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EXAM IN THE SUBJECT TFY 4320 MEDICAL PHYSICS

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Aid: Simple calculator permitted

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Task 1:

a) Explain how the bremsstrahlung spectrum of a diagnostic X-ray tube can be derived from a thin target spectrum. Why does the beam need filtering before entering the patient? What is meant by the phrase 'beam hardening'? How does the automatic exposure control in a diagnostic X-ray unit work?
The spectrum from a thin target has constant intensities (photon energy times number of photons) up to energy of each electron (corresponding to kV_{peak}*)that hits the target. In the case of a thick target (anode) one can imagine that the anode consists of several layers and that the electron looses a fixed amount of energy in each layer. From each of these layers one gets a thin layer spectrum, adding up to form the theoretical linear increase of intensity with decreasing photon energy.Beam hardening means that the low energy photons because of a higher attenuation will be absorber, resulting in a shift toward higher photon energies in the spectrum. Automatic exposure control is based on an ionization chamber in the beam. The charge collectedt from this chamberis amplified and serves to stop exposure when a preset dose to the detector is reached.*

b) Explain how a digital detector based on amorphous selenium is constructed and how it works. Another kind of digital detector is based on the use of silicon to collect the charge. Which are the differences between these two detectors?
The amorphous selenium acts as a semiconductor in which electron-hole pairs are formed in proportion to the deposited radiation energy. The detector surface is

divided into pixels by small charge collecting metal areas forming a lattice. The charge collecting capacitor in each such pixel is connected to a field effect transistor which can be opened by a signal on a gate line. A gate line signal is applied simultaneously to aall the pixels in a pixel row, and the charges that flows out of the capacitors are connected to leads running in the column direction. These signals are amplified for each row position by a charge sensitive amplifier and in turn transferred to analog to digital converters. The digital values are then stored into the corresponding positions in an image memory.

The other kind of flat panel detectors uses needles of the scintillator CsI,The light produced is converted to charges under each pixel by small photodioded and collecting capacitros. The layout of the gate lines and signal lines are the same as in the selenium detector.

c) Assume that a digital detector can be used to acquire dynamic series of images. Explain how such series in combination with an injection of contrast medium can be used to produce net images of arteries and veins.

The digital subtraction technique is based on a precontrast and a contrast image.The logarithm to the transmission of radiation before injection of iodine contrast may be expressed as follows

$$\ln(I_0 / I) = \int \mu(l)dl$$

And afterwards by the expression

$$\ln(I_0 / I) = \int \mu(l)dl + m \cdot (\mu / \rho)_I$$

Where m is the surface weight (g/cm^2) of iodine and $(\mu/\rho)_I$ is the mass attenuation coefficient for iodine. The integral runs along the entire path from the x-ray source to the detector. By subtraction of the two equations a net image of the vessels is produced.

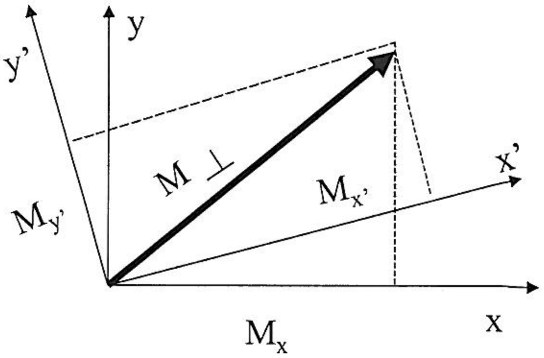
d) Assume that a digital detector has an efficiency of 60 percent. During the expose 400 monoenergetc photons move towards a certain pixel. What would be the signal dependant relative standard deviation of the output signal from that pixel?

*The number of photons that interact is 0.6*400m and the standard deviation is the (because of a Gaussian distribution) the square root of this number or approximately 16. The relative standard deviation would thus be 1/16=6.6 per cent.*

Task 2:

a) How should a receiver coil be constructed and positioned within the scanner to pick up signals form the rotating M-vectors after excitation in an MR-scanner?
It is important that the direction of the coil is such that the M-vectors after excitation rotate in such a way that they induce a maximum signal in the coil. This is obtained if the axis of the coil is perpendicular to the B0 field. A saddle coil is one of the most common constructions.

b) Explain how the signal from the coil is used in a quadrature demodulator to obtain a complex signal that represents the rotation of all the M-vectors in the excited plane. Which is the frame of reference?
After amplification the signal is fed into two branches in which the signal is multiplied by a cosine and a sine version of the reference frequency (Larmour frequency of the B_0 field). Refer to the figure that shows the M-vector rotating relative to the reference. The x',y' system is rotating with an angular frequency $\omega_0 t$



*The recorded signal is proportional to the projection of **M** on the x axis:*

$M_{x'} \cos(\omega_0 t)$ - $M_{y'} \sin(\omega_0 t)$.

The signals from the demodulator (after multiplication with the reference waves) are

$$\begin{aligned}
\mathrm{Re} &= (M_{x'} \cos(\omega_0 t) - M_{y'} \sin(\omega_0 t)\ ) \cos(\omega_0 t) \\
&= 0.5\, M_{x'} (\cos^2(\omega_0 t) + \sin^2(\omega_0 t)) + 0.5\, (\cos^2(\omega_0 t) - \sin^2(\omega_0 t)) - M_{y'} \sin(\omega_0 t) \cos(\omega_0 t) \\
&= 0.5\, M_{x'} + 0.5\, M_{x'} \cos(2\omega_0 t) -0.5\, M_{y'} \sin(2\omega_0 t)
\end{aligned}$$

$$\begin{aligned}
\mathrm{Im} &= (M_{x'} \cos(\omega_0 t) - M_{y'} \sin(\omega_0 t)\ ) \sin(\omega_0 t) \\
&= 0.5\, M_{y'} (\sin^2 (\omega_0 t) + \cos^2(\omega_0 t)) +0.5\, M_{y'} (\sin^2 (\omega_0 t)- \cos^2(\omega_0 t)) - M_{x'} \sin(\omega_0 t) \cos(\omega_0 t) \\
&= \quad 0.5\, M_{y'} + 0.5\, M_{y'} \cos(2\omega_0 t) -0.5\, M_{x'} \sin(2\omega_0 t)
\end{aligned}$$

An analogue low pass filter is then used to remove the high frequency components.

The frame of reference is a coordinate system that rotates with the Larmour *frequency of the* B_0 *field*

c) How is the local angular frequency of a rotating M-vector related to the magnetic field gradient G_x and the coordinate x? Express the phase angle that a rotating M-vector in position x acquires after t time units with this gradient. How is this phase related to a one of the spatial frequency components of the M-vector distribution?
Relative to the x',y' system, the angular frequency is given by
$\omega_x = \gamma x G_x$ *leading to a phase* $\varphi_x = \omega_x t = \gamma t G_x x = k_x x$*. The quantity* k_x *is a spatial angular frequency. This can be seen by a dimension analysis:*
$[k_x]=[radians/sT].[T/m].[s]=[radians/m]$

d) In a spin echo sequence, the M-vectors have to be excited several times to fill the k-space. Explain how it is possible to perform simultaneous acquisitions of data for multiple planes. *The echo is read within a maximum of 100 ms after the excitation of a plane. The time between excitations (the repetition time) may be as long as 2000 ms. While the M-vectors of that plane undergo relaxation, a second plane may be excited (using a slightly different frequency) and then a third etc. The signals read are store into the same line of different k-spaces (one space for each plane). After all excitation have been performed, these k-spaces have been filled, enabling the reconstruction of one image per k-space.*

Task 3:

a) Derive the formula for the shift of frequency in ultrasound measurement of blood flow velocity. Assume that the ultrasound field hits the blood vessel at an angle θ, and that the velocity of the blood cells is v. Which are the differences between continuous and pulsed Doppler measurements? Which are the advantages and disadvantages of the two techniques? Explain how a continuous demodulated signal is formed in the case of a pulsed Doppler technique.
The round trip time from a ultrasound transmitter to the receiving piezoelectric element is

In the case of a stationary ultrasound reflector

$t = 2x/c$ *(c is the speed of the ultrasound wave)*
and

$t' = 2(x \pm v\cos\theta)/c$

in the case of a reflector that moves at a velocity v at en angle θ *relative to the direction of the ultrasound beam. This means that (apart for a phase difference) the reflected signal can be expressed as* $r(t)=q\cos(t')=q\cos((\omega \pm 2v\cos\theta\omega/c)t)$
A pulsed Doppler measurement can be fixed to a range, while this is not possible for the continous Doppler technique. The continuos Doppler technology is usually very cheap. A continuous signal is formed by demodulating each short pulse (signed)and linking the levels together along the time axis with' sample and hold' circuits. This signal is smoothed before further processing and analysis.

b) The central slice theorem may be written F(k,θ) = P(k, θ) where k is the angular spatial frequency and F(k,θ) and P(k, θ) represent Fourier transforms of the function f(x,y) and its projection p(t, θ) on an axis t at an angle θ relative to the x,y-system.

Explain briefly what is the significance of this formula and how it can be used to derive an expression of the function f(x,y).

The theorem states that information on the Fourier transform along a given direction in space can be found by Fourier transforming the projection perpendicular to this direction in space. An inverse Fourier transform may then in principle be used to obtain f(x,y) from the projections. Specifically this can be done by using a polar coordinate system for the inverse transform.