

TFY 4240 ELEKTROMAGNETISK TEORI
LØSNINGSFORSLAG, EXAMEN 8 DES
OPPGAVE 1. 2007

a) Fluksen er gitt av

$$\Phi = \int_0^x B \, dA = \int_0^x B \cdot \frac{1}{2}\sqrt{2} \cdot l \cdot ds$$
$$= \frac{1}{2}\sqrt{2} l \int_0^x B \cdot ds$$

$$\underline{\underline{\Phi(x) = \frac{1}{2}\sqrt{2} l B \cdot x}}$$

Den induerte ems er gitt av

$$\mathcal{E} = - \frac{d\Phi}{dt} = - \frac{d\Phi}{dx} \cdot \frac{dx}{dt} = -\frac{1}{2}\sqrt{2} l B \cdot v$$

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Retningen av strømmen er med-urs, og
strømstyrken er:

$$\underline{\underline{I = \frac{|\mathcal{E}|}{R} = \frac{\frac{1}{2}\sqrt{2} l B \cdot v}{R}}}$$

b) Kraften er gitt av

$$\vec{F} = I \vec{l} \times \vec{B}$$

$$|\vec{F}| = I \cdot l \cdot B = \frac{\frac{1}{2}\sqrt{2} l^2 B^2 \cdot v}{R}$$

og har komponentene

$$F_y = -F_x = \frac{\frac{1}{2} l^2 B^2 \cdot v}{R}$$

Den mekaniske effekt er gitt av

$$P = |F_x| \cdot v = \frac{\frac{1}{2} l^2 B^2 \cdot v^2}{R}$$

Den elektriske effekt i motstanden R er gitt av

$$P_E = RI^2 = R \cdot \left(\frac{\frac{1}{2} \sqrt{2} l B v}{R} \right)^2$$
$$= \frac{\frac{1}{2} l^2 B^2 \cdot v^2}{R}$$

$$\Rightarrow \underline{P = P_E}$$

c) Vertikalkomponenten av kraften er gitt av:

$$F_y = \frac{\frac{1}{2} l^2 B^2 \cdot v}{R}$$

Staven løfter fra skinnene når denne

kraften er større/lik ~~den~~ tyngdekraften

Dette opptrer når:

$$\frac{\frac{1}{2} l^2 B^2 v}{R} \geq mg$$

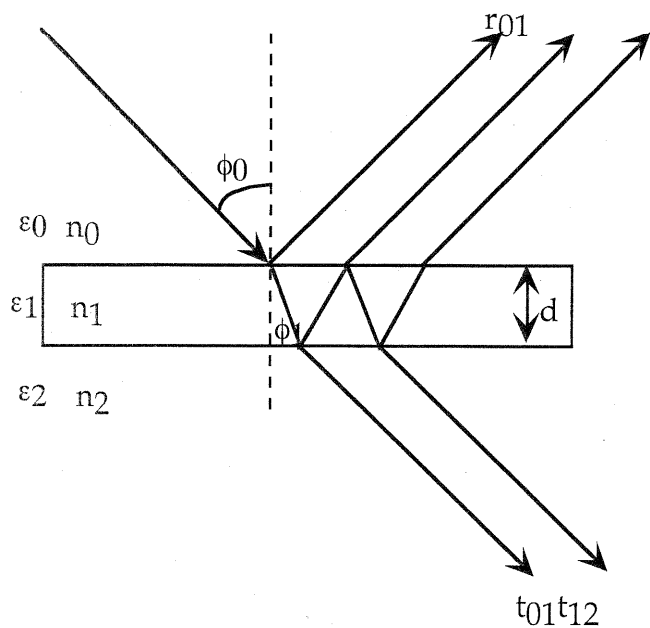
$$B^2 \geq \frac{2mgR}{l^2 v}$$

d) Anta $\vec{B} \perp \vec{I}$ og at orienteringen er slik at kraften virker vertikalt

$$\Rightarrow |F| = B \cdot I \cdot l \geq mg \Rightarrow I \geq \frac{mg}{Bl} = 4905 \text{ A}$$

Oppgave 2.

Vi skal nå se litt på refleksjon og transmisjon til en tynn film på en overflate. Systemet er illustrert i figuren under.



Den innfallende bølgen E_i reflekteres ved den første flaten. Noe transmitteres, reflekteres ved den indre flaten, transmitteres ved flate 1 etc. Vi får en sum av delbølger: (NB! Vi summerer feltamplituder)

$$\begin{aligned}
 r &= r_{01} + t_{01}r_{12}t_{10}e^{2i\delta_1} + t_{01}r_{12}r_{10}r_{12}t_{10}e^{4i\delta_1} + \dots \\
 &= r_{01} + \frac{r_{12}t_{01}t_{10}e^{2i\delta_1}}{1 + r_{12}r_{01}e^{2i\delta_1}}
 \end{aligned}$$

$$\begin{aligned}
 t &= t_{01}t_{12}e^{i\delta_1} + t_{01}r_{12}r_{10}t_{12}e^{3i\delta_1} + \dots \\
 &= \frac{t_{01}t_{12}e^{i\delta_1}}{1 + r_{12}r_{01}e^{2i\delta_1}}
 \end{aligned}$$

Ved å bruke Fresnels formler kan r-ligningen forenkles,

$$r = \frac{r_{01} + (r_{01}^2 r_{12} + r_{12} t_{01} t_{10} e^{2i\delta_1})}{1 + r_{12} r_{01} e^{2i\delta_1}}$$

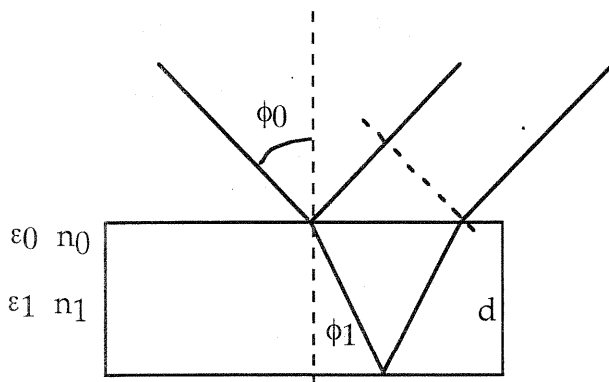
Fra Fresnels formler fås at $r_{01}^2 + t_{01} t_{10} = 1$. Dette fører da til

$$r = \frac{r_{01} + r_{12} e^{2i\delta_1}}{1 + r_{12} r_{01} e^{2i\delta_1}} \quad \text{Vis dette!}$$

$2\delta_1$ er faseforskjellen mellom bølge 1 og 2. Ut fra geometri/optisk veilengdeforskjell blir den

$$\delta_1 = \frac{2\pi}{\lambda} n_1 d_1 \cos \phi_1$$

Bevis:



Forskjellen i optisk veilengde er gitt av figuren:

$$\begin{aligned} & \frac{n_1 2d}{\cos \phi_1} - 2d \operatorname{tg} \phi_1 \sin \phi_0 n_0 \\ &= 2d \left(\frac{n_1}{\cos \phi_1} - \operatorname{tg} \phi_1 \sin \phi_0 n_0 \right) \\ &= 2d \left(\frac{n_1}{\cos \phi_1} - \frac{\sin \phi_1}{\cos \phi_1} n_1 \sin \phi_1 \right) = 2d n_1 \cos \phi_1 \end{aligned}$$

Dette gir da $\delta_1 = \frac{2\pi}{\lambda} n_1 d_1 \cos \phi_1$ som angitt ovenfor.

b) We assume normal incidence
The reflection coefficients are then

$$r_{01} = \frac{n_0 - n_1}{n_0 + n_1} \quad r_{12} = \frac{n_1 - n_2}{n_1 + n_2}$$

Insert this into

$$\begin{aligned} \frac{r_{01} + r_{12}}{1 + r_{01}r_{12}} &= \frac{\left(\frac{n_0 - n_1}{n_0 + n_1}\right) + \left(\frac{n_1 - n_2}{n_1 + n_2}\right)}{1 + \left(\frac{n_0 - n_1}{n_0 + n_1}\right)\left(\frac{n_1 - n_2}{n_1 + n_2}\right)} \\ &= \frac{2n_1(n_0 - n_2)}{2n_1(n_0 + n_2)} = \frac{n_0 - n_2}{n_0 + n_2} = r_{02} \end{aligned}$$

This must also be so from the equation for a thin film on a surface

$$r = \frac{r_{01} + r_{12} e^{2i\delta_1}}{1 + r_{01}r_{12} e^{2i\delta_1}}$$

If the film thickness is zero $\delta_1 = 0$

$$r = \frac{r_{01} + r_{12}}{1 + r_{01}r_{12}}$$

and this corresponds to the reflection at the 0-2 interface.

c)

Dersom $2\delta_1$ er lik 2π "5er"
ikke strålingen boksen

$$\delta_1 = \frac{2\pi}{\lambda} n_1 d_1 \cos\phi_1 \quad \phi_1 = 0$$

$$\Rightarrow 2\pi = 2 \cdot \frac{2\pi}{\lambda} n_1 d_1$$

$$d_1 = \frac{\lambda}{2n_1} = \frac{c}{2\nu n_1} = \frac{3 \cdot 10^8}{2 \cdot 10^{10} \cdot 2,15} = 0,6 \cdot 10^{-2} \text{ m} \\ = 6 \text{ mm}$$

Problem 3.

a) The time variation of the source is $e^{+i\omega t}$, but at the observation point we have to replace this by $e^{i\omega t_r}$ where t_r is the retarded time $t_r = t - r/c$

$$\text{Thus } e^{i\omega t_r} = e^{i\omega(t - r/c)} = e^{i(\omega t - kr)}$$

$$\text{Since } \omega = kc$$

the conditions are those given in the textbook p 445, 446

1) $d \ll r$; The size of the dipole small compared to the distance r

2) $d \ll \lambda$; λ is the wavelength of the emitted radiation ($d \ll \frac{c}{\omega}$ in the book)

3) $\lambda \ll r$ Wavelength should be small compared to the distance from the dipole to the observation point.

b) The vector potential is now given by

$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \frac{e^{i(\omega t - kr)}}{r} \int \vec{J}(\vec{r}') e^{ik\vec{r}'\cdot\vec{r}} d\vec{r}'$$

Let the antenna be located along the z -axis. We have to evaluate the integral

$$\begin{aligned} I &= \int \vec{J}(r') e^{ikz'\cos\theta} dz' \\ &= I_0 \int_0^{\lambda/2} \frac{e^{ikz'} - e^{-ikz'}}{2} e^{ikz'\cos\theta} dz' \\ &\quad + I_0 \int_{-\lambda/2}^0 \frac{e^{ikz'} - e^{-ikz'}}{2} e^{ikz'\cos\theta} dz' \end{aligned}$$

These integrals are given:

$$I = I_0 \left(\frac{1 + e^{i\pi\cos\theta}}{k\sin^2\theta} + \frac{1 + e^{-i\pi\cos\theta}}{k\sin^2\theta} \right)$$

which leads to a vector potential \vec{A}

$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \frac{e^{i(\omega t - kr)}}{r} \frac{I_0 \cdot 2}{k\sin^2\theta} (1 + \cos(\pi\cos\theta)) \hat{z}$$

c)

$$\begin{aligned} \vec{S} &= \sqrt{\frac{\epsilon_0}{\mu_0}} \omega^2 (\vec{r} \times \vec{A})^2 \hat{r} \\ &= \sqrt{\frac{\epsilon_0}{\mu_0}} \omega^2 \sin^2 \theta \left(\frac{\mu_0 c}{2\pi \omega} \right)^2 \frac{(1 + \cos(\pi \cos \theta))^2}{\sin^4 \theta} \frac{1}{r} I_0^2 \end{aligned}$$

and the time averaged Poynting vector becomes

$$\langle \vec{S} \rangle = \frac{1}{2} \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{I_0^2}{4\pi^2 r^2} \frac{(1 + \cos(\pi \cos \theta))^2}{\sin^2 \theta} \hat{r}$$

and

$$\frac{dP}{d\Omega} = \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{I_0^2}{8\pi^2} \frac{(1 + \cos(\pi \cos \theta))^2}{\sin^2 \theta}$$

The total radiated power becomes

$$\begin{aligned} P &= \int \frac{dP}{d\Omega} d\Omega = \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{I_0^2}{8\pi^2} \int \frac{(1 + \cos(\pi \cos \theta))^2}{\sin^2 \theta} \sin \theta d\theta d\phi \\ &= \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{I_0^2}{4\pi} 1,66 \end{aligned}$$

The radiation is max for $\theta = \pi/2$.

$$S_{\max} = \frac{1}{2} \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{I_0^2}{4\pi r^2} \cdot 4$$

and the directivity

$$D = \frac{4\pi r^2 S_{\max}}{P} = \frac{\frac{1}{2} \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{I_0^2}{4\pi^2 r^2} \cdot 4 \cdot 4\pi r^2}{\sqrt{\frac{\mu_0}{\epsilon_0}} \frac{I_0^2}{4\pi} \cdot 1,66} = 4,82$$

The radiation resistance:

$$P = \frac{1}{2} I_0^2 R_{\text{rad}}$$

$$\Rightarrow R_{\text{rad}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{I_0^2}{4\pi} \cdot 1,66 \cdot 2 / I_0^2$$

$$= \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{3,32}{4\pi} = 377 \cdot \frac{3,32}{4\pi} = \underline{\underline{99,6 \Omega}}$$

2)

i)

Vi ser direkte fra uttrykket for \vec{A}
at $\vec{A} = 0$

$\vec{r}' \times \vec{J}(\vec{r}') = 0$ siden \vec{r}' i under
integrasjonen ligger langs z -aksen
og $\vec{J} \parallel \hat{z}$ -aksen.

$$M_0 = 0$$

ii)

Now remember that the magnetization due to a current \vec{J} was $\vec{M} = \frac{1}{2}(\vec{r}' \times \vec{J})$

and the total moment: $\vec{M} = \frac{1}{2} \int (\vec{r}' \times \vec{J}) d\vec{z}'$ (prob. 5.60)

The magnetic moment of a current loop was $\vec{m}_0 = \pi a^2 \cdot I_0 \hat{z}$ and the expression above is the magnetic moment/volume of a distributed current

Example: Current loop. Use cylindrical coordinates

$$\vec{J}(\vec{r}') = I_0 \delta(r' - a) \delta(z' - z_0) \hat{\phi}$$

$$\vec{m} = \frac{1}{2} \int (\vec{r}' \times \hat{\phi}) I_0 \delta(r' - a) \delta(z' - z_0) r' dr' d\phi dz'$$

$$= \frac{1}{2} 2\pi I_0 a^2 \hat{z} = I_0 \pi a^2 \hat{z} = \text{moment of current loop}$$