FY8201 / TFY8 **Nanoparticle and polymer physics I**
SOLUTION of EXERCISE 8

Eq. (x.x) refers to version AM24nov05 of lecture notes: "Nano-particle and polymer physics". Equations pertinent to this exercise you will find in Ch. 5.1.4.

A)

$$\begin{aligned} \text{Displacement :} \quad x(t) &= x_0 \sin\omega t && (1) \\ \text{Friction force :} \quad F^{(\mathrm{h})} &= -\zeta\dot{x} = -6\pi\eta R\omega x_0 \cos\omega t && (2) \\ \text{Inertia force :} \quad F^{(\mathrm{m})} &= m\ddot{x} = -\frac{4}{3}\pi R^3 \rho_\mathrm{s}\omega^2 x_0 \sin\omega t\,, && (3) \end{aligned}$$

where η is the viscosity of the fluid, ρ_s is the mass density of the sphere and ζ is the friction coefficient for spheres (Stokes law). Then

$$\frac{|F^{(\mathrm{m})}|}{|F^{(\mathrm{h})}|} = \frac{\frac{4}{3}\pi R^3\rho_\mathrm{s}\omega^2 x_0}{6\pi\eta R\omega x_0} = \underline{\frac{2}{9}\frac{\rho_\mathrm{s}}{\eta}\omega R^2}. \qquad (4)$$

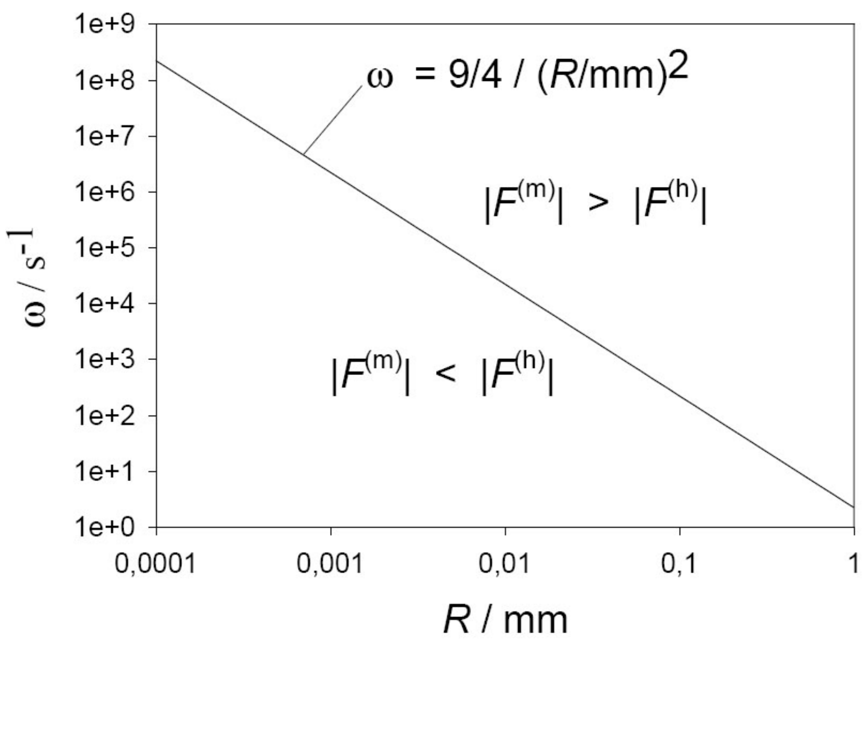
B) The result shows that $|F^{(\mathrm{m})}| \le |F^{(\mathrm{h})}|$ when

$$\omega \le \frac{9}{2}\frac{\eta}{\rho_\mathrm{s}}\frac{1}{R^2}.$$

For $\rho_\mathrm{s} = 2000$ kg/m^3 and $\eta = 1.0\cdot 10^{-3}$ Ns/m^2, this yields

$$\omega \le \frac{9}{2}\cdot\frac{1\cdot 10^{-3}\ \mathrm{Ns/m^2}}{2000\ \mathrm{kg/m^3}}\frac{1}{R^2} = \frac{9/4\cdot 10^{6}\ \mathrm{m^2\ s^{-1}}}{R^2} = \frac{9/4}{(R/\mathrm{mm})^2}\ \mathrm{s}^{-1}. \qquad (5)$$

A log-log-plot of $\omega = \frac{9/4}{(R/\mathrm{mm})^2}\ \mathrm{s}^{-1}$ as function of R is shown below. In the graph note that for small particles $|F^{(\mathrm{m})}| \le |F^{(\mathrm{h})}|$ also for very rapid oscillations.



C) The Reynold number is defined (Eq. 5.10) $Re := R\frac{\rho_0}{\eta}|\dot{x}|$ and is an indicator whether the flow is laminar or not. For $Re \leq 0.1$ the Stokes law is well satisfied. For the conditions described above ($\rho_0 = 1.00$ g/cm$^3 = 1.00 \cdot 10^3$ kg/m^3 and $\eta = 1.0 \cdot 10^{-3}$ Ns/m^2) and with $x_0 = 1.0$ μm, we get

$$\begin{aligned} Re = R\frac{\rho_0}{\eta}\omega x_0 &\leq 0.1 \\ \omega &\leq \frac{1}{10}\frac{\eta}{x_0\rho_0}\cdot\frac{1}{R} = \frac{1}{10}\cdot\frac{1\cdot 10^{-3}\ \mathrm{Ns/m^2}}{1\ \mu\mathrm{m}\cdot 1\cdot 10^3\ \mathrm{kg/m^3}}\cdot\frac{1}{R} = \frac{1}{10\,\mathrm{s/m}}\cdot\frac{1}{R} \\ \omega &\leq \frac{100}{R/\mathrm{mm}}\ \mathrm{s}^{-1}\,. \end{aligned} \tag{6}$$

A log-log-plot of $\omega = \frac{100}{R/\mathrm{mm}}$ s^{-1} as function of R is shown below. Note that the condition for laminar flow is fulfilled also for very rapid oscillations when the particles are very small. For larger particles ($R >\sim 0.1$ mm) the figure shows that the condition $|F^{(\mathrm{m})}| \leq |F^{(\mathrm{h})}|$ is more strickt than the laminar Reynold-condition.