Ordinary problems.

Problem 1. Discrete time filters.

a. A moving average filter is described by the difference equation

$$y[n] = \tfrac{1}{4}x[n-1] + \tfrac{1}{2}x[n] + \tfrac{1}{4}x[n+1]$$

Assume an input signal to the filter

$$x[n] = \cos(\alpha\, n)$$

The output may then be written on form

$$y[n] = x[n]\, f(\alpha)$$

Determine the function $f(\alpha)$.

b. A discrete time LTI highpass filter has a transfer function

$$H(\mathrm{e}^{j\Omega}) = \begin{cases} 1 & \pi - \Omega_c \le |\Omega| \le \pi \\ 0 & |\Omega| < \pi - \Omega_c \end{cases}$$

Assume an input signal to the filter

$$x[n] = \begin{cases} 1 & |n| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

What is the output signal $y[n]$?

Problem 2. Correlation functions and power spectra

a. Assume some stationary stochastic process $Y(t)$, with realisations $y(t)$, and assume also that the mean value is $< y(t) >= 0$.
The autocorrelation function $R_{yy}(\tau)$

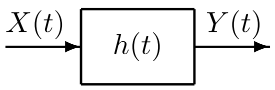
$$R_{yy}(\tau) =< y(t)y(t-\tau) >$$

and the power spectrum $G_{yy}(j\omega)$, defined by

$$< y^2 >= \int_{-\infty}^{\infty} G_{yy}(j\omega)\, \mathrm{d}\omega/2\pi$$

are related through what are called 'the Wiener-Khinchin relations'.
State these relations, in words and as mathematical expressions.
Also, comment on what kind of assumptions must be made to arrive at the expressions.

b.



Consider an LTI system as shown, with random processes $X(t)$ and $Y(t)$ as input and output, respectively, and an impulse response function $h(t)$.

Show first that the mean square of the output signal may be written as

$$< y^2 >= \int \int \mathrm{d}\tau_1 \, \mathrm{d}\tau_2 \, h(\tau_1) h(\tau_2) R_{xx}(\tau_2 - \tau_1)$$

where $R_{xx}(\tau)$ is the autocorrelation function of the input signal.
Then, use this result to prove the relation

$$G_{yy}(j\omega) = |H(j\omega)|^2 \, G_{xx}(j\omega)$$

where G_{xx} and G_{yy} are the input and output power spectra, and $H(j\omega)$ is the transfer function.

Fourier series and Fourier transforms.

We list below various Fourier series and Fourier transform pairs, as they have been used in this course. (The placements of N's and 2π's vary between different text-books.)

Periodic signals.

$\underline{x(t+T_0) = x(t).}$

$$\boxed{\begin{aligned} x(t) &= \textstyle\sum_{k=-\infty}^{\infty} X[k]\,\mathrm{e}^{jk\omega_0 t} \\ X[k] &= \tfrac{1}{T_0}\textstyle\int_{-T_0/2}^{T_0/2} x(t)\,\mathrm{e}^{-jk\omega_0 t}\,\mathrm{d}t \end{aligned}}$$

Fourier series (FS)
$\omega_0 = 2\pi/T_0$

$\underline{x[n+N] = x[n].}$

$$\boxed{\begin{aligned} x[n] &= \textstyle\sum_{k=0}^{N-1} X[k]\,\mathrm{e}^{jk\Omega_0 n} \\ X[k] &= \tfrac{1}{N}\textstyle\sum_{n=0}^{N-1} x[n]\,\mathrm{e}^{-jk\Omega_0 n} \end{aligned}}$$

Discrete time Fourier series (DTFS)
$\Omega_0 = 2\pi/N$

Aperiodic signals.

(Periodic signals with $T_0 \to \infty$ or $N \to \infty$.)

$$\boxed{\begin{aligned} x(t) &= \textstyle\int_{-\infty}^{\infty} X(j\omega)\,\mathrm{e}^{j\omega t}\,\mathrm{d}\omega/2\pi \\ X(j\omega) &= \textstyle\int_{-\infty}^{\infty} x(t)\,\mathrm{e}^{-j\omega t}\,\mathrm{d}t \end{aligned}}$$

Fourier transform (FT)

$$\boxed{\begin{aligned} x[n] &= \tfrac{1}{2\pi}\textstyle\int_{-\pi}^{\pi} X(\,\mathrm{e}^{j\Omega})\,\mathrm{e}^{j\Omega n}\,\mathrm{d}\Omega \\ X(\mathrm{e}^{j\Omega}) &= \textstyle\sum_{n=-\infty}^{\infty} x[n]\,\mathrm{e}^{-j\Omega n} \end{aligned}}$$

Discrete time Fourier transform (DTFT)

Finite length discrete datasets.

$$\boxed{\begin{aligned} x[n] &= \tfrac{1}{N}\textstyle\sum_{k=0}^{N-1} X[k]\,\mathrm{e}^{j\cdot 2\pi kn/N} \\ X[n] &= \textstyle\sum_{n=0}^{N-1} x[n]\,\mathrm{e}^{-j\cdot 2\pi kn/N} \end{aligned}}$$

Discrete Fourier transform (DFT)

Student no:

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SIF 4076/MNFFY 308 Signal processing
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Multiple choice questions.

1. Discrete time LTI system.

A discrete time LTI (linear time independent) system with input $x[n]$ and output $y[n]$ is described by the difference equation

$$y[n] = \tfrac{6}{5}\, y[n-1] + x[n]$$

The system is

A:	An IIR (infinite impulse response) filter	☐
B:	A FIR (finite impulse response) filter	☐
C:	An unstable configuration	☐
D:	Neither of those	☐

2. Continuous time LTI system.

A continuous time LTI system has input signal $x(t)$ and output signal $y(t)$. The response to an input $x(t) = \delta(t)$ is

$$y(t) = \mathrm{e}^{-a(t-t_0)}\, u(t-t_0)$$

where $u(t)$ is the unit step function, $u(t<0) = 0$ and $u(t \geq 0) = 1$.
If the input is $x(t) = \mathrm{e}^{j\omega t}$, the output will be

A:	$y(t) = x(t)\mathrm{e}^{-at_0}$	☐
B:	$y(t) = x(t)/(a + j\omega)$	☐
C:	$y(t) = x(t)\mathrm{e}^{-j\omega t_0}/(a + j\omega)$	☐
D:	Neither of those	☐

3. Analog filter

An analog filter has a transfer function

$$H(j\omega) = 1/\left(1 + j\frac{a\omega}{1 - b^2\omega^2}\right)$$

This transfer functions represents

A:	A lowpass filter	☐
B:	A bandpass filter	☐
C:	A bandstop filter	☐
D:	Neither of those	☐

4. Mathematical sampling

A signal $x(t)$ is sampled at regular intervals, giving a sampled signal

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(t)\,\delta(t - nT_s)$$

The signal x(t) is band-limited to $f_{\max} = 4.2\,kHz$, i.e. the signal does not contain frequency components with frequencies $f \geq f_{max}$.
The maximum (longest) sampling interval T_s that will allow perfect reconstruction of the input signal $x(t)$ from $x_s(t)$ is then

A: $T_s^{\max} \approx 40\,\mu s$ ☐
B: $T_s^{\max} \approx 120\,\mu s$ ☐
C: $T_s^{\max} \approx 240\,\mu s$ ☐
D: Neither of those ☐

5. Discrete time Fourier transform

A discrete time signal $x[n]$ has the values

$$x[n] = \begin{cases} 1 & |n| \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

The discrete time Fourier transform (DTFT) of this signal is

A: $X(e^{j\Omega}) = \sin(3\Omega)/(3\Omega)$ ☐
B: $X(e^{j\Omega}) = \cos(5\Omega/2)/\cos(\Omega/2)$ ☐
C: $X(e^{j\Omega}) = \sin(5\Omega/2)/\sin(\Omega/2)$ ☐
D: Neither of those ☐

6. Fourier transform of a square pulse

A continuous time signal $x(t)$ has the values

$$x(t) = \begin{cases} 1 & 0 \leq t < T \\ 0 & \text{otherwise} \end{cases}$$

The Fourier transform of the signal is

A: $T\sin(\omega T)/(\omega T)$ ☐
B: $e^{-j\omega T}\sin(\omega T)/(\omega T)$ ☐
C: $T\sin(\omega T/2)/(\omega T/2)$ ☐
D: Neither of those ☐

7. Synchronous demodulation.

A signal $x(t)$ modulates a carrier wave $w(t) = \cos \omega_c t$ to give an amplitude modulated wave $s(t) = x(t) \cos \omega_c t$. This wave is subsequently demodulated by synchronous demodulation, to give an output signal

$$y(t) = s(t) \cos \omega_c t$$

The Fourier transform $Y(j\omega) = \mathcal{F}(y(t))$ may be written as

A:	$X(j\omega) * X^*(j\omega)$	☐
B:	$X(j(\omega + \omega_c)) + X(j(\omega - \omega_c))$	☐
C:	$X(j\omega) + X(j(\omega + 2\omega_c)) + X(j(\omega - 2\omega_c))$	☐
D:	Neither of those	☐

8. Quadrature modulated process

A quadrature modulated process has in-phase and quadrature components

$$\begin{aligned} X_1(t) &= X(t) \cos(\omega_c t + \theta) \\ X_2(t) &= X(t) \sin(\omega_c t + \theta) \end{aligned}$$

where θ is randomly distributed on $(0, 2\pi)$, i.e.

$$p(\theta)\mathrm{d}\theta = \begin{cases} \mathrm{d}\theta/2\pi & \theta \in (0, 2\pi) \\ 0 & \text{otherwise} \end{cases}$$

The cross correlation function $R_{x_1 x_2}(\tau) =< x_1(t) x_2(t-\tau) >$ is then

A:	$R_{x_1 x_2}(\tau) = 0$	☐
B:	$R_{x_1 x_2}(\tau) = -\frac{1}{2} R_{xx}(\tau) \cos \omega_c \tau$	☐
C:	$R_{x_1 x_2}(\tau) = -\frac{1}{2} R_{xx}(\tau) \sin \omega_c \tau$	☐
D:	Neither of those	☐

9. Information entropy

An alphabet $\mathcal{S}$ consists of 4 letters, $\mathcal{S} = \{s_0, s_1, s_2, s_3\}$, with probabilities of occurrence $p_0 = 1/2, p_1 = 1/4, p_2 = p_3 = 1/8$.

The information entropy of this alphabet is

A:	1.75 dB	☐
B:	1.25 bits	☐
C:	1.75 bits	☐
D:	Neither of those	☐

10. Shannon's 3. theorem

Shannon's 3. theorem (the channel capacity theorem) may be formulated as

$$C = B\ \log_2(1 + P/N_0B)$$

In this equation, the meanings of (some of) the symbols are

A: N_0 is the number of channels, and P is the channel probability ☐

B: N_0 is the number of channels, and B is the bandwidth per channel ☐

C: N_0 is the noise power, and B is the bandwidth ☐

D: Neither of those ☐

11. Discrete wavelet- and scaling functions

Discrete wavelet functions $\psi(t)$ and scaling functions $\phi(t)$ are connected through relations of form

A: $\phi(t) = \sum_k p_k \psi(2t - k)$ ☐

B: $\psi(2t) = \sum_k q_k \phi(t - k)$ ☐

C: $\psi(t) = \sum_k r_k \phi(2t - k)$ ☐

D: Neither of those ☐

12. About the questions

You don't have to answer this – but if you do, it is fine:
This exam was

A: About as expected ☐

B: Much worse ☐

C: Much easier ☐

D: Neither of those ☐