Norges teknisk–naturvitenskapelige universitet NTNU

Institutt for fysikk
Fakultet for naturvitenskap og teknologi

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Exam in TFY4305 Non-Linear Dynamics
Friday, December 10, 2010
09:00–13:00

Allowed help: Alternativ B
  Approved pocket calculator.
  K. Rottman: *Matematisk formelsamling* (All editions)
  O.H. Jahren og K.J. Knudsen: *Formelsamling i matematikk*.

This problem set consists of 2 pages.

Problem 1

Consider the system of differential equations

$$\dot{x} = x^2 - y - 1\,, \tag{1}$$
$$\dot{y} = (x-2)y\,. \tag{2}$$
$$\tag{3}$$

a) Find the fixed points of the system.

b) Determine the Jacobian matrix for the system.

c) Determine the type and stability of the fixed points.

d) Draw the nullclines of the system.

e) Sketch its phase portrait.

Problem 2

Consider the one-dimensional differential equation

$$\dot{x} = r^2 x - 2x^2 + x^3 \,, \tag{4}$$

where $r \in (-\infty, \infty)$.

a) Find the fixed points.

b) Identify a bifurcation point and find the critical value of r, r_c.

c) What kind of bifurcation takes place for $r = r_c$?

d) Determine the stability of all the fixed points.

e) Sketch the bifurcation diagram, i.e., fixed points x^* as function of r. Distinguish between the stable and unstable fixed points.

Problem 3

Consider the logistic map

$$x_{n+1} = r x_n (1 - x_n) \,. \tag{5}$$

a) Show that by the transformation $x_n = a\tilde{x}_n + b$, the logistic equation can be transformed into

$$\tilde{x}_n = R - \tilde{x}_n^2 \,. \tag{6}$$

Determine a, b and R as functions of r for this transformation.

b) Consider now the map $x_{n+1} = f_R(x_n) = R - x_n^2$. We assume that R has been chosen so that the system is in a two cycle with x_0 being one of the values it oscillates between. Write a fourth order equation in x_0 for the period two cycle of the map.

c) Write the derivative of the second iterate ${f^2}_R{}'(x_0)$.

d) What is the condition for the period two cycle to undergo a period doubling bifurcation?