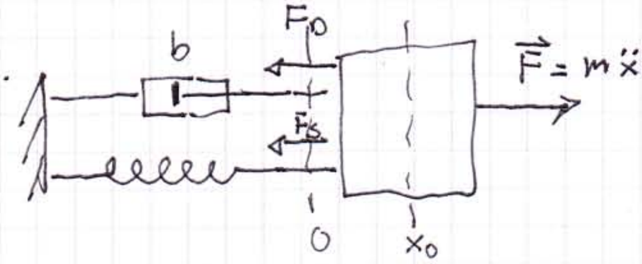
Oppg. 1

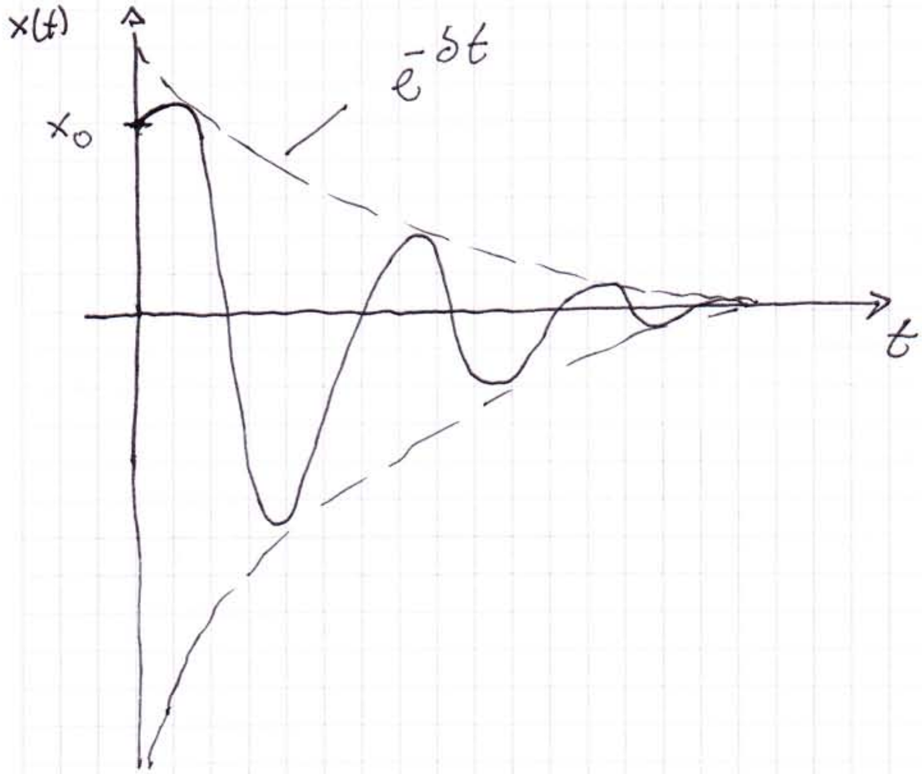
a)



$$m\ddot{x} = -b\dot{x} - kx \quad \Rightarrow \quad \ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$$

homogen løsning gitt av formen

$$x(t) = e^{-\delta t} \cos(\omega t + \alpha)$$



1b)

Ber. lign. Homogen løsning:

$$m\ddot{x} = -sx \Rightarrow \ddot{x} + \frac{s}{m}x = 0$$ (homogenløsn.)

Denne drar vi ut, slik som i oppgave 1

Partikulærløsningen:

$$m\ddot{x} + sx = -eE_0\cos(\omega t)$$

$$\ddot{x} + \frac{s}{m}x = -\frac{eE_0}{m}\cos(\omega t)$$

$$\Rightarrow \quad x(t) = A\cos(\omega t + \varphi)$$

ser på kompleks løsn.

$$\tilde{x}(t) = \hat{A}e^{i\omega t}, \quad x(t) = \mathrm{Re}(\tilde{x}(t))$$

$$\ddot{\tilde{x}} + \frac{s}{m}\tilde{x} = -\frac{eE_0}{m}e^{i\omega t}$$

$$-\omega^2\hat{A}e^{i\omega t} + \frac{s}{m}\hat{A}e^{i\omega t} = -\frac{eE_0}{m}e^{i\omega t}$$

$$\Rightarrow \quad \hat{A}\Big(\underbrace{\frac{s}{m}}_{=\omega_0^2} - \omega^2\Big) = -\frac{eE_0}{m}$$

$$\hat{A} = \frac{-eE_0/m}{(\omega_0^2 - \omega^2)} \qquad \omega_0^2 << \omega^2$$

$$|\hat{A}| = A = \frac{eE_0/m}{\omega_0^2 - \omega^2}, \quad \varphi = \pi$$

$$x(t) = A(\omega)\cos(\omega t + \pi) = -A(\omega)\cos(\omega t)$$

$$\vec{p}(t) = -e\,x(t)\,\hat{x} = \frac{e^2 E_0/m}{\omega_0^2 - \omega^2}\cos(\omega t)\,\hat{x}$$ [dipolmom.]

$$\vec{P}(t) = N\cdot\vec{p}(t) = \underbrace{\frac{Ne^2 E_0/m}{\omega_0^2 - \omega^2}}_{P(\omega)}\cos(\omega t)\,\hat{x}$$ [Polarisering til dielektrikum]

Oppgitt $\vec{P}(\omega) \approx \varepsilon_0 \chi_e \vec{E_0}$

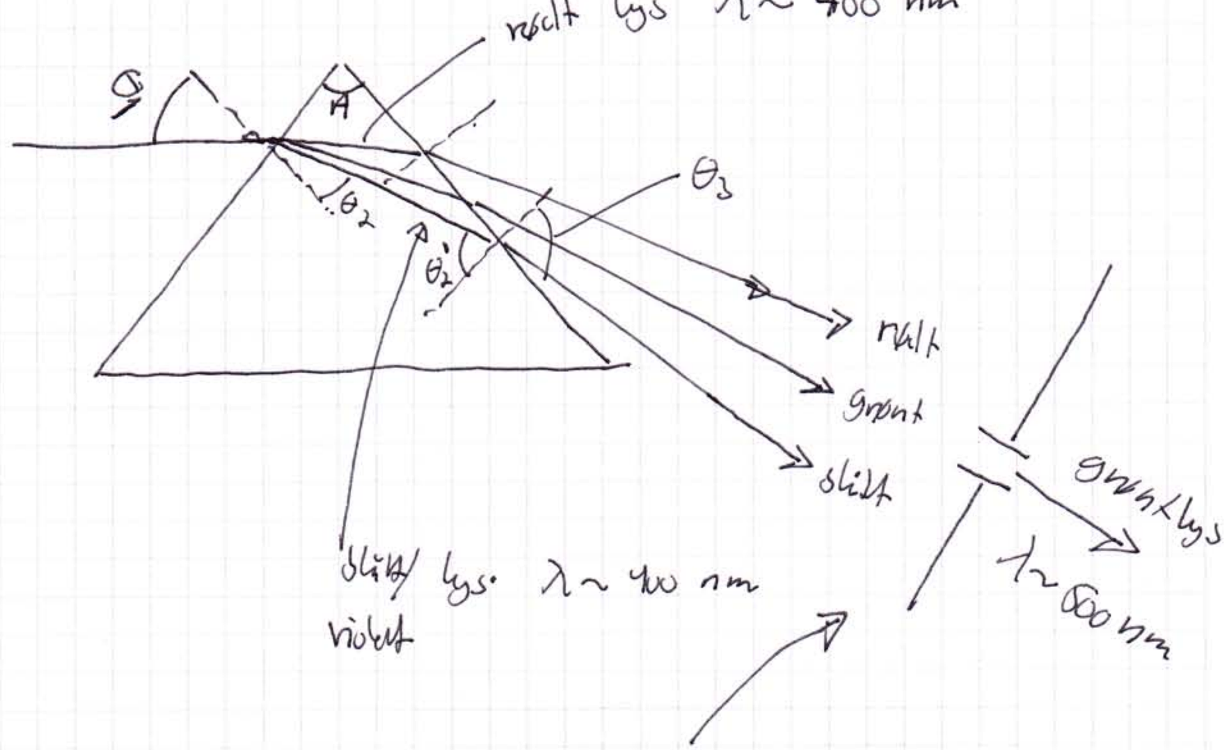
Videre, fra formelsamling har vi at

$$\vec{D}(\omega) \approx \varepsilon_0 \vec{E_0} + \vec{P}(\omega) = \varepsilon_0 (1 + \chi_e(\omega))\vec{E_0}$$
$$= \varepsilon_0 \varepsilon_r(\omega)\vec{E_0}$$

$$\Rightarrow \varepsilon_r(\omega) = (1 + \chi_e(\omega)) = 1 + \frac{P(\omega)}{\varepsilon_0 E_0}$$

$$= \underline{\underline{1 + \frac{Ne^2/(m\varepsilon_0)}{\omega_0^2 - \omega^2}}}$$

c) Gitt prisme med brytn. indeks $n(\lambda)$, i luft.



Setter inn hull (aperture) i utgående stråle ⇒ velger ut en enkelt bølgelengde λ, f.eks grønt lys

relevante formler vil være brytningsloven:

$$\sin\theta_1 = n(\lambda)\sin\theta_2(\lambda)$$

$$\Rightarrow \theta_2(\lambda) = \sin^{-1}\left[\frac{\sin\theta_1}{n(\lambda)}\right]$$

$$\theta_2(700\ \text{nm}) < \theta_2(400\ \text{nm})$$

Denne har lignende uttrykk på andre flate, men da trenger en vinkelen A. ~~For et rettvinklet prisme A = 90°~~

$$\cancel{\theta_2' = \theta_3} \Rightarrow \sin\theta_3(\lambda) = n(\lambda)\sin\theta_2'(\lambda)$$

$$\Leftrightarrow \theta_3(\lambda = 400\ \text{nm}) > \theta_3(\lambda = 700\ \text{nm}),\ \text{siden}\ \theta_2'(700) > \theta_2'(400)$$

Oppg. 2

2a) $\overset{\text{total bølge}}{y(x,t)} = \underbrace{y_0 \sin(kx-\omega t)}_{\text{innkommende bølge}} + \underbrace{y_0 \sin(kx+\omega t)}_{\text{reflektert bølge}}$

bruker $\sin(a\pm b) = \sin a \cos b \pm \cos a \sin b$

$$y(x,t) = y_0\Big[\sin kx \cos\omega t - \cos kx \sin\omega t$$

$$+ \sin kx \cos\omega t + \cos kx \sin\omega t\Big]$$

$$= 2y_0 \sin kx \cos\omega t$$

Siden strengen er festet i begge ender, ~~for~~ kreves det at for $x=L$:

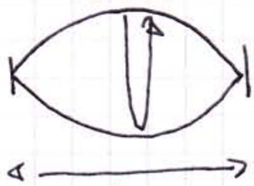
$$k\cancel{L} = n\pi$$

$$\frac{2\pi}{\lambda_n} L = n\pi \iff \lambda_n = \frac{2L}{n}$$

Videre har vi, siden $\frac{\omega}{k} = v$ at $\lambda f = v \left(= \sqrt{\frac{S}{\mu}}\right)$

$$\Rightarrow \quad f_n = \frac{v}{\lambda_n} = \frac{n}{2L} v = \frac{n}{2L}\sqrt{\frac{S}{\mu}}, \text{ der } \mu = m/L$$

2b)



$$T_1 = 5\,ms = \frac{1}{f_1} \quad , \quad \underline{f_1 = 200\,Hz}$$

$$v = f_1 \cdot \frac{2L}{1} = \frac{1}{T_1} \cdot 2L$$

$$= \frac{2\,m}{5 \times 10^{-3}\,s}$$

$$\underline{\underline{v = 400\,m/s}}$$

$$v = \sqrt{\frac{S}{m/L}} \Rightarrow v^2\left(\frac{m}{L}\right) = S$$

$$S = 8000\,N = \underline{\underline{8\,kN}}$$

2c)

$$\Delta p(x,t) = \Delta p_0 \sin(\underbrace{k_1 x - \omega_1 t}_{a}) + \Delta p_0 \sin(\underbrace{k_2 - \omega_2 t}_{b})$$

Bruker $\sin a + \sin b = 2 \sin \frac{1}{2}(a+b) \cos \frac{1}{2}(a-b)$

$$\Rightarrow \Delta p(x,t) = \Delta p_0 \sin\Big(\underbrace{\frac{(k_1+k_2)}{2}}_{\bar{k}} x - \underbrace{\frac{(\omega_1+\omega_2)}{2}}_{\bar{\omega}} t\Big) \cos\Big(\underbrace{\frac{(k_1-k_2)}{2}}_{\frac{\Delta k}{2}} x - \underbrace{\frac{(\omega_1-\omega_2)}{2}}_{\frac{\Delta\omega}{2}} t\Big)$$

Intensitet er gitt ved

$$\langle I \rangle = \langle \varepsilon \rangle \cdot v \qquad \text{(fra appendix)}$$

gitt at $\varepsilon(x,t) = \rho v^2 \left(\frac{\partial \xi}{\partial x}\right)^2$

Fra appendix vet vi at

$$\Delta p(x,t) = -B \frac{\partial \xi}{\partial x}$$

$$\Rightarrow \varepsilon(x,t) = \rho v^2 \left(-\frac{\Delta p(x,t)}{B}\right)^2 = \frac{\rho v^2}{B^2} (\Delta p(x,t))^2$$

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$$\langle \varepsilon(x,t) \rangle = \frac{\rho v^2}{B^2} \Delta p_0^2 \langle \sin^2(\bar{k}x - \bar{\omega} t) \rangle \cos^2\left(\frac{\Delta k x}{2} - \frac{\Delta \omega}{2} t\right)$$

tidsmidler over en periode $\bar{\omega}$, og antar $\Delta\omega << \bar{\omega}$

$$\Rightarrow \langle \varepsilon(x,t) \rangle = \frac{1}{2} \frac{\rho v^2}{B} \Delta p_0^2 \cos^2\left(\frac{\Delta k}{2} x - \frac{\Delta \omega}{2} t\right)$$

Svevning oppstår da intensiteten som vi hører med frekvens $\bar{\omega}$ vil variere mellom I_{maks} og 0.

$$I(x,t) = \cancel{\tfrac{4}{2}} \frac{\rho v^3}{B} \Delta p_0^2 \underbrace{\cos^2\left(\frac{\Delta k}{2} x - \frac{\Delta \omega}{2} t\right)}_{\text{svevning}} \underbrace{\sin^2\left(\bar{k} x - \bar{\omega} t\right)}_{\text{bærebølge med frekvens } \bar{\omega}}$$

tidsmidlet intensitet

$$\langle I(x,t) \rangle \approx \frac{1}{2} \frac{\rho v^3}{B} \Delta p_0^2 \cos^2\left(\frac{\Delta k}{2} x - \frac{\Delta \omega}{2} t\right)$$

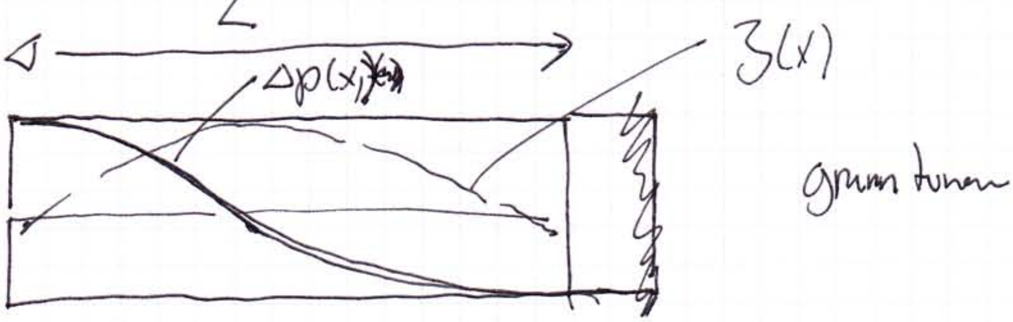
2a) To lukkede ender

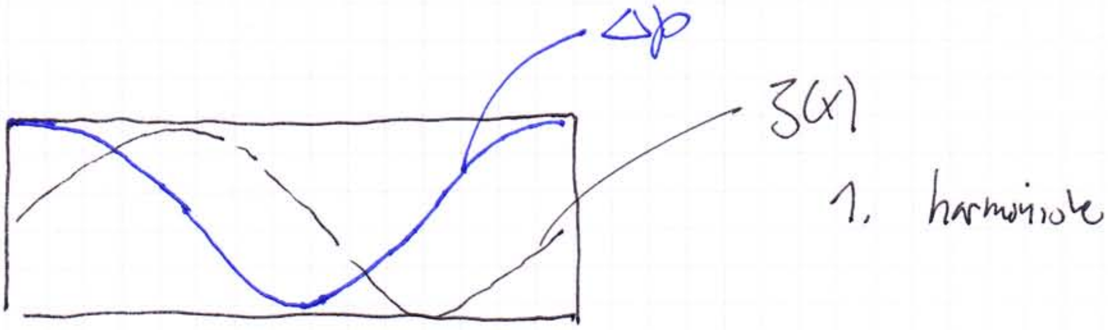
$\Rightarrow$ partikkelutsving $\zeta(x=0,t) = 0$

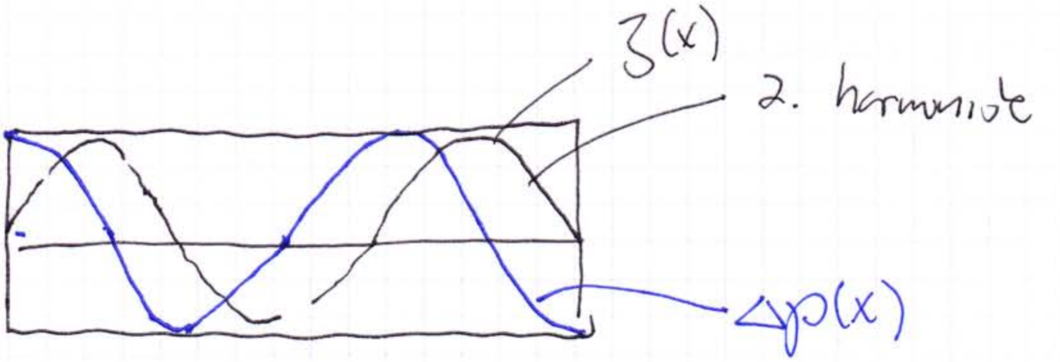
$\zeta(x=L,t) = 0$

Siden $\Delta p = -B\dfrac{\partial \zeta}{\partial x} \iff \Delta p(x=0,t) = MAX$

$\Delta p(x=L,t) = MAX$







Oppg. 3

Ikke dispersivt medium!

a) Bruker Gauss-E: $\vec{\nabla} \cdot \vec{D} = \varepsilon_0 \varepsilon_r \vec{\nabla} \cdot \vec{E} = 0$

$$\Rightarrow \quad i\vec{k} \cdot \vec{E} = 0 \quad \Leftrightarrow \quad \hat{k}\left(= \frac{\vec{k}}{|\vec{k}|}\right) \perp \vec{E}_0$$

$$\Rightarrow \quad \hat{k} \cdot \vec{E}_0 = 0 \quad \Leftrightarrow \hat{k} \perp \vec{E}_0$$

Bruker Faraday's lov:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \to i\vec{k} \quad , \quad \frac{\partial}{\partial t} \to -i\omega$$

$$\Rightarrow \quad i\vec{k} \times \vec{E} = +i\omega \vec{B}$$

$$\hat{k} \times \vec{E} = \underbrace{\frac{\omega}{k}}_{v} \vec{B}$$

$$\vec{B} = \frac{1}{v}(\hat{k} \times \vec{E})$$

Refleksjonsloven:

3b) $\vec{k}_{\parallel,i} = \vec{k}_{\parallel,r} \iff$ (i) $\frac{2\pi}{\lambda}\sin\theta_i = \frac{2\pi}{\lambda}\sin\theta_r$

$\Rightarrow \theta_i = \theta_r$

(ii) reflekterte strålen i samme plan som $\vec{k}$ & $\hat{z}$ ≡ innfallsplanet

Brytningsloven:

$\vec{k}_{\parallel,i} = \vec{k}_{\parallel,t} \iff$ (i) $\frac{2\pi}{\lambda_0} n_1 \sin\theta_i = \frac{2\pi}{\lambda_0} n_2 \sin\theta_t$,

der λ_0 = vakuum bølgelengde,

dette kommer av at

$\frac{\omega}{k} = \frac{c}{n} = v$

$\Rightarrow n_1 \sin\theta_i = n_2 \sin\theta_2$

(ii) transmitterte stråle ligger i innfallsplanet!

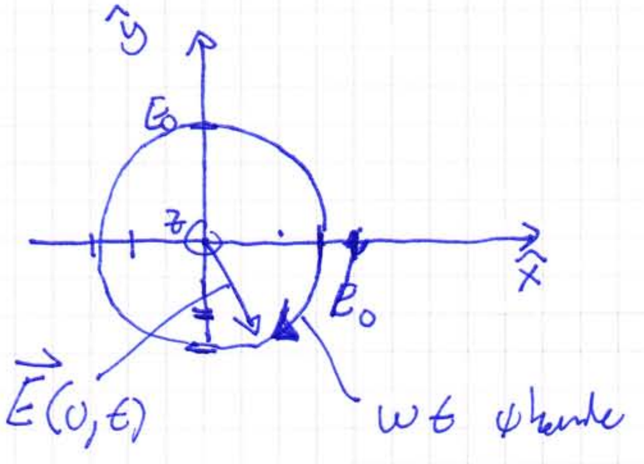
Oppg. 3c)

$$\vec{E}(z,t) = E_0 \cos(kz-\omega t)\hat{x} + E_0 \cos(kz-\omega t - \frac{\pi}{2})\hat{y}$$

ser på $\vec{E}(z=0,t) = E_0 \cos(\omega t)\hat{x} + E_0 \cos(\omega t + \frac{\pi}{2})\hat{y}$

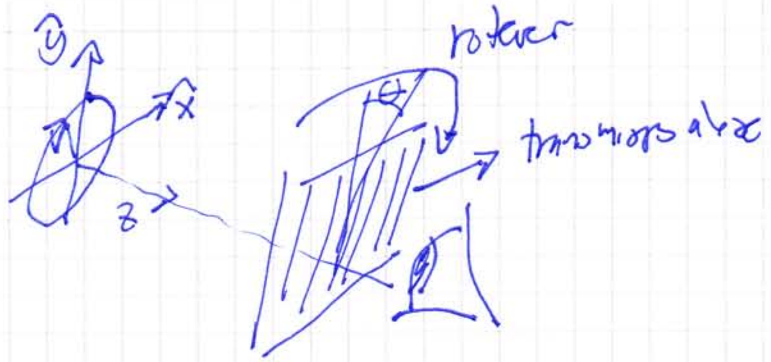
$$= E_0 \cos(\omega t)\hat{x} + -E_0 \sin(\omega t)\hat{y}$$

(bruker $\cos(\omega t + \frac{\pi}{2}) = \cos\omega t \cos\frac{\pi}{2} - \sin\omega t \sin\frac{\pi}{2} = -\sin\omega t$)



Resultant feltet $\vec{E}(0,t)$ ~~roterer~~ følger en sirkulær bane som funksjon av tiden t. Dreier med klokka.

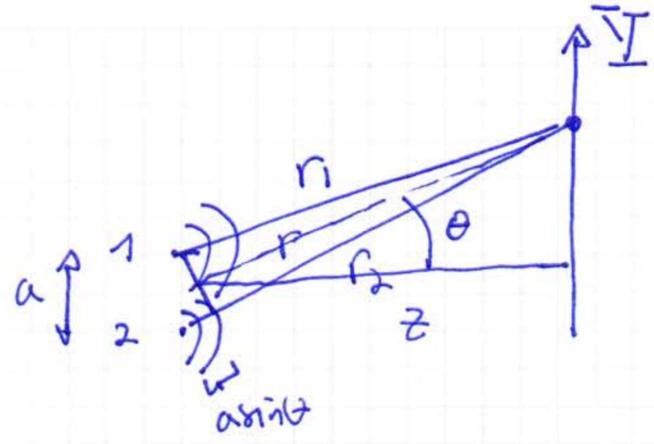
3c)



Siden $\vec{E}(0,t)$ har lik sannsynlighet for alle lineære pol. tilstander $\Rightarrow$ I = constant som funksjon av θ

Oppg. 4

a)



$\theta \approx \frac{Y}{z}$

Dette er et standard Youngs eksperiment. ~~Para~~

Feltet ved Y, z:

$$E(r,z) = E_0 \cos(kr_1 - \omega t + \varphi_1) + E_0 \cos(kr_2 - \omega t + \varphi_2)$$

Metode 1:

$$E(r,z) = E_0 \, Re \left\{ \underbrace{e^{i(kr_1 - \omega t + \varphi_1)} + e^{i(kr_2 - \omega t + \varphi_2)}}_{} \right\}$$

$$e^{-i\omega t} e^{i(kr_1 + \varphi_1)} \left[\underbrace{1 + e^{i(k(r_2 - r_1) + (\varphi_2 - \varphi_1))}}_{} \right]$$

$(\varphi_2 - \varphi_1) = \Delta\varphi$

bruker $(r_2 - r_1) = a\sin\theta$, ~~og~~

og $(1 + e^{ix}) = 2e^{ix/2} \cos x/2$

$$e^{i(ka\sin\theta + \Delta\varphi)/2} \left(\cos\left(\frac{k\Delta r}{2} + \frac{\Delta\varphi}{2} \right) \right)$$

$$\Rightarrow E(r,z) = 2E_0 \cos\left(kr_1 + \frac{ka\sin\theta}{2} + \cancel{\Delta\varphi} + \frac{\Delta\varphi}{2} - \omega t \right) \cdot \cos\left(k\frac{\Delta r}{2} + \frac{\Delta\varphi}{2} \right)$$

$$I = \epsilon_0 c \left(E(r,z) \right)^2 = \epsilon_0 c 4 E_0^2 \cos^2(\text{am} - \omega t) \cos^2\left(\frac{k\Delta r}{2} + \frac{\Delta\varphi}{2} \right)$$

$$\langle I\rangle = \underbrace{\tfrac{1}{2}\epsilon_0 c 4E_0^2}_{4I_0} \cos^2\left(ka\sin\theta/2 + \tfrac{\Delta\varphi}{2}\right) \equiv \text{interferens}$$

$$\approx \tfrac{1}{2}\epsilon_0 c E_0^2 \cos^2\left(\frac{\pi a Y}{\lambda Z} + \frac{(\varphi_2-\varphi_1)}{2}\right) = 4I_0\cos^2(\ )$$

Metode 2:

Bruker oppgitt trig. formel $\cos a + \cos b = 2\cos\frac{1}{2}(a+b)\cos\frac{1}{2}(a-b)$

$a = kr_1 - \omega t + \varphi_1$

$b = kr_2 - \omega t + \varphi_2$

$$\Rightarrow E(r,t) = E_0\left(2\cos\left(\frac{k(r_1+r_2)}{2} - \frac{(\omega+\omega)}{2}t + \frac{(\varphi_1+\varphi_2)}{2}\right)\right.$$

$$\left.\times \cos\left(k\frac{(r_1-r_2)}{2} + \frac{(\varphi_1-\varphi_2)}{2}\right)\right)$$

$$= 2E_0\cos(\ \ -\ \omega t)\cdot\cos\left(k\frac{\Delta r}{2} + \Delta\varphi/2\right)$$

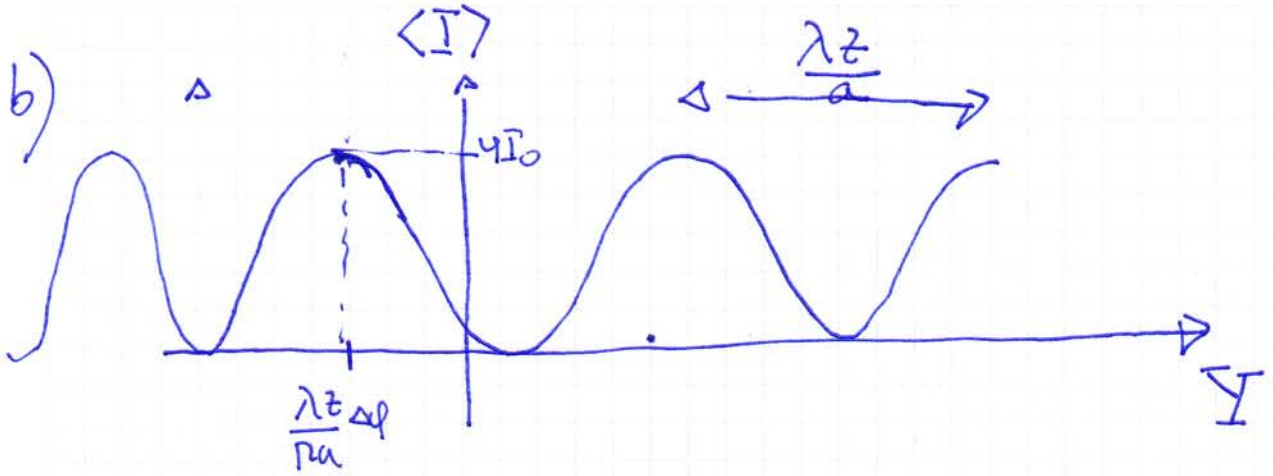
$$I = \epsilon_0 c\left(E(r,t)\right)^2 = 4\epsilon_0 c E_0^2 \cos^2(\ \ -\omega t)\cos^2\left(k\frac{\Delta r}{2} + \frac{\Delta\varphi}{2}\right)$$

$$\langle I\rangle = 4I_0\cos^2\left(k\frac{\Delta r}{2} + \frac{\Delta\varphi}{2}\right),$$

der $\Delta r = a\sin\theta \approx \frac{aY}{Z}$, $\Delta\varphi = \varphi_2 - \varphi_1$

og $k = \frac{2\pi}{\lambda}$

b)



$$\langle I(Y, z_0)\rangle = 4 I_0 \cos^2\left(\frac{\pi a Y}{\lambda z_0} + \frac{\Delta\varphi}{2}\right)$$

MAX FOR $\frac{\pi a Y}{\lambda z_0} = -\frac{\Delta\varphi}{2} + m\pi$, $m = 0, 1, 2$

$$Y = \cancel{\frac{\Delta\varphi\,\lambda z}{2\pi\,\alpha}}\ \frac{\lambda z_0}{\pi a}\left(m\pi - \Delta\varphi\right),\quad m = 0, 1, 2$$

$z = z_0 = 10$ km, $\lambda = c/f = \frac{3\times10^8}{2.1\times10^9} = 0.1428$ m

$$\Rightarrow Y_{m=1} - Y_{m=0} = \frac{\lambda z_0}{a} = \lambda \cdot \frac{10\times10^{2}}{10} $$

$$= \underline{14.2857\,m} = 100\,\lambda$$

c)

φνάκε $\langle I(\bar{Y} = \bar{Y}_0 = 150m, z_0) \rangle = MAX = 4I_0$

$$\Rightarrow \quad \frac{\pi a \bar{Y}_0}{\lambda z_0} = -\frac{\Delta\varphi}{2} + m\pi$$

$$\Delta\varphi = (\varphi_2 - \varphi_1) = 2m\pi - \frac{2\pi a \bar{Y}_0}{\lambda z_0}$$

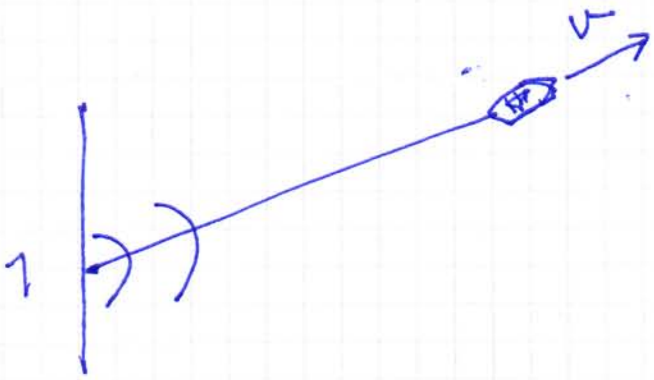
$$= 2\pi\Big(m - \underbrace{\frac{a\bar{Y}_0}{\lambda z_0}}_{\underbrace{\frac{10^2 \cdot 1.5\times10^2}{\lambda \cdot 10\times10^3}}_{10.5}}\Big)$$

$$= 2\pi(m - 10.5)$$

$$\varphi_2 - \varphi_1 < 2\pi \Rightarrow (\varphi_2 - \varphi_1) = 2\pi(11 - 10.5)$$

$$= \underline{\pi} \quad , \text{ valgt } m = 11.$$

d)



$f_0 = 200$ GHz

Fra formelsamling finner vi

$$\underbrace{\bar{\nu}}_{=f} = \underbrace{\nu}_{=f_0}\left(\frac{(c-v)}{(c+v)}\right)^{\frac{1}{2}}$$

Dette er det relativistiske uttrykket, slik at det kan forenkles til

$$\frac{\nu\, c^{\frac{1}{2}}(1-v/c)^{\frac{1}{2}}}{c^{\frac{1}{2}}(1+v/c)^{\frac{1}{2}}} \simeq \nu \left(1-\frac{1}{2}\frac{v}{c}\right)\cdot\left(1-\frac{1}{2}\frac{v}{c}\right)$$

$$\approx \nu\left(1-\frac{v}{c}\right) = \underline{\nu\left(\frac{c-v}{c}\right)}$$

som lyd med kilde i ro og observatør i bevegelse!!

$\Rightarrow$ f_1 på båten blir:

$$f_1 = f_0\left(\frac{c-v}{c}\right)$$, v = båtens hastighet

f_2 ved ~~på~~ antenne 1:

$$f_2 = f_1\left(\frac{c+v}{c}\right)$$, Antenne 1 har hastighet $+v$ med hensyn til båten

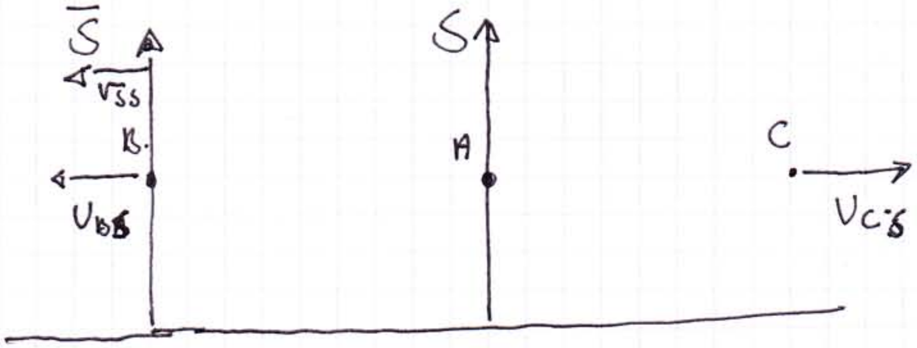
$$\Rightarrow f_2 = f_0\left(\frac{c(c-v)}{c}\right)^2 = f_0 \frac{\cancel{c^2}}{\cancel{c^2}}\left(1-\frac{v}{c}\right)^2 \approx f_0 \cancel{c}\left(1-2\frac{v}{c}\right)$$

$$\frac{(f_2-f_0)}{f_0} \approx \underline{\underline{-\frac{2v}{c}}} = \cancel{\ldots}$$

f_2 er til lavere frekvens (rødskiftet) med hensyn til f_0.

Oppg. 5

Skal transformere mellom systemet til A som vi kaller S og systemet til B, som vi kaller $\bar{S}$. Systemet $\bar{S}$ har hastighet $v_{\bar{S}S} = -0.7c$ relativt til A.



$u_{BS} = -0.7c$ hastighet til B målt i S (av A)

$\bar{u}_{B\bar{S}}$ = hastighet til B målt i $\bar{S}$ (av B)

u_{CS} = hastighet til C —||— S (av A)

$u_{C\bar{S}}$ = —|| ———— $\bar{S}$ (av B)

$\bar{u}_{A\bar{S}}$ = hastighet til A målt i $\bar{S}$ (av B)

$$\bar{u}_{A\bar{S}} = \frac{d\bar{x}}{d\bar{t}} = \frac{\gamma(dx - v\,dt)}{\gamma(dt - \frac{v}{c^2}dx)} = \frac{(u_{AS} - v_{\bar{S}S})}{(1 - \frac{v_{\bar{S}S}}{c^2}u_{AS})}$$

$$= -0.7c = \frac{(u_{AS} - (-0.7c))}{1 - \frac{(-0.7c)}{c^2}\cdot(0)} = \underline{0.7c} = \underline{\underline{-v_{\bar{S}S}}}$$

$$\bar{u}_{C\bar{S}} = \frac{u_{CS} - v_{\bar{S}S}}{1 - \frac{v_{\bar{S}S}}{c^2}u_{CS}} = \frac{0.7c - (-0.7c)}{1 - \frac{(-0.7c)\cdot(0.7c)}{c^2}}$$

$$= \frac{1.4c}{1 + 0.7^2} = 0.939c \simeq \underline{\underline{0.94c}}$$