. Reciprocal lattice points are shown as solid dots. Consider the second Brillouin zone (2BZ). One segment of it is marked "2" in the figure.

a) a) We assume that the Fermi-surface lies very close to the incribed circle. Sketch it directly on the figure or on a redrawing of it in the NFL ("Near Free Lattice") approximation.

b) What is the direction of the group velocity $\vec{v}_g$ for electrons with $\vec{k}$ vectors in the vicinity of point P just inside 2BZ.

c) Draw in all of the E_F contour lines of 2BZ in the reduced zone scheme showing how you determined where one of the elements went. Show point P in the reduced zone scheme.

d) Draw in the segments of the Fermi surface of the third Brillouin zone (3BZ) in the periodic zone scheme.


k
y
2
P
k
x