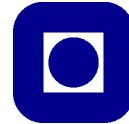


Norges teknisk–naturvitenskapelige
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Exam in TFY4305 Non-Linear Dynamics

Friday, December 10, 2010
09:00–13:00

Allowed help: Alternativ B

Approved pocket calculator.

K. Rottman: *Matematisk formelsamling* (All editions)

O.H. Jähren og K.J. Knudsen: *Formelsamling i matematikk*.

This problem set consists of 2 pages.

Problem 1

Consider the system of differential equations

$$\dot{x} = x^2 - y - 1, \quad (1)$$

$$\dot{y} = (x - 2)y. \quad (2)$$

(3)

- a) Find the fixed points of the system.
- b) Determine the Jacobian matrix for the system.
- c) Determine the type and stability of the fixed points.
- d) Draw the nullclines of the system.
- e) Sketch its phase portrait.

Problem 2

Consider the one-dimensional differential equation

$$\dot{x} = r^2x - 2x^2 + x^3, \quad (4)$$

where $r \in (-\infty, \infty)$.

- a) Find the fixed points.
- b) Identify a bifurcation point and find the critical value of r , r_c .
- c) What kind of bifurcation takes place for $r = r_c$?
- d) Determine the stability of all the fixed points.
- e) Sketch the bifurcation diagram, i.e., fixed points x^* as function of r . Distinguish between the stable and unstable fixed points.

Problem 3

Consider the logistic map

$$x_{n+1} = rx_n(1 - x_n). \quad (5)$$

- a) Show that by the transformation $x_n = a\tilde{x}_n + b$, the logistic equation can be transformed into

$$\tilde{x}_n = R - \tilde{x}_n^2. \quad (6)$$

Determine a , b and R as functions of r for this transformation.

- b) Consider now the map $x_{n+1} = f_R(x_n) = R - x_n^2$. We assume that R has been chosen so that the system is in a two cycle with x_0 being one of the values it oscillates between. Write a fourth order equation in x_0 for the period two cycle of the map.
- c) Write the derivative of the second iterate $f_R^2(x_0)$.
- d) What is the condition for the period two cycle to undergo a period doubling bifurcation?