

FY8201 / TFY8 Nanoparticle and polymer physics I

EXERCISE 1

A) Write down the equation of motion for the one-dimensional Rouse chain using Newton's law and Hooke's law. Write on vector form the set of differential equations describing the dynamics of the individual beads of the chain. By introducing the relative coordinate $Q_\nu = R_{\nu+1} - R_\nu$ show how the *Rouse matrix* enters.

HINT: It will prove convenient to introduce and define $R_0 = R_1$ and $R_{N+1} = R_N$, and consequently $Q_0 = 0$ and $Q_N = 0$.

B) Study the limit when the number of beads in the Rouse chain $N \rightarrow \infty$ keeping the length L constant. Show how the equation of motion can be interpreted as the difference form of the differential equation describing the pressure waves in a one-dimensional continuous rod.

C) From the results above, show that the eigenvalues of the Rouse matrix in the continuous case $N \rightarrow \infty$ equals

$$a_j = \frac{\pi^2 j^2}{N^2}, \quad j = 1, 2, \dots$$

D) Also prove that the the eigenvalues of the Rouse matrix in the discrete case (finite N) equals

$$a_j = 4 \sin^2 \left(\frac{j\pi}{2N} \right), \quad j = 1, 2, \dots$$