NORGES TEKNISK-
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Contact during the exam:
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EXAM
TFY4340 MESOSCOPIC PHYSICS
Friday June 1 2012, 0900 - 1300
English

Remedies: C

- K. Rottmann: Mathematical formulae
- Calculator with empty memory

Pages 2 – 5: Problems 1 – 5. The five problems are relatively unrelated and may be answered in any order. Also, many of the subquestions within a given problem may be answered independently from the others.

Notation: Vectors are given in bold italic. Unit vectors are given with a hat above the symbol.

Some constants:
Electron mass: $m_e = 9.1 \cdot 10^{-31}$ kg. Elementary charge: $e = 1.6 \cdot 10^{-19}$ C.
Boltzmann constant: $k_B = 1.38 \cdot 10^{-23}$ J/K. Planck constant: $\hbar = h/2\pi = 1.05 \cdot 10^{-34}$ Js.

The grades will be available around June 10.

PROBLEM 1: Historical. **[Weight: 10%. Suggested timing: 5 – 20 minutes.]**

Connect Nobel laureate(s) with topic and year for receiving the prize:

A Albert Einstein
B Albert Fert/Peter Grünberg
C Andre Geim/Konstantin Novoselov
D Ivar Giæver/Leo Esaki/Brian Josephson
E Klaus von Klitzing

1 Giant magnetoresistance
2 Graphene
3 Photoelectric effect
4 Quantum Hall effect
5 Tunneling phenomena in semiconductors and superconductors

1921 1973 1985 2007 2010

PROBLEM 2: Short qualitative questions. **[20%. 20 – 50 min.]**

Use 1 – 3 sentences to answer each of the subquestions **a** – **e** below.

a) Explain briefly the difference between Bloch states and surface states.

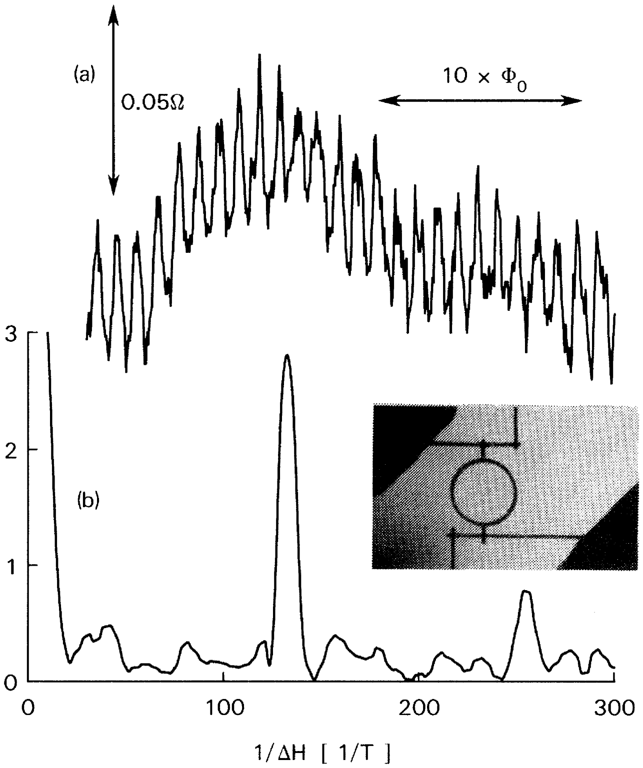
b) To the question ”What is mesoscopic physics?”, what would your answer be?

c) Assuming isotropic conditions (i.e., no directional dependence of relevant properties), how is the electron effective mass m^* defined in terms of the electronic band structure $E(k)$?

d) How would you define, and hence distinguish between, a metal and an insulator, in terms of the electronic band structure?

e) Explain briefly, with a figure and a few sentences, how the controlled layer-by-layer growth of GaAs and $\mathrm{Al_{0.3}Ga_{0.7}As}$ can be used to construct a resonant tunneling device.

PROBLEM 3: Aharonov – Bohm effect. **[25%. 30 – 70 min.]**



The figure is taken from R. A. Webb et al, *Phys Rev Lett* **54**, 2696 (1985). Figure (a) displays the magnetoresistance of the metallic (gold) ring shown in the inset, i.e., $R(H)$, with H the (uniform) applied magnetic field perpendicular to the plane of the ring. (No distinction between H and B here!) The inside diameter of the ring is $d = 784$ nm and its wire width is $w = 41$ nm. Figure (b) shows the Fourier spectrum of R, with the most prominent peak at $1/\Delta H = 131$ T^{-1}. The temperature in the experiment was 10 mK.

a) It follows directly from Feynman's path integral approach to quantum mechanics that phase coherent electron transport in the presence of a magnetic field $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ manifests itself as a phase factor $\Delta\phi$ in the wavefunction,

$$\psi \sim \exp(i\Delta\phi) = \exp(-\frac{ie}{\hbar}\int \boldsymbol{A}\cdot d\boldsymbol{l}),$$

where the ("path") integral is taken along the path traversed by the electron. Show that quantum interference is responsible for the observed oscillatory behavior of the magnetoresistance $R(B)$ in the experiment by Webb et al. (*Aharonov - Bohm effect.*) In other words, show that the transmission probability T of the ring contains a B-dependent factor,

$$T \sim |\psi_T|^2 \sim 1 + \cos(2\pi B/B_0),$$

and find an expression for the "period" B_0 (in terms of fundamental constants and the ring geometry).

b) Verify that the peak in the Fourier spectrum of R at $1/B_0 = 1/\Delta H = 131\ \mathrm{T}^{-1}$ is consistent with the size of the ring.

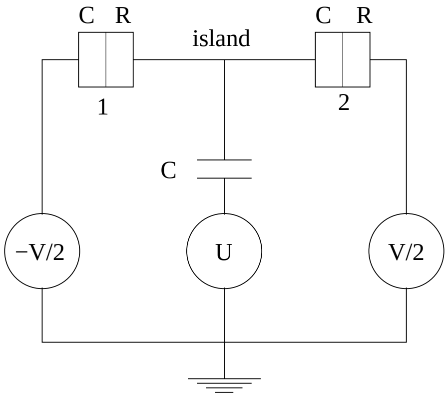
c) Explain briefly why the Aharonov - Bohm effect may be regarded as a *nonlocal* effect of the electromagnetic field. Explain also what is meant by *gauge invariance*, and argue that the Aharonov - Bohm effect is indeed gauge invariant.

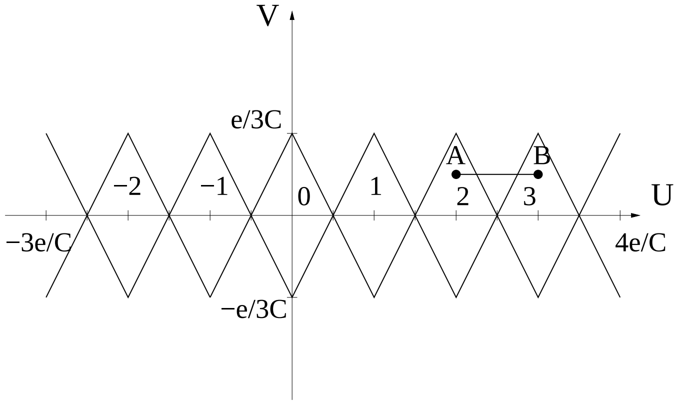
d) In *The Big Bang Theory*, who is performing experiments related to the Aharonov - Bohm effect?

Given information: Stokes' theorem (for arbitrary vector field $\boldsymbol{G}$):

$$\int (\nabla \times \boldsymbol{G}) \cdot d\boldsymbol{S} = \oint \boldsymbol{G} \cdot d\boldsymbol{l}$$

PROBLEM 4: Single electron transistor. [15%. 10 – 30 min.]





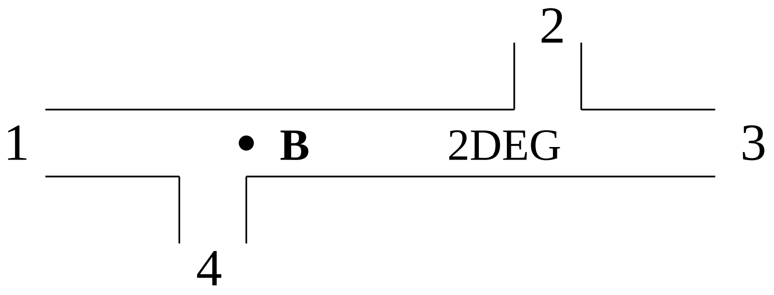
The single electron transistor shown in the left figure has a metallic island coupled to the outside world through two identical tunneling junctions 1 and 2, both with capacitance C and resistance R ($R \gg R_Q = h/e^2$), and an ideal capacitor C. The island has a net charge $q = -ne$, corresponding to n "extra" electrons. Constant bias voltages $\pm V/2$ are applied to the tunneling junctions, as illustrated in the figure. A time dependent gate voltage $U(t) = U_0 + U_1 \cos(2\pi f t)$ is connected to the ideal capacitor. The right figure shows the UV plane with diamond shaped areas, inside which a certain number of extra electrons (n) on the island corresponds to the equilibrium situation (i.e., the lowest energy). The appropriate value of n is indicated inside each diamond shaped area.

a) Assume a value of V corresponding to the horizontal line between A and B in the figure. Explain what happens when the gate voltage is set to oscillate with frequency f between the minimum value $U_0 - U_1$ at A and the maximum value $U_0 + U_1$ at B. Specifically, what is the resulting current I?

b) If the system capacitance C is in the fF (femtofarad) range, at what temperature T should experiments be performed, in order to reveal clear single electron charging effects?

PROBLEM 5: Büttiker – Landauer and quantum Hall effect. [30%. 30 – 70 min.]

Consider the following ideal 4-terminal device:



A strong uniform magnetic field $\boldsymbol{B}$ is applied perpendicular to the 2DEG and points out of the plane.

The Büttiker–Landauer equations,

$$I_\alpha = \sum_{\beta \neq \alpha} G_{\alpha\beta} \left(V_\alpha - V_\beta \right),$$

with conductances

$$G_{\alpha\beta} = \frac{2e^2}{h} T_{\alpha\beta},$$

relate the net current entering into terminal α to the potentials at the various terminals. Here, $T_{\alpha\beta}$ denotes the "direct transmission sum" from terminal β to terminal α.

a) Assume that only the lowest Landau level lies below the Fermi energy E_F in the bulk region of the 2DEG. Express the relation between currents and potentials as

$$I_\alpha = \frac{2e^2}{h} \sum_{\beta=1}^{4} \gamma_{\alpha\beta} V_\beta,$$

and write down the 4×4 matrix $\boldsymbol{\gamma}$.

b) Let terminals 1 and 3 be the "source" and the "drain", respectively, whereas terminals 2 and 4 are ideal voltage probes. Find the Hall resistance $R_H = R_{13,24} = (V_2 - V_4)/I_1$ and the 2-terminal resistance $R_{2t} = R_{13,13} = (V_1 - V_3)/I_1$.

c) Interchange the roles of terminals 2 and 3 and find the 2-terminal resistance $R_{2t} = R_{12,12} = (V_1 - V_2)/I_1$ and the longitudinal resistance $R_L = R_{12,34} = (V_3 - V_4)/I_1$.

d) Qualitatively: With terminals 1 and 3 as source and drain, and terminals 2 and 4 as voltage probes (as in **b)**), explain how R_H will change as we decrease the magnetic field strength. Explain also how R_L depends on B when terminals 3 and 4 are used as voltage probes (as in **c)**).