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Exam in TFY4305 IKKELINEÆR DYNAMIKK
Onsdag, 2. desember, 2009
09:00–13:00

Allowed help: Alternativ B
Godkjent lommekalkulator.
K. Rottman: *Matematisk formelsamling* (alle sprogutgaver).
O.H. Jahren og K.J. Knudsen: *Formelsamling i matematikk*.

This problem set consists of 2 pages.

Problem 1

Consider the system of differential equations

$$\dot{x} = y(1+x)\,, \quad (1)$$
$$\dot{y} = x(1+y^3)\,. \quad (2)$$
$$(3)$$

a) Find the fixed points of the system.

b) Determine the type and stability of the fixed points.

c) Use the nullclines and the eigendirections of the fixed points together with common sense to sketch phase space.

Problem 2

Consider the system of differential equations

$$\dot{x} = -x-2y^2\,, \quad (4)$$
$$\dot{y} = xy-x^2y\,. \quad (5)$$
$$(6)$$

a) Show that there is only one fixed point.

b) Construct a Liapunov function $V(x,y)$ to show that the fixed point is stable. Hint: It might be a good idea to try out $V(x,y)=x^2+ay^2$ where a is a parameter.

Problem 3

We will here consider a linear chain of N particles. Each particle is connected to its two nearest neighbors through a linear spring. In addition, each particle is subject to an external "ϕ^4" potential. The hamiltonian (i.e., energy function) for such a system is given by

$$H = \sum_n \frac{1}{2}\,(\phi_{n+1} - \phi_n)^2 + \frac{a}{4}(\phi_n^2 - 1)^2\,, \tag{7}$$

where a is a positive constant.

a) Show that the equillibrium positions of the particles, determined by

$$\frac{\partial H}{\partial \phi_n} = 0\,, \tag{8}$$

are given by the relation

$$\phi_{n+1} + \phi_{n-1} - 2\phi_n = a\phi_n(\phi_n^2 - 1)\,. \tag{9}$$

b) Show that by substituting $x_n = \phi_n$ and $y_n = \phi_{n-1}$, relation (9) can be transformed into a two-dimensional mapping $(x_{n+1}, y_{n+1}) = T(x_n, y_n)$ on the form

$$T(x_n, y_n) = (2x_n + ax_n(x_n^2 - 1) - y_n, x_n)\,. \tag{10}$$

c) Show that T is area preserving.

d) Find the fixed points for T.

e) Determine the eigenvalues and the stability of the fixed points.

f) Show that the fixed point $(0, 0)$ bifurcates at $a = 4$. What kind of bifurcation is this?