TFY4102 Løsningsforslag Eksamen 19. mai 2009. ①

Oppgave 1.

a) Horisontal bevegelse: $v_x = v \cdot \cos\theta$ $\quad x(t) = v_x \cdot t$

Vertikal bevegelse: $v_y = v \cdot \sin\theta - gt$

$$y(t) = v_y t - \frac{1}{2} g t^2$$

Varighet av flytur: t_1

Fra horisontal bevegelse: $\underline{t_1} = \frac{x(t_1)}{v_x} = \underline{\frac{s_0}{v \cdot \cos\theta}}$

Fra vertikal bevegelse: $y(t_1) = h_0$

$$\Rightarrow h_0 = v_y \cdot t_1 - \frac{1}{2} g t_1^2$$

$$h_0 = \cancel{v} \cdot \sin\theta \cdot \frac{s_0}{\cancel{v} \cdot \cos\theta} - \frac{1}{2} g \frac{s_0^2}{v^2 \cos^2\theta} \qquad \Big| \, v^2 \cdot \cos^2\theta \cdot 2$$

$$2v^2 \cdot h_0 \cos^2\theta = 2v^2 \cdot s_0 \sin\theta \cdot \cos\theta - g s_0^2$$

$$\Rightarrow v^2 = \frac{s_0^2 \cdot g}{2 \cos\theta (s_0 \sin\theta - h_0 \cos\theta)}$$

$$\Rightarrow v = s_0 \sqrt{\frac{g}{2\cos\theta (s_0 \sin\theta - h_0 \cos\theta)}} \qquad \text{q.e.d.}$$

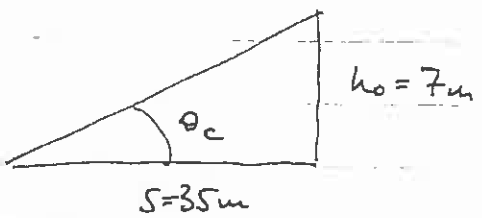
b) $v(\theta) \to \infty$ for $\theta \to \theta_c$

θ_c gitt ved at $s_0 \sin\theta_c - h_0 \cos\theta_c = 0$

$$\Rightarrow \quad \underline{\underline{\tan\theta_c = \frac{h_0}{s_0}}} \quad \Rightarrow \quad \underline{\underline{\theta_c = \arctan\left(\frac{h_0}{s_0}\right)}}$$

Tallverdi: $\underline{\underline{\theta_c}} = \arctan\left(\frac{7.0}{35.0}\right) = \underline{\underline{11,3^\circ}}$

Tolkning:



For $\theta = \theta_c$ må bilen "fly" rettlinjet fra 0 mot treffpunktet, dvs. det kan ikke tillates noen avbøyning pga. tyngdens akselerasjon, dvs. "flytiden" må gå mot null, og tilsvarende $v \to \infty$.

Hvis $\theta < \theta_c$ vil bilen treffe $y < h_0$ selv om $v \to \infty$. Matematisk blir v da gitt som $\sqrt{-\text{tall}}$ dvs. et imaginært tall. Begge deler angir at problemet ikke har løsning.

c) Fra grafen estimeres $v(\theta)$ å ha minimalverdi for $\theta \simeq 50^\circ$

$$v_{min} = 35m \sqrt{\frac{9{,}81\ m/s^2}{2 \cdot \cos 50^\circ [35m \cdot \sin 50^\circ - 7m \cdot \cos 50^\circ]}} = 35 \sqrt{\frac{9{,}81}{28{,}68}}\ m/s$$

$$\underline{\underline{v_{min}}} = 35{,}0 \cdot 0{,}585 \ \mathrm{m/s} = \underline{\underline{20{,}5 \ \mathrm{m/s} = 74 \ \mathrm{km/t}}}$$

Dette synes ikke å være en "svært høy fart", men er allikevel langt over fartsgrensen for tettbygd strøk. Fartsgrensen på stedet er ukjent. Farten på veien var nødvendigvis høyere enn v_{min}.

d) Kollisjonskraften F stanser bilen over strekningen $s = 2\mathrm{m}$.
Hastighet like før kollisjonen setter til v_1.

$$\Rightarrow \quad F \cdot s = \tfrac{1}{2} m v_1^2$$ fra energibetraktning.

Finner v_1 fra energibetraktning:

$$\tfrac{1}{2} m v_{min}^2 = \tfrac{1}{2} m v_1^2 + m g h_0$$

$$\Rightarrow \quad \tfrac{1}{2} m v_1^2 = \tfrac{1}{2} m v_{min}^2 - m g h_0$$

$$\Rightarrow \quad F = \tfrac{1}{2} m v_1^2 / s = \frac{m}{s}\left[\tfrac{1}{2} v_{min}^2 - g h_0\right] = \frac{1400\,\mathrm{kg}}{2\mathrm{m}}\left[\tfrac{1}{2} 20{,}5^2 - 9{,}81 \cdot 7\right]\frac{\mathrm{m}^2}{\mathrm{s}^2}$$

$$\underline{\underline{F \doteq 99 \ \mathrm{kN}}}$$

Alternativt: $\underline{t_1} = \dfrac{s_0}{v_{min}\cos\theta} = \dfrac{35\,\mathrm{m}}{20{,}5\,\mathrm{m/s} \cdot \cos 50°} = 2{,}65\,\mathrm{s}$

$v_x = v_{min}\cos\theta = 20.5 \cdot \cos 50 = 13{,}2 \ \mathrm{m/s}$

$v_y = v \cdot \sin\theta - g \cdot t_1 = (20.5 \sin 50° - 9{,}81 \cdot 2{,}65)\ \mathrm{m/s} = -\underline{10{,}3 \ \mathrm{m/s}}$

$v_1 = \sqrt{v_x^2 + v_y^2} = \sqrt{174 + 106}\ \mathrm{m/s} = \sqrt{280}\ \mathrm{m/s} = \underline{\underline{16{,}7 \ \mathrm{m/s}}}$

Energibetraktning som før:

$$F \cdot s = \frac{1}{2} m v_1^2$$

$$\Rightarrow \underline{F} = \frac{m}{2s} \cdot v_1^2 = \frac{1400\,kg}{2 \cdot 2\,m} \cdot 280 \frac{m^2}{s^2} \approx \underline{98\,kN}$$

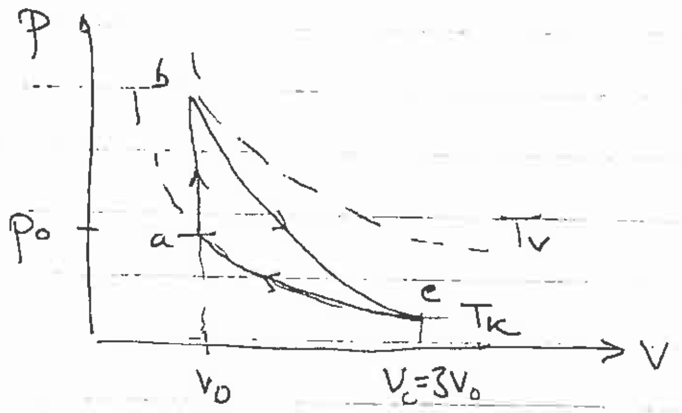
$$\underline{\underline{F \simeq 98\,kN}} \quad \text{(mot fartsretningen)}$$

Kollisjonstiden t_2 bestemmes fra overført impuls

$$F \cdot t_2 = |\Delta p| = m v_1$$

$$\underline{\underline{t_2}} = \frac{m v_1}{F} = \frac{1400\,kg \cdot 16{,}7\,m/s}{98500} = \underline{\underline{0{,}24\,s}}$$

Oppgave 2.



Diatomær gass, $\gamma = \frac{c_p}{c_v} = \frac{7}{5} = 1{,}4.$

a) Gitt $p_0, V_0, V_c = 3V_0, T_K = T_0$

Bestem T_V, p_b, p_c

Isoterm c–a : $\frac{p_a V_a}{T_a} = \frac{p_c V_c}{T_c}$

Innsatte, kjente verdier: $\frac{p_0 V_0}{T_0} = \frac{p_c \cdot 3V_0}{T_0}$

$$\Rightarrow \quad \underline{\underline{p_c = \frac{1}{3} p_0}}$$

Adiabat b–c : $T_b V_b^{(\gamma-1)} = T_c V_c^{(\gamma-1)}$

Innsatt kjente verdier $T_b \cdot V_0^{(\gamma-1)} = T_0 \cdot V_0^{(\gamma-1)} \cdot 3^{(\gamma-1)}$

$$\Rightarrow \quad \underline{\underline{T_b \equiv T_V = T_0 \cdot 3^{(\gamma-1)}}}$$

Tilstandsligningen: $\frac{p_b \cdot V_b}{T_b} = \frac{p_c V_c}{T_c}$

Innsatt kjente verdier: $\frac{p_b \cdot V_0}{T_0 \cdot 3^{(\gamma-1)}} = \frac{\frac{1}{3} p_0 \cdot 3 \cdot V_0}{T_0}$

$$\Rightarrow \quad \underline{\underline{p_b = p_0 \cdot 3^{(\gamma-1)}}}$$

b) Arbeid utført i hver av delprosessene:

Isokor a→b: $\underline{\underline{W_{ab}}} = \int_a^b p \cdot dV = \underline{\underline{0}}$ fordi V = konstant

Adiabat b→c: $Q_{bc} = \Delta U + W_{bc} = 0$

$$\Rightarrow W_{bc} = -\Delta U = -\left[nC_V \cdot \Delta T\right] = nC_V\left[T_b - T_c\right]$$

Fra hint: $C_V = \frac{R}{(\gamma-1)}$

$$\Rightarrow \underline{\underline{W_{bc}}} = \frac{nRT_b - nRT_c}{(\gamma-1)} = \underline{\underline{\frac{1}{(\gamma-1)}\left[p_b V_b - p_c V_c\right]}}$$

$$W_{bc} = \frac{1}{(\gamma-1)}\left[p_0 \cdot 3^{(\gamma-1)} \cdot V_0 - \frac{1}{3} p_0 \cdot 3 V_0\right]$$

$$\underline{\underline{W_{bc} = \frac{p_0 V_0}{(\gamma-1)}\left[3^{(\gamma-1)} - 1\right]}}$$

Isoterm c→a: $W_{ca} = \int_c^a p \cdot dV$

$$p \cdot V = nRT = \text{konstant} \Rightarrow p = \frac{nRT_0}{V}$$

$$\underline{\underline{W_{ca}}} = \int_c^a nRT_0 \cdot \frac{dV}{V} = nRT_0 \cdot \ln\frac{V_a}{V_c} = \underline{\underline{-nRT_0 \cdot \ln 3}}$$

$$p_0 \cdot V_0 = nRT_0$$

$$\Rightarrow \underline{\underline{W_{ca} = -p_0 V_0 \cdot \ln 3}}$$

c) Totalarbeid for et helt omløp:

$$W_{ut} = W_{ab} + W_{bc} + W_{ca} = 0 + \frac{p_0 V_0}{(\gamma-1)}\left[3^{(\gamma-1)} - 1\right] - p_0 V_0 \ln 3$$

$$\underline{\underline{W_{ut} = p_0 V_0 \left[\frac{(3^{(\gamma-1)} - 1)}{(\gamma-1)} - \ln 3\right]}} \qquad \text{q.e.d.}$$

Tallverdier:

$$\left\{\frac{3^{(\gamma-1)} - 1}{\gamma - 1} - \ln 3\right\} = \left[\frac{3^{0,4} - 1}{0,4} - 1,0986\right] = 0,281$$

$$\Rightarrow \underline{W_{ut}} = 2,0 \cdot 10^5 \frac{N}{m^2} \cdot 2,0 \cdot 10^{-3} m^3 \cdot 0,281 = 1,12 \cdot 10^2 Nm = \underline{112 J}$$

$$P = \frac{W_{ut}}{T} = \frac{W_{ut}}{1/f} = f \cdot W_{ut} = \frac{1000 \frac{syk}{min}}{60 \frac{s}{min}} \cdot 112 J = \underline{\underline{1873 W}}$$

d) Tilførte varmemengder:

Bruker termodynamikkens 1. lov. $\underline{Q = \Delta U + W}$

$$Q_{ab} = \Delta U + W_{ab} \qquad W_{ab} = 0$$

$$\Rightarrow Q_{ab} = \Delta U = U_b - U_a = nC_v(T_b - T_a) = nC_v(T_v - T_0)$$

$$Q_{ab} = n\frac{R}{(\gamma-1)}(T_v - T_o) = \frac{1}{(\gamma-1)}(nRT_v - nRT_0)$$

$$Q_{ab} = \frac{1}{(\gamma-1)}[p_b V_0 - p_0 V_0] = \frac{p_0V_0}{(\gamma-1)}[3^{(\gamma-1)} - 1]$$

$$Q_{ab} = \frac{400\,J}{0.4}[0.5518] = 551,8\,J \quad \text{(absorbert)}$$

$Q_{bc} = 0$ pr. def. av adiabat.

$Q_{ca} = \Delta U + W_{ca}$ $\quad \Delta U = 0$ pga. isoterm.

$\Rightarrow Q_{ca} = W_{ca} = -p_0V_0 \cdot \ln 3 \quad (<0)$

$Q_{ca} = -400\,J \cdot \ln 3 = -439,4\,J$ (dvs: avgitt varme)

e) Virkningsgrad: $\quad \varepsilon = \frac{W_{ut}}{Q_H} = \frac{W_{ut}}{Q_{ab}}$

$$\varepsilon = \frac{p_0V_0\left[\frac{3^{(\gamma-1)}-1}{(\gamma-1)} - \ln 3\right]}{p_0V_0\left[\frac{3^{(\gamma-1)}-1}{(\gamma-1)}\right]} = 1 - \frac{\ln 3(\gamma-1)}{3^{(\gamma-1)}-1}$$

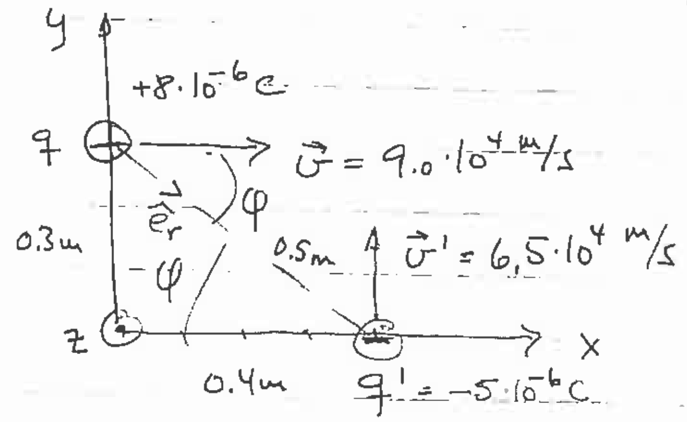
$$\varepsilon = 1 - \frac{\ln 3 \cdot 0,4}{0,5518} = 0,20$$

For Carnot prosess:

$$\varepsilon_c = 1 - \frac{T_c}{T_H} = 1 - \frac{T_0}{T_0 \cdot 3^{(\gamma-1)}} = 1 - \frac{1}{3^{(\gamma-1)}} = 1 - 3^{(1-\gamma)}$$

$$\varepsilon_c = 1 - 0,644 = 0,36$$

Oppgave 3.



r = Avst. qq'

$r = \sqrt{0{,}3^2+0{,}4^2}\,m = \underline{0{,}5\,m}$

a) Magnetfelt B generert av q i pos. for q':

$$\vec{B} = \frac{\mu_0}{4\pi}\cdot q\frac{(\vec{v}\times\hat{e}_r)}{r^2}$$

$\hat{e}_r$ enhetsvektor fra kilde (q) i retning P (dvs. q')

$$|\vec{v}\times\hat{e}_r| = v\cdot1\cdot\sin\varphi = \frac{3}{5}v$$

Tallverdi: $|B| = \frac{\mu_0}{4\pi}\cdot\frac{qv\sin\varphi}{r^2}$, $\quad \sin\varphi = \frac{3}{5}$

$$|B| = \frac{\cancel{4\pi}\cdot10^{-7}\,N/A^2}{\cancel{4\pi}}\cdot\frac{8\cdot10^{-6}\,As\cdot9\cdot10^{4}\,m/s\cdot\frac{3}{5}}{(0.5\,m)^2} \quad \underline{173\cdot10^{-9}\,T}$$

Retning: ved høyre hånds regel blir $\vec{v}\times\hat{e}_r = -v\cdot k$

dvs. retning inn i papirplanet $(q>0)$

→ I posisjon for q': $\underline{\underline{\vec{B} = -173\cdot10^{-9}\,T\ k}}$

Alternativt :

$$\hat{e}_r = \frac{1}{0,5}\left(0,4\,\mathbf{i} - 0,3\,\mathbf{j}\right) = \underline{0,8\,\mathbf{i} - 0,6\,\mathbf{j}}$$

$$\underline{\vec{v} \times \hat{e}_r} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 9\cdot 10^4\,\mathrm{m/s} & 0 & 0 \\ 0,8 & -0,6 & 0 \end{vmatrix} = \underline{-5,4\cdot 10^4\,\mathrm{m/s}\ \mathbf{k}}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\, q\,\frac{(\vec{v}\times\hat{e}_r)}{r^2} = \frac{4\pi\cdot 10^{-7}\,\frac{N}{A^2}}{4\pi}\cdot\frac{8\cdot 10^{-6}\,A\,s\cdot(-5,4\cdot 10^4\,\mathrm{m/s})\,\mathbf{k}}{(0,5\,m)^2}$$

$$\underline{\underline{\vec{B} = -173\cdot 10^{-9}\,T\ \mathbf{k}}}$$ ⇒ inn i papirplanet.

Kraftvirkning på q' pga magnetfeltet B :

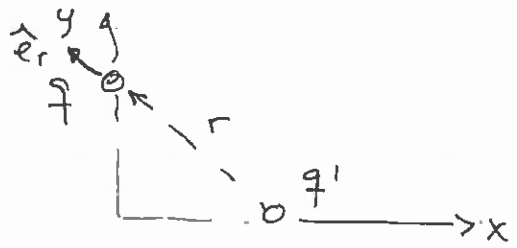
$$\vec{F} = q'(\vec{v}\,' \times \vec{B}) \qquad \vec{v}\,' \perp \vec{B}$$

$$\vec{F} = q'|v'B|\,[\mathbf{j}\times(-\mathbf{k})] = -5\cdot 10^{-6}\,C\cdot 6,5\cdot 10^4\,\tfrac{m}{s}\cdot 173\cdot 10^{-9}\,T\,[-\mathbf{i}]$$

$$\underline{\underline{\vec{F}}} = 5,6\cdot 10^3\cdot 10^{-11}\,N\,\mathbf{i} = \underline{\underline{56\,nN\ \mathbf{i}}}$$ ⇒ i pos. x-retn.

b) Elektrostatisk kraft

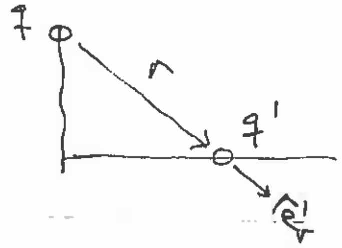
Kraft fra q' på q: $\vec{F}_q = \frac{q'}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \hat{e}_r$



$$\underline{\underline{\vec{F}_q}} = \frac{-5\cdot 10^{-6}\cdot 8\cdot 10^{-6}\,C^2}{4\pi\cdot 8{,}85\cdot 10^{-12}\,C^2/Nm^2}\,\frac{1}{(0{,}5m)^2}\,\hat{e}_r = \underline{\underline{-1{,}44\,N\,\hat{e}_r}}$$

Kraften $\vec{F}_q$ har retning $-\hat{e}_r$ dvs. fra q mot q' (langs forbindelseslinjen qq'). Tiltrekkende kraft.

Kraft fra q på q': $\vec{F}_{q'} = \frac{q}{4\pi\varepsilon_0} \cdot \frac{q'}{r^2} \hat{e}_r'$



$\underline{\underline{\vec{F}_{q'} = -1{,}44\,N\,\hat{e}_r'}}$ rettet fra q' mot q.

Kreftene er like store, men motsatt rettet $\vec{F}_q = -\vec{F}_{q'}$, begge virker tiltrekkende.

c) Verdien av elektrostatisk potensial generert av q i posisjon fra q' (i: i avstand r)

$$V(r) = \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{r}$$

$$= \frac{8 \cdot 10^{-6} C}{4\pi \cdot 8{,}85 \cdot 10^{-12} \frac{C^2}{Nm^2}} \cdot \frac{1}{0{,}5m}$$

$$\underline{\underline{V(r)}} = 0{,}144 \cdot 10^{6} V = \underline{\underline{0{,}144 \; MV}}$$

Elektrostatisk potensiell energi av systemet

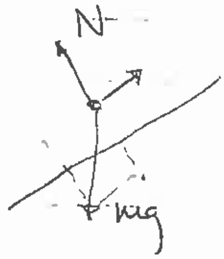
$$U = \sum_{i<j} \frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}} = \frac{qq'}{4\pi\varepsilon_0 r} = q' \cdot V$$

$$\Rightarrow \underline{\underline{U}} = 0{,}144 \cdot 10^{6} V \cdot (-5{,}0 \cdot 10^{-6} C) = \underline{\underline{-0{,}72 \; J}} < 0$$

Potensiell energi er negativ fordi ladningene har motsatt fortegn og ikke er i ∞ avstand fra hverandre.

Oppgave 4. Flervalgsspørsmål

1)



Svar: C (3)

2) $W = \vec{F} \cdot \vec{s}$

$W = (-3\mathbf{i} + 6\mathbf{j} - 9\mathbf{k}) \cdot (6\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})\ \text{J}$

$W = (-18 - 24 - 18)\ \text{J} = -60\ \text{J}$ Svar: E

3) $E = \frac{1}{2}kA^2$

$\Rightarrow 2E = \frac{1}{2}k(A\sqrt{2})^2$ Svar: C faktor $1.4 = \sqrt{2}$

4) $F = ma = -kx$

maks $|a|$ for maks $|x|$ Svar: D (4)

5) $\frac{1}{2}m\bar{v}^2 = \frac{3}{2}kT$

$\langle K \rangle = \frac{3}{2}kT$ Svar: B

$m_{O_2} > m_{N_2}$

$\Rightarrow v_{O_2} < v_{N_2}$

(14)

6) $\langle K \rangle = \frac{3}{2} kT$

$\frac{\langle K_1 \rangle}{\langle K_2 \rangle} = \frac{T_1}{T_2} = \frac{293}{333} = \underline{0,88}$ Svar: D

7) U, S, T uavh. av prosessen
Q, W avh. –"–

$\Rightarrow$ Svar: A

8) $\Phi_E = \int \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} = \frac{2q - q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$

$\Rightarrow$ Svar: A

9) $V_{ab} = 12\,V$ $t = 2,5\,s$ $I = 60\,A$

$W = P \cdot t = V_{ab} \cdot I \cdot t = 12 \cdot 60 \cdot 2,5\,J = \underline{1800\,J}$

Svar: A

10) $\Phi_m = 6t^2 + 7t + 1$ $\varepsilon = -\frac{d\Phi_m}{dt}$

$\varepsilon = -\frac{d\Phi_m}{dt} = -[12t + 7]$

$t = 2\,s \Rightarrow$

$\Rightarrow |\varepsilon| = +[12 \cdot 2 + 7]\,V = 31\,V$

Svar: D