

NTNU

Institutt for fysikk

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LØSNINGSFORSLAG

TFY 4145 Mekanisk fysikk og FY1001 Generell fysikk 1

Onsdag 14 desember 2005

Tid: 09.00-13.00

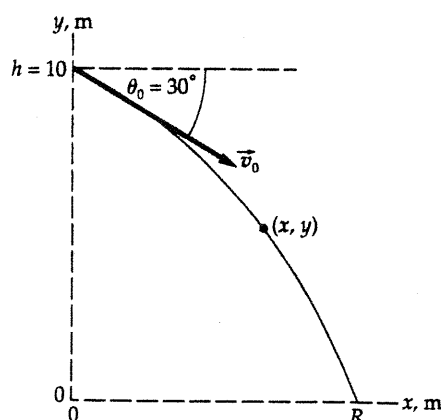
Bedømmelseskriterier:

Oppgavesettet består av 4 oppgaver som vektes i følge prosentangivelse gitt i parentes bak oppgavennummeret. Innen hver oppgave vektes deloppgavene likt.

Oppgave 1 (20%)

a)

Picture the Problem In the absence of air resistance, the hammer experiences constant acceleration as it falls. Choose a coordinate system with the origin and coordinate axes as shown in the figure and use constant-acceleration equations to describe the x and y coordinates of the hammer along its trajectory. We'll use the equation describing the vertical motion to find the time of flight of the hammer and the equation describing the horizontal motion to determine its range.



Using a constant-acceleration equation, express the x coordinate of the hammer as a function of time:

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $v_{0x} = v_0 \cos \theta_0$, and $a_x = 0$,

$$x = (v_0 \cos \theta_0)t$$

Using a constant-acceleration equation, express the y coordinate of the hammer as a function of time:

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

or, because $y_0 = h$, $v_{0y} = v_0 \sin \theta$, and $a_y = -g$,

$$y = h + (v_0 \sin \theta)t - \frac{1}{2}gt^2$$

Substitute numerical values to obtain:

$$y = 10 \text{ m} + (4 \text{ m/s})(\sin 30^\circ)t - \frac{1}{2}(9.81 \text{ m/s}^2)t^2$$

Substitute the conditions that exist when the hammer hits the ground:

$$0 = 10 \text{ m} - (4 \text{ m/s})\sin 30^\circ t - \frac{1}{2}(9.81 \text{ m/s}^2)t^2$$

Solve for the time of fall to obtain:

$$t = 1.24 \text{ s}$$

Use the x -coordinate equation to find the horizontal distance traveled by the hammer in 1.24 s:

$$R = (4 \text{ m/s})(\cos 30^\circ)(1.24 \text{ s})$$

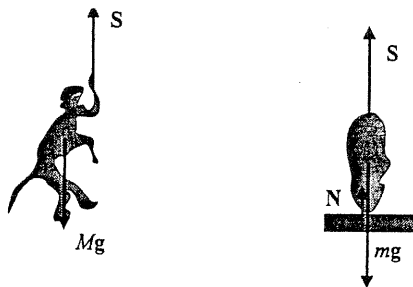
$$= \boxed{4.29 \text{ m}}$$

hammer
= apehatt

Vi lager frilegemediagram for henholdsvis apen og bananklassen.
Siden trinsen og tauet er masseløse, er $S_1 = S_2 = S$. Idet klasen akkurat letter fra underlaget er $N = 0$, og derfor er det kritiske snordraget

b)

$$S = mg$$



For apen har vi

$$S - Mg = Ma$$

Det gir

$$a_{\text{kritisk}} = \frac{m - M}{M} g = \frac{16 \text{ kg} - 12 \text{ kg}}{12 \text{ kg}} \times 9.8 \text{ m/s}^2 = \underline{3.27 \text{ m/s}^2}$$

c)

Picture the Problem Let the system consist of you, the extended weights, and the platform. Because the net external torque acting on this system is zero, its angular momentum remains constant during the pulling in of the weights.

Using conservation of angular momentum, relate the initial and final angular speeds of the system to its initial and final moments of inertia:

$$I_i \omega_i = I_f \omega_f$$

Solve for ω_f :

$$\omega_f = \frac{I_i}{I_f} \omega_i$$

Substitute numerical values and evaluate ω_f :

$$\omega_f = \frac{6 \text{ kg} \cdot \text{m}^2}{1.8 \text{ kg} \cdot \text{m}^2} (1.5 \text{ rev/s}) = \boxed{5.00 \text{ rev/s}}$$

Express the change in the kinetic energy of the system:

$$\Delta K = K_f - K_i = \frac{1}{2} I_f \omega_f^2 - \frac{1}{2} I_i \omega_i^2$$

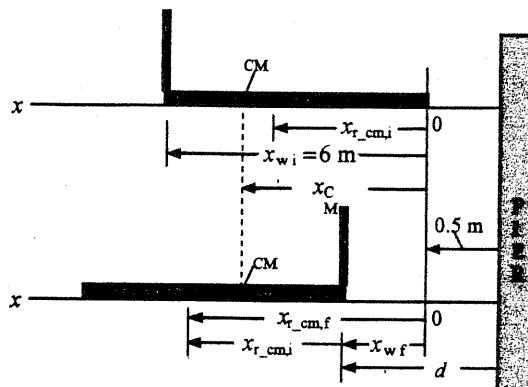
Substitute numerical values and evaluate ΔK :

$$\begin{aligned} \Delta K &= \frac{1}{2} (1.8 \text{ kg} \cdot \text{m}^2) \left(5 \frac{\text{rev}}{\text{s}} \times \frac{2\pi \text{ rad}}{\text{rev}} \right)^2 \\ &\quad - \frac{1}{2} (6 \text{ kg} \cdot \text{m}^2) \left(1.5 \frac{\text{rev}}{\text{s}} \times \frac{2\pi \text{ rad}}{\text{rev}} \right)^2 \\ &= \boxed{622 \text{ J}} \end{aligned}$$

Because no external agent does work on the system, the energy comes from the internal energy of the man.

Oppgave 2 (25%)

Picture the Problem Take the origin to be at the initial position of the right-hand end of raft and let the positive x direction be to the left. Let " w " denote the woman and " r " the raft, d be the distance of the end of the raft from the pier after the woman has walked to its front. The raft moves to the left as the woman moves to the right; with the center of mass of the woman-raft system remaining fixed (because $F_{\text{ext,net}} = 0$). The diagram shows the initial ($x_{w,i}$) and final ($x_{w,f}$) positions of the woman as well as the initial ($x_{r,cm,i}$) and final ($x_{r,cm,f}$) positions of the center of mass of the raft both before and after the woman has walked to the front of the raft.



a)

Express the distance of the raft from the pier after the woman has walked to the front of the raft:

$$d = 0.5 \text{ m} + x_{r,w} \quad (1)$$

Express x_{cm} before the woman has walked to the front of the raft:

$$x_{\text{cm}} = \frac{m_w x_{w,i} + m_r x_{r,cm,i}}{m_w + m_r}$$

Express x_{cm} after the woman has walked to the front of the raft:

$$x_{\text{cm}} = \frac{m_w x_{w,f} + m_r x_{r,cm,f}}{m_w + m_r}$$

Because $F_{\text{ext,net}} = 0$, the center of mass remains fixed and we can equate these two expressions for x_{cm} to obtain:

$$m_w x_{w,i} + m_r x_{r,cm,i} = m_w x_{w,f} + m_r x_{r,cm,f}$$

Solve for $x_{w,f}$:

$$x_{w,f} = x_{w,i} - \frac{m_r}{m_w} (x_{r,cm,f} - x_{r,cm,i})$$

From the figure it can be seen that $x_{r,cm,f} - x_{r,cm,i} = x_{w,f}$. Substitute $x_{w,f}$ for $x_{r,cm,f} - x_{r,cm,i}$ and to obtain:

$$x_{w,f} = \frac{m_w x_{w,i}}{m_w + m_r}$$

Substitute numerical values and evaluate $x_{w,f}$:

$$x_{w,f} = \frac{(60 \text{ kg})(6 \text{ m})}{60 \text{ kg} + 120 \text{ kg}} = 2.00 \text{ m}$$

Substitute in equation (1) to obtain:

$$d = 2.00 \text{ m} + 0.5 \text{ m} = \boxed{2.50 \text{ m}}$$

b)

Express the total kinetic energy of the system:

$$K_{\text{tot}} = \frac{1}{2} m_w v_w^2 + \frac{1}{2} m_r v_r^2$$

Noting that the elapsed time is 2 s, find v_w and v_r :

$$v_w = \frac{x_{w,f} - x_{w,i}}{\Delta t} = \frac{2 \text{ m} - 6 \text{ m}}{2 \text{ s}} = -2 \text{ m/s}$$

relative to the dock, and

$$v_r = \frac{x_{r,f} - x_{r,i}}{\Delta t} = \frac{2.50 \text{ m} - 0.5 \text{ m}}{2 \text{ s}} = 1 \text{ m/s},$$

also relative to the dock.

Substitute numerical values and evaluate K_{tot} :

$$\begin{aligned} K_{\text{tot}} &= \frac{1}{2} (60 \text{ kg}) (-2 \text{ m/s})^2 \\ &\quad + \frac{1}{2} (120 \text{ kg}) (1 \text{ m/s})^2 \\ &= \boxed{180 \text{ J}} \end{aligned}$$

c)

Evaluate K with the raft tied to the pier:

$$\begin{aligned} K_{\text{tot}} &= \frac{1}{2} m_w v_w^2 = \frac{1}{2} (60 \text{ kg}) (3 \text{ m/s})^2 \\ &= \boxed{270 \text{ J}} \end{aligned}$$

All the kinetic energy derives from the chemical energy of the woman and, assuming she stops via static friction, the kinetic energy is transformed into her internal energy.

Oppgave 3 (20%)

a)

Siden massene faller samtidig fra samme høyde y_0 og har samme vertikale akselerasjon g , vil de alltid ha *samme høyde*. De kolliderer derfor når de har samme x -koordinat. Bevegelsen av de to kulene som funksjon av tiden er

$$m_1 : \quad x_1 = v_{01} t \quad \text{og} \quad y_1 = y_0 - \frac{1}{2} g t^2$$

$$m_2 : \quad x_2 = 10 \text{ m} \quad \text{og} \quad y_2 = y_0 - \frac{1}{2} g t^2$$

I kollisjonsøyeblikket er

$$\begin{aligned} x_1 &= x_2 \\ v_{01} t &= 10 \text{ m} \\ t &= \frac{10 \text{ m}}{5 \text{ m/s}} = \underline{2 \text{ s}} \end{aligned}$$

Koordinatene er da

$$\begin{aligned} x_1 &= x_2 = \underline{10 \text{ m}} \\ y_1 &= y_2 = 50 \text{ m} - \frac{1}{2} \times (9.8 \text{ m/s}^2) \times (2 \text{ s})^2 = \underline{30.4 \text{ m}} \end{aligned}$$

Oppgave 3 (20%)

Massesenterhastigheten er uforandret ved kollisjonen. Det felles legemet fortsetter med samme massesenterhastigheten som før kollisjonen. Den er

b)

$$V_x = \frac{m_1 v_{01}}{m_1 + m_2} = \frac{(2 \text{ kg}) \times (5 \text{ m/s})}{(2 \text{ kg} + 3 \text{ kg})} = 2 \text{ m/s}$$

$$V_y = gt = (9.8 \text{ m/s}^2)t$$

Posisjonen til massesenteret idet massene starter er

$$X_0 = \frac{m_1 x_{01} + m_2 x_{02}}{m_1 + m_2} = \frac{(3 \text{ kg}) \times (10 \text{ m})}{(2 \text{ kg} + 3 \text{ kg})} = 6 \text{ m}$$

$$Y_0 = 50 \text{ m}$$

Bevegelsen av massesenteret er da gitt ved

$$X = X_0 + V_x t$$

$$Y = Y_0 - \frac{1}{2} g t^2$$

Det treffer bakken, $Y = 0$, etter tiden

$$t = \sqrt{\frac{2Y_0}{g}} = \sqrt{\frac{2 \times (50 \text{ m})}{(9.8 \text{ m/s}^2)}} = 3.19 \text{ s}$$

X -posisjonen er da

$$X = X_0 + V_x t = (6 \text{ m}) + (2 \text{ m/s}) \times (3.19 \text{ s})$$

$$\underline{X = 12.4 \text{ m/s}}$$

En *alternativ måte* å finne hvor det felles legemet treffer bakken, er å finne hvor de to partiklene ville truffet bakken hvis de fortsetter i sine opprinnelige baner uten å kolliderer. For m_1 har vi

$$y_1 = y_0 - \frac{1}{2} g t^2 = 0$$

$$t = \sqrt{\frac{2y_0}{g}} = 3.19 \text{ s}$$

$$x_1 = v_{01} t = (5 \text{ m/s}) \times (3.19 \text{ s}) = 15.95 \text{ m}$$

Massesenterets X -posisjon idet begge massene treffer bakken er da

$$X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(2 \text{ kg}) \times (15.95 \text{ m}) + (3 \text{ kg}) \times (10 \text{ m})}{(2 \text{ kg} + 3 \text{ kg})} = \underline{12.4 \text{ m}}$$

Oppgave 4 (35%)

a)

Picture the Problem Assume that $m_1 > m_2$. Choose a coordinate system in which the positive y direction is downward for the block whose mass is m_1 and upward for the block whose mass is m_2 and draw free-body diagrams for each block. Apply Newton's 2nd law of motion to both blocks and solve the resulting equations simultaneously.

Draw a FBD for the block whose mass is m_2 :



Draw a FBD for the block whose mass is m_1 :



Apply $\sum F_y = ma_y$ to this block:

$$T - m_2 g = m_2 a_2$$

Because the blocks are connected by a taut string, let a represent their common acceleration:

$$a = a_1 = a_2$$

Add the two force equations to eliminate T and solve for a :

$$m_1 g - m_2 g = m_1 a + m_2 a$$

and

$$a = \frac{m_1 - m_2}{m_1 + m_2} g$$

Substitute for a in either of the force equations and solve for T :

$$T = \frac{2m_1 m_2 g}{m_1 + m_2}$$

b)

Festet ved bakken løses.

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$\frac{1}{2} m_{30} v^2 + \frac{1}{2} m_{20} v^2$$

$$+ m_{20} g \Delta h - m_{30} g \Delta h = 0$$

$$v = \sqrt{\frac{2g\Delta h(m_{30} - m_{20})}{m_{20} + m_{30}}}$$

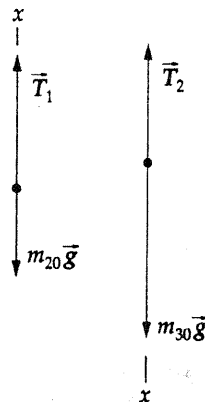
$$v = \sqrt{\frac{2(9.81 \text{ m/s}^2)(2 \text{ m})(30 \text{ kg} - 20 \text{ kg})}{20 \text{ kg} + 30 \text{ kg}}}$$

$$= 2.8 \text{ m/s}$$

alternativt: bruk a)

c)

Picture the Problem Let the system include the blocks, the pulley and the earth. Choose the zero of gravitational potential energy to be at the ledge and apply energy conservation to relate the impact speed of the 30-kg block to the initial potential energy of the system. We can use a constant-acceleration equations and Newton's 2nd law to find the tensions in the strings and the descent time.



(a) Use conservation of energy to relate the impact speed of the 30-kg block to the initial potential energy of the system:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$\frac{1}{2} m_{30} v^2 + \frac{1}{2} m_{20} v^2 + \frac{1}{2} I_p \omega_p^2$$

$$+ m_{20} g \Delta h - m_{30} g \Delta h = 0$$

Substitute for ω_p and I_p to obtain:

$$\frac{1}{2}m_{30}v^2 + \frac{1}{2}m_{20}v^2 + \frac{1}{2}\left(\frac{1}{2}M_p r^2\right)\left(\frac{v^2}{r^2}\right) + m_{20}g\Delta h - m_{30}g\Delta h = 0$$

Solve for v :

$$v = \sqrt{\frac{2g\Delta h(m_{30} - m_{20})}{m_{20} + m_{30} + \frac{1}{2}M_p}}$$

Substitute numerical values and evaluate v :

$$v = \sqrt{\frac{2(9.81 \text{ m/s}^2)(2 \text{ m})(30 \text{ kg} - 20 \text{ kg})}{20 \text{ kg} + 30 \text{ kg} + \frac{1}{2}(5 \text{ kg})}} = \boxed{2.73 \text{ m/s}}$$

d)

Find the angular speed at impact from the tangential speed at impact and the radius of the pulley:

$$\omega = \frac{v}{r} = \frac{2.73 \text{ m/s}}{0.1 \text{ m}} = \boxed{27.3 \text{ rad/s}}$$

e)

Apply Newton's 2nd law to the blocks:

$$\sum F_x = T_1 - m_{20}g = m_{20}a \quad (1)$$

$$\sum F_x = m_{30}g - T_2 = m_{30}a \quad (2)$$

Using a constant-acceleration equation, relate the speed at impact to the fall distance and the

$$v^2 = v_0^2 + 2a\Delta h$$

or, because $v_0 = 0$,

acceleration and solve for and evaluate a :

$$a = \frac{v^2}{2\Delta h} = \frac{(2.73 \text{ m/s})^2}{2(2 \text{ m})} = 1.87 \text{ m/s}^2$$

Substitute in equation (1) to find T_1 :

$$T_1 = m_{20}(g + a) = (20 \text{ kg})(9.81 \text{ m/s}^2 + 1.87 \text{ m/s}^2) = \boxed{234 \text{ N}}$$

Substitute in equation (2) to find T_2 :

$$T_2 = m_{30}(g - a) = (30 \text{ kg})(9.81 \text{ m/s}^2 - 1.87 \text{ m/s}^2) = \boxed{238 \text{ N}}$$

Noting that the initial speed of the 30-kg block is zero, express the time-of-fall in terms of the fall distance and the block's average speed:

$$\Delta t = \frac{\Delta h}{v_{av}} = \frac{\Delta h}{\frac{1}{2}v} = \frac{2\Delta h}{v}$$

Substitute numerical values and evaluate Δt :

$$\Delta t = \frac{2(2 \text{ m})}{2.73 \text{ m/s}} = \boxed{1.47 \text{ s}}$$