

Using a constant-acceleration equation, express the final velocity of the bullet in terms of its acceleration and solve for the acceleration:

$$v^2 = v_0^2 + 2a\Delta x$$

and

$$a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{-v_0^2}{2\Delta x}$$

Substitute to obtain:

$$F_{\text{wood}} = -\frac{mv_0^2}{2\Delta x}$$

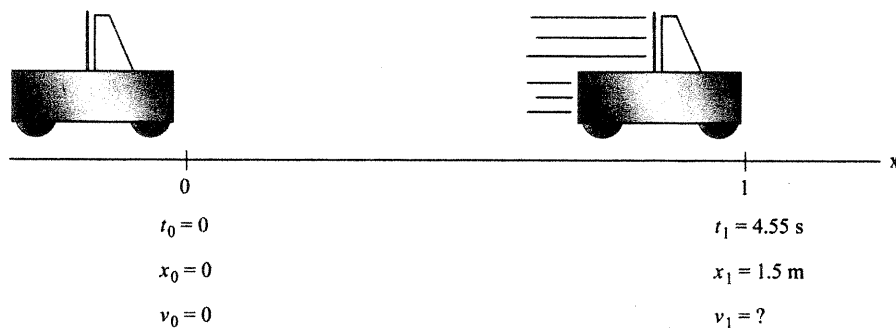
Substitute numerical values and evaluate  $F_{\text{wood}}$ :

$$\begin{aligned} F_{\text{wood}} &= -\frac{(1.8 \times 10^{-3} \text{ kg})(500 \text{ m/s})^2}{2(0.06 \text{ m})} \\ &= \boxed{-3.75 \text{ kN}} \end{aligned}$$

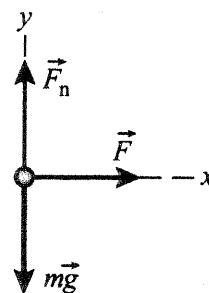
where the negative sign means that the direction of the force is opposite the velocity.

\*30 ••

**Picture the Problem** The pictorial representation summarizes what we know about the motion. We can find the acceleration of the cart by using a constant-acceleration equation.



The free-body diagram shows the forces acting on the cart as it accelerates along the air track. We can determine the net force acting on the cart using Newton's 2<sup>nd</sup> law and our knowledge of its acceleration.



(a) Apply  $\sum F_x = ma_x$  to the cart to obtain an expression for the net force  $F$ :

$$F = ma$$

Using a constant-acceleration equation, relate the displacement of the cart to its acceleration, initial speed, and travel time:

$$\Delta x = v_0 \Delta t + \frac{1}{2} a (\Delta t)^2$$

or, because  $v_0 = 0$ ,

$$\Delta x = \frac{1}{2} a (\Delta t)^2$$

Solve for  $a$ :

$$a = \frac{2\Delta x}{(\Delta t)^2}$$

Substitute for  $a$  in the force equation to obtain:

$$F = m \frac{2\Delta x}{(\Delta t)^2} = \frac{2m\Delta x}{(\Delta t)^2}$$

Substitute numerical values and evaluate  $F$ :

$$F = \frac{2(0.355 \text{ kg})(1.5 \text{ m})}{(4.55 \text{ s})^2} = \boxed{0.0514 \text{ N}}$$

(b) Using a constant-acceleration equation, relate the displacement of the cart to its acceleration, initial speed, and travel time:

$$\Delta x = v_0 \Delta t + \frac{1}{2} a' (\Delta t)^2$$

or, because  $v_0 = 0$ ,

$$\Delta x = \frac{1}{2} a' (\Delta t)^2$$

Solve for  $\Delta t$ :

$$\Delta t = \sqrt{\frac{2\Delta x}{a'}}$$

If we assume that air resistance is negligible, the net force on the cart is still 0.0514 N and its acceleration is:

$$a' = \frac{0.0514 \text{ N}}{0.722 \text{ kg}} = 0.0713 \text{ m/s}^2$$

Substitute numerical values and evaluate  $\Delta t$ :

$$\Delta t = \sqrt{\frac{2(1.5 \text{ m})}{0.0713 \text{ m/s}^2}} = \boxed{6.49 \text{ s}}$$

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**Picture the Problem** The acceleration of an object is related to its mass and the *net* force acting on it according to  $\vec{F}_{\text{net}} = m\vec{a}$ . Let  $m$  be the mass of the object and choose a coordinate system in which the direction of  $2F_0$  in (b) is the positive  $x$  and the direction of the left-most  $F_0$  in (a) is the positive  $y$  direction. Because both force and acceleration are vector quantities, find the resultant force in each case and then find the resultant acceleration.

(a) Calculate the acceleration of the object from Newton's 2<sup>nd</sup> law of motion:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

Express the net force acting on the object:

$$\vec{F}_{\text{net}} = F_x \hat{i} + F_y \hat{j} = F_0 \hat{i} + F_0 \hat{j}$$

Apply  $\sum F_x = ma_x$  to the 0.250-kg block and solve for the tension  $T_2$ :

$$T_3 \cos \theta - T_2 = 0 \text{ since } a = 0.$$

and

$$T_2 = T_3 \cos \theta$$

Substitute numerical values and evaluate  $T_2$ :

$$T_2 = (3.43 \text{ N}) \cos 45.7^\circ = \boxed{2.40 \text{ N}}$$

By symmetry:

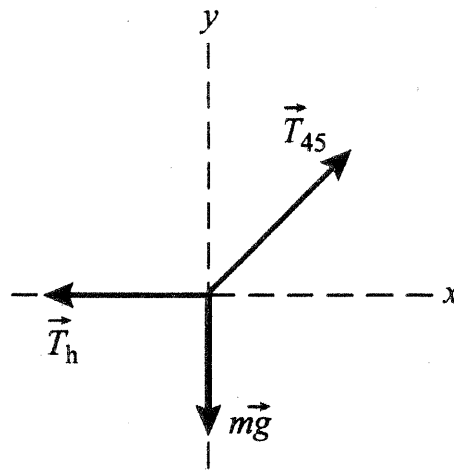
$$T_1 = T_3 = \boxed{3.43 \text{ N}}$$

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**Picture the Problem** The suspended body is in equilibrium under the influence of the forces  $\vec{T}_h$ ,  $\vec{T}_{45}$ , and  $m\vec{g}$ ;

$$\text{i.e., } \vec{T}_h + \vec{T}_{45} + m\vec{g} = 0$$

Draw the free-body diagram of the forces acting on the knot just above the 100-N body. Choose a coordinate system with the positive  $x$  direction to the right and the positive  $y$  direction upward. Apply the conditions for translational equilibrium to determine the tension in the horizontal cord.



If the system is to remain in static equilibrium, the vertical component of  $T_{45}$  must be exactly balanced by, and therefore equal to, the tension in the string suspending the 100-N body:

$$T_v = T_{45} \sin 45^\circ = mg$$

Express the horizontal component of  $T_{45}$ :

$$T_h = T_{45} \cos 45^\circ$$

Because  $T_{45} \sin 45^\circ = T_{45} \cos 45^\circ$ :

$$T_h = mg = \boxed{100 \text{ N}}$$

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**Picture the Problem** The acceleration of *any* object is directly proportional to the *net* force acting on it. Choose a coordinate system in which the positive  $x$  direction is the same as that of  $\vec{F}_1$  and the positive  $y$  direction is to the right. Add the two forces to determine the net force and then use Newton's 2<sup>nd</sup> law to find the acceleration of the object. If  $\vec{F}_3$  brings the system into equilibrium, it must be true that  $\vec{F}_3 + \vec{F}_1 + \vec{F}_2 = 0$ .

(a) Find the components of  $\vec{F}_1$  and  $\vec{F}_2$ :

$$\begin{aligned}\vec{F}_1 &= (20\text{ N})\hat{i} \\ \vec{F}_2 &= \{(-30\text{ N})\sin 30^\circ\}\hat{i} \\ &\quad + \{(30\text{ N})\cos 30^\circ\}\hat{j} \\ &= (-15\text{ N})\hat{i} + (26\text{ N})\hat{j}\end{aligned}$$

Add  $\vec{F}_1$  and  $\vec{F}_2$  to find  $\vec{F}_{\text{tot}}$ :

$$\vec{F}_{\text{tot}} = (5\text{ N})\hat{i} + (26\text{ N})\hat{j}$$

Apply  $\sum \vec{F} = m\vec{a}$  to find the acceleration of the object:

$$\begin{aligned}\vec{a} &= \frac{\vec{F}_{\text{tot}}}{m} \\ &= \boxed{(0.500\text{ m/s}^2)\hat{i} + (2.60\text{ m/s}^2)\hat{j}}\end{aligned}$$

(b) Because the object is in equilibrium under the influence of the three forces, it must be true that:

$$\begin{aligned}\vec{F}_3 + \vec{F}_1 + \vec{F}_2 &= 0 \\ \text{and} \\ \vec{F}_3 &= -(\vec{F}_1 + \vec{F}_2) \\ &= \boxed{(-5.00\text{ N})\hat{i} + (-26.0\text{ N})\hat{j}}\end{aligned}$$

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**Picture the Problem** The acceleration of the object equals the net force,  $\vec{T} - m\vec{g}$ , divided by the mass. Choose a coordinate system in which upward is the positive  $y$  direction. Apply Newton's 2<sup>nd</sup> law to the forces acting on this body to find the acceleration of the object as a function of  $T$ .

(a) Apply  $\sum F_y = ma_y$  to the object:

$$T - w = T - mg = ma_y$$

Solve this equation for  $a$  as a function of  $T$ :

$$a_y = \frac{T}{m} - g$$

Substitute numerical values and evaluate  $a_y$ :

$$a_y = \frac{5\text{ N}}{5\text{ kg}} - 9.81\text{ m/s}^2 = \boxed{-8.81\text{ m/s}^2}$$

(b) Proceed as in (a) with  $T = 10\text{ N}$ :

$$a = \boxed{-7.81\text{ m/s}^2}$$

(c) Proceed as in (a) with  $T = 100\text{ N}$ :

$$a = \boxed{10.2\text{ m/s}^2}$$

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**Picture the Problem** The picture is in equilibrium under the influence of the three forces shown in the figure. Due to the symmetry of the support system, the vectors  $\vec{T}$  and  $\vec{T}'$  have the same magnitude  $T$ . Choose a coordinate system in which the positive  $x$

direction is to the right and the positive  $y$  direction is upward. Apply the condition for translational equilibrium to obtain an expression for  $T$  as a function of  $\theta$  and  $w$ .

(a) Referring to Figure 4-37, apply the condition for translational equilibrium in the vertical direction and solve for  $T$ :

$$\sum F_y = 2T \sin \theta - w = 0$$

and

$$T = \frac{w}{2 \sin \theta}$$

$T_{\min}$  occurs when  $\sin \theta$  is a maximum:

$$\theta = \sin^{-1}(1) = 90^\circ$$

$T_{\max}$  occurs when  $\sin \theta$  is a minimum. Because the function is undefined when  $\sin \theta = 0$ , we can conclude that:

$$T \rightarrow T_{\max} \text{ as } \theta \rightarrow 0^\circ$$

(b) Substitute numerical values in the result in (a) and evaluate  $T$ :

$$T = \frac{(2 \text{ kg})(9.81 \text{ m/s}^2)}{2 \sin 30^\circ} = 19.6 \text{ N}$$

**Remarks:**  $\theta = 90^\circ$  requires wires of infinite length; therefore it is not possible. As  $\theta$  gets small,  $T$  gets large without limit.

\*50 ...

**Picture the Problem** In part (a) we can apply Newton's 2<sup>nd</sup> law to obtain the given expression for  $F$ . In (b) we can use a symmetry argument to find an expression for  $\tan \theta_0$ . In (c) we can use our results obtained in (a) and (b) to express  $x_i$  and  $y_i$ .

(a) Apply  $\sum F_y = 0$  to the balloon:

$$F + T_i \sin \theta_i - T_{i-1} \sin \theta_{i-1} = 0$$

Solve for  $F$  to obtain:

$$F = T_{i-1} \sin \theta_{i-1} - T_i \sin \theta_i$$

(b) By symmetry, each support must balance half of the force acting on the entire arch. Therefore, the vertical component of the force on the support must be  $N/2$ . The horizontal component of the tension must be  $T_H$ . Express  $\tan \theta_0$  in terms of  $N/2$  and  $T_H$ :

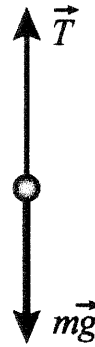
$$\tan \theta_0 = \frac{NF/2}{T_H} = \frac{NF}{2T_H}$$

By symmetry,  $\theta_{N+1} = -\theta_0$ . Therefore, because the tangent function is odd:

$$\tan \theta_0 = -\tan \theta_{N+1} = \frac{NF}{2T_H}$$

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**Picture the Problem** We know, because the speed of the load is changing in parts (a) and (c), that it is accelerating. We also know that, if the load is accelerating in a particular direction, there must be a *net* force in that direction. A free-body diagram for part (a) is shown to the right. We can apply Newton's 2<sup>nd</sup> law of motion to each part of the problem to relate the tension in the cable to the acceleration of the load. Choose the upward direction to be the positive *y* direction.



(a) Apply  $\sum F_y = ma_y$  to the load and solve for  $T$ :

$$T - mg = ma$$

and

$$T = ma_y + mg = m(a_y + g) \quad (1)$$

Substitute numerical values and evaluate  $T$ :

$$T = (1000 \text{ kg})(2 \text{ m/s}^2 + 9.81 \text{ m/s}^2)$$

$$= \boxed{11.8 \text{ kN}}$$

(b) Because the crane is lifting the load at constant speed,  $a = 0$ :

$$T = mg = \boxed{9.81 \text{ kN}}$$

(c) Because the acceleration of the load is downward,  $a$  is negative.

$$T - mg = ma_y$$

Apply  $\sum F_y = ma_y$  to the load:

Substitute numerical values in equation (1) and evaluate  $T$ :

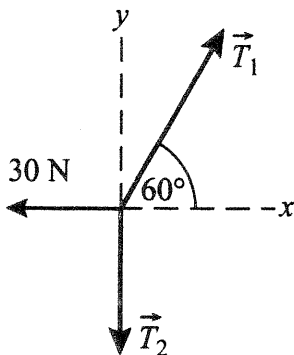
$$T = (1000 \text{ kg})(9.81 \text{ m/s}^2 - 2 \text{ m/s}^2)$$

$$= \boxed{7.81 \text{ kN}}$$

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**Picture the Problem** Draw a free-body diagram for each of the depicted situations and use the conditions for translational equilibrium to find the unknown tensions.

(a)



$$\sum F_x = T_1 \cos 60^\circ - 30 \text{ N} = 0$$

and

$$T_1 = (30 \text{ N}) / \cos 60^\circ = \boxed{60.0 \text{ N}}$$

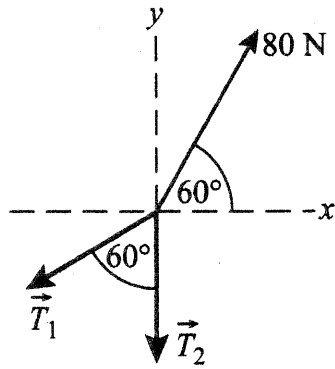
$$\sum F_y = T_1 \sin 60^\circ - T_2 = 0$$

and

$$T_2 = T_1 \sin 60^\circ = \boxed{52.0 \text{ N}}$$

$$\therefore m = T_2 / g = \boxed{5.30 \text{ kg}}$$

(b)



$$\Sigma F_x = (80 \text{ N})\cos 60^\circ - T_1 \sin 60^\circ = 0$$

and

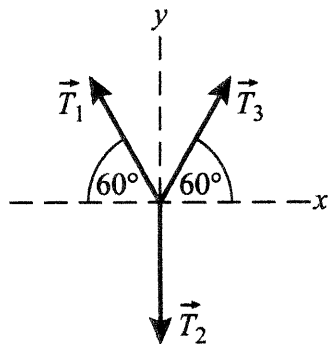
$$T_1 = (80 \text{ N})\cos 60^\circ / \sin 60^\circ = \boxed{46.2 \text{ N}}$$

$$\Sigma F_y = (80 \text{ N})\sin 60^\circ - T_2 - T_1 \cos 60^\circ = 0$$

$$T_2 = (80 \text{ N})\sin 60^\circ - (46.2 \text{ N})\cos 60^\circ = \boxed{46.2 \text{ N}}$$

$$m = T_2/g = \boxed{4.71 \text{ kg}}$$

(c)



$$\Sigma F_x = -T_1 \cos 60^\circ + T_3 \cos 60^\circ = 0$$

and

$$T_1 = T_3$$

$$\Sigma F_y = 2T_1 \sin 60^\circ - mg = 0$$

and

$$T_1 = T_3 = (58.9 \text{ N}) / (2 \sin 60^\circ) = \boxed{34.0 \text{ N}}$$

$$\therefore m = T_1/g = \boxed{3.46 \text{ kg}}$$

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**Picture the Problem** Construct the free-body diagram for that point in the rope at which you exert the force  $\vec{F}$  and choose the coordinate system shown on the FBD. We can apply Newton's 2<sup>nd</sup> law to the rope to relate the tension to  $F$ .

(a) Noting that  $T_1 = T_2 = T$ , apply

$$\Sigma F_y = ma_y \text{ to the car:}$$

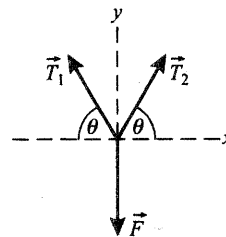
Solve for and evaluate  $T$ :

$2T \sin \theta - F = ma_y = 0$  because the car's acceleration is zero.

$$T = \frac{F}{2 \sin \theta} = \frac{400 \text{ N}}{2 \sin 3^\circ} = \boxed{3.82 \text{ kN}}$$

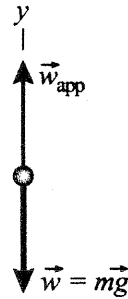
(b) Proceed as in part (a):

$$T = \frac{600 \text{ N}}{2 \sin 4^\circ} = \boxed{4.30 \text{ kN}}$$



\*65 •

**Picture the Problem** The sketch to the right shows a person standing on a scale in the elevator immediately after the cable breaks. To its right is the free-body diagram showing the forces acting on the person. The force exerted by the scale on the person,  $\vec{w}_{\text{app}}$ , is the person's apparent weight.



From the free-body diagram we can see that  $\vec{w}_{\text{app}} + m\vec{g} = m\vec{a}$  where  $\vec{g}$  is the local gravitational field and  $\vec{a}$  is the acceleration of the reference frame (elevator). When the elevator goes into free fall ( $\vec{a} = \vec{g}$ ), our equation becomes  $\vec{w}_{\text{app}} + m\vec{g} = m\vec{a} = m\vec{g}$ . This tells us that  $\vec{w}_{\text{app}} = 0$ . (e) is correct.

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**Picture the Problem** The free-body diagram shows the forces acting on the 10-kg block as the elevator accelerates upward. Apply Newton's 2<sup>nd</sup> law of motion to the block to find the minimum acceleration of the elevator required to break the cord.



Apply  $\sum F_y = ma_y$  to the block:

$$T - mg = ma_y$$

Solve for  $a_y$  to determine the minimum breaking acceleration:

$$a_y = \frac{T - mg}{m} = \frac{T}{m} - g$$

Substitute numerical values and evaluate  $a_y$ :

$$a_y = \frac{150 \text{ N}}{10 \text{ kg}} - 9.81 \text{ m/s}^2 = \boxed{5.19 \text{ m/s}^2}$$

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**Picture the Problem** The free-body diagram shows the forces acting on the 2-kg block as the elevator ascends at a constant velocity. Because the acceleration of the elevator is zero, the block is in equilibrium under the influence of  $\vec{T}$  and  $m\vec{g}$ . Apply Newton's 2<sup>nd</sup> law of motion to the block to determine the scale reading.



(a) Apply  $\sum F_y = ma_y$  to the block to obtain:

$$T - mg = ma_y \quad (1)$$



For motion with constant velocity,  
 $a_y = 0$  and:

$$T - mg = 0 \text{ and } T = mg$$

Substitute numerical values and  
 evaluate  $T$ :

$$T = (2 \text{ kg})(9.81 \text{ m/s}^2) = \boxed{19.6 \text{ N}}$$

(b) As in part (a), for constant  
 velocity,  $a = 0$ :

$$T - mg = ma_y$$

and

$$T = (2 \text{ kg})(9.81 \text{ m/s}^2) = \boxed{19.6 \text{ N}}$$

(c) Solve equation (1) for  $T$  and  
 simplify to obtain:

$$T = mg + ma_y = m(g + a_y) \quad (2)$$

Because the elevator is ascending  
 and its speed is increasing, we have  
 $a_y = 3 \text{ m/s}^2$ . Substitute numerical  
 values and evaluate  $T$ :

$$T = (2 \text{ kg})(9.81 \text{ m/s}^2 + 3 \text{ m/s}^2) \\ = \boxed{25.6 \text{ N}}$$

(d) For  $0 < t < 5 \text{ s}$ :  $a_y = 0$  and

$$T_{0 \rightarrow 5 \text{ s}} = \boxed{19.6 \text{ N}}$$

Using its definition, calculate  $a$  for  
 $5 \text{ s} < t < 9 \text{ s}$ :

$$a = \frac{\Delta v}{\Delta t} = \frac{0 - 10 \text{ m/s}}{4 \text{ s}} = -2.5 \text{ m/s}^2$$

Substitute in equation (2) and  
 evaluate  $T$ :

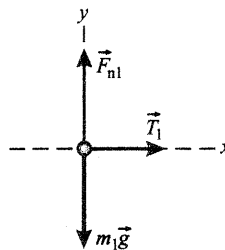
$$T_{5 \text{ s} \rightarrow 9 \text{ s}} = (2 \text{ kg})(9.81 \text{ m/s}^2 - 2.5 \text{ m/s}^2) \\ = \boxed{14.6 \text{ N}}$$

## Free-Body Diagrams: Ropes, Tension, and Newton's Third Law

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**Picture the Problem** Draw a free-body diagram for each object and apply Newton's 2<sup>nd</sup> law of motion. Solve the resulting simultaneous equations for the ratio of  $T_1$  to  $T_2$ .

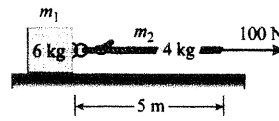
Draw the FBD for the box to the left  
 and apply  $\sum F_x = ma_x$ :



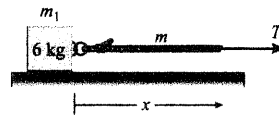
$$T_1 = m_1 a_1$$

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**Picture the Problem** The pictorial representations shown to the right summarize the information given in this problem. While the mass of the rope is distributed over its length, the rope and the 6-kg block have a common acceleration. Choose a coordinate system in which the direction of the 100-N force is the positive  $x$  direction. Because the surface is horizontal and frictionless, the only force that influences our solution is the 100-N force.



Part (a)



Part (b)

(a) Apply  $\sum F_x = ma_x$  to the objects shown for part (a):

$$100 \text{ N} = (m_1 + m_2)a$$

Solve for  $a$  to obtain:

$$a = \frac{100 \text{ N}}{m_1 + m_2}$$

Substitute numerical values and evaluate  $a$ :

$$a = \frac{100 \text{ N}}{10 \text{ kg}} = \boxed{10.0 \text{ m/s}^2}$$

(b) Let  $m$  represent the mass of a length  $x$  of the rope. Assuming that the mass of the rope is uniformly distributed along its length:

$$\frac{m}{x} = \frac{m_2}{L_{\text{rope}}} = \frac{4 \text{ kg}}{5 \text{ m}} \text{ and } m = \left( \frac{4 \text{ kg}}{5 \text{ m}} \right) x$$

Apply  $\sum F_x = ma_x$  to the block in part (b) to obtain:

$$T = (m_1 + m)a$$

Substitute numerical values and evaluate  $T$ :

$$\begin{aligned} T &= \left[ 6 \text{ kg} + \left( \frac{4 \text{ kg}}{5 \text{ m}} \right) x \right] (10 \text{ m/s}^2) \\ &= \boxed{60 \text{ N} + (8 \text{ N/m})x} \end{aligned}$$

\*81 ••

**Picture the Problem** Choose a coordinate system in which upward is the positive  $y$  direction and draw the free-body diagram for the frame-plus-painter. Noting that  $\vec{F} = -\vec{T}$ , apply Newton's 2<sup>nd</sup> law of motion.



(a) Letting  $m_{\text{tot}} = m_{\text{frame}} + m_{\text{painter}}$ , apply  $\sum F_y = ma_y$  to the frame-plus-painter and solve  $T$ :

$$2T - m_{\text{tot}}g = m_{\text{tot}}a$$

and

$$T = \frac{m_{\text{tot}}(a + g)}{2}$$

Substitute numerical values and evaluate  $T$ :

$$T = \frac{(75 \text{ kg})(0.8 \text{ m/s}^2 + 9.81 \text{ m/s}^2)}{2}$$

$$= 398 \text{ N}$$

Because  $F = T$ :

$$F = \boxed{398 \text{ N}}$$

(b) Apply  $\sum F_y = ma_y$  with  $a = 0$  to obtain:

$$2T - m_{\text{tot}}g = 0$$

Solve for  $T$ :

$$T = \frac{1}{2} m_{\text{tot}}g$$

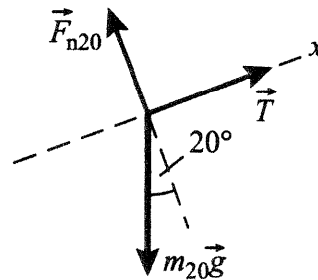
Substitute numerical values and evaluate  $T$ :

$$T = \frac{1}{2}(75 \text{ kg})(9.81 \text{ m/s}^2) = \boxed{368 \text{ N}}$$

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**Picture the Problem** Choose a coordinate system in which up the incline is the positive  $x$  direction and draw free-body diagrams for each block. Noting that  $\vec{a}_{20} = -\vec{a}_{10}$ , apply Newton's 2<sup>nd</sup> law of motion to each block and solve the resulting equations simultaneously.

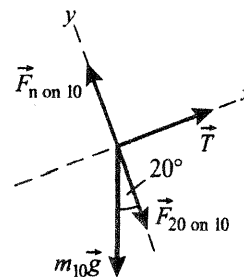
Draw a FBD for the 20-kg block:



Apply  $\sum F_x = ma_x$  to the block to obtain:

$$T - m_{20}g \sin 20^\circ = m_{20}a_{20}$$

Draw a FBD for the 10-kg block. Because all the surfaces, including the surfaces between the blocks, are frictionless, the force the 20-kg block exerts on the 10-kg block must be normal to their surfaces as shown to the right.



Using a constant-acceleration equation, relate the displacement of the 5-kg block to its acceleration and the time during which it is accelerated:

$$\Delta x_5 = \frac{1}{2} a_5 (\Delta t)^2$$

Using a constant-acceleration equation, relate the displacement of the 20-kg block to its acceleration and the time during which it is accelerated:

$$\Delta x_{20} = \frac{1}{2} a_{20} (\Delta t)^2$$

Divide the first of these equations by the second to obtain:

$$\frac{\Delta x_5}{\Delta x_{20}} = \frac{\frac{1}{2} a_5 (\Delta t)^2}{\frac{1}{2} a_{20} (\Delta t)^2} = \frac{a_5}{a_{20}}$$

Use the result of part (a) to obtain:

$$a_5 = 2a_{20}$$

Let  $a_{20} = a$ . Then  $a_5 = 2a$  and the force equations become:

$$2T = m_{20}a$$

and

$$m_5 g - T = m_5(2a)$$

Eliminate  $T$  between the two equations to obtain:

$$a = a_{20} = \frac{m_5 g}{2m_5 + \frac{1}{2}m_{20}}$$

Substitute numerical values and evaluate  $a_{20}$  and  $a_5$ :

$$a_{20} = \frac{(5 \text{ kg})(9.81 \text{ m/s}^2)}{2(5 \text{ kg}) + \frac{1}{2}(20 \text{ kg})} = \boxed{2.45 \text{ m/s}^2}$$

and

$$a_5 = 2(2.45 \text{ m/s}^2) = \boxed{4.91 \text{ m/s}^2}$$

Substitute for either of the accelerations in either of the force equations and solve for  $T$ :

$$T = \boxed{24.5 \text{ N}}$$

## Free-Body Diagrams: The Atwood's Machine

\*84 ••

**Picture the Problem** Assume that  $m_1 > m_2$ . Choose a coordinate system in which the positive  $y$  direction is downward for the block whose mass is  $m_1$  and upward for the block whose mass is  $m_2$  and draw free-body diagrams for each block. Apply Newton's 2<sup>nd</sup> law of motion to both blocks and solve the resulting equations simultaneously.

Draw a FBD for the block whose mass is  $m_2$ :



Apply  $\sum F_y = ma_y$  to this block:

$$T - m_2g = m_2a_2$$

Draw a FBD for the block whose mass is  $m_1$ :



Apply  $\sum F_y = ma_y$  to this block:

$$m_1g - T = m_1a_1$$

Because the blocks are connected by a taut string, let  $a$  represent their common acceleration:

$$a = a_1 = a_2$$

Add the two force equations to eliminate  $T$  and solve for  $a$ :

$$m_1g - m_2g = m_1a + m_2a$$

and

$$a = \frac{m_1 - m_2}{m_1 + m_2} g$$

Substitute for  $a$  in either of the force equations and solve for  $T$ :

$$T = \frac{2m_1m_2g}{m_1 + m_2}$$

85 ..

**Picture the Problem** The acceleration can be found from the given displacement during the first second. The ratio of the two masses can then be found from the acceleration using the first of the two equations derived in Problem 89 relating the acceleration of the Atwood's machine to its masses.

Using a constant-acceleration equation, relate the displacement of the masses to their acceleration and solve for the acceleration:

$$\Delta y = v_0t + \frac{1}{2}a(\Delta t)^2$$

or, because  $v_0 = 0$ ,

$$\Delta y = \frac{1}{2}a(\Delta t)^2$$

Solve for and evaluate  $a$ :

$$a = \frac{2\Delta y}{(\Delta t)^2} = \frac{2(0.3\text{ m})}{(1\text{ s})^2} = 0.600\text{ m/s}^2$$

Substitute numerical values and evaluate  $\Delta t$ :

$$\Delta t = \sqrt{\frac{2(1\text{ m})}{1.71\text{ m/s}^2}} = \boxed{1.08\text{ s}}$$

97 ..

**Picture the Problem** Note that, while the mass of the rope is distributed over its length, the rope and the block have a common acceleration. Because the surface is horizontal and smooth, the only force that influences our solution is  $\vec{F}$ . The figure misrepresents the situation in that each segment of the rope experiences a gravitational force; the combined effect of which is that the rope must sag.

(a) Apply  $\vec{a} = \vec{F}_{\text{net}} / m_{\text{tot}}$  to the rope-block system to obtain:

$$a = \frac{F}{m_1 + m_2}$$

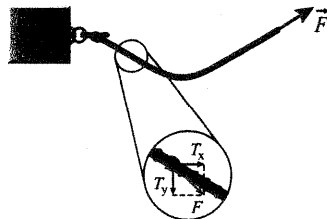
(b) Apply  $\sum \vec{F} = m\vec{a}$  to the rope, substitute the acceleration of the system obtained in (a), and simplify to obtain:

$$\begin{aligned} F_{\text{net}} &= m_2 a = m_2 \left( \frac{F}{m_1 + m_2} \right) \\ &= \frac{m_2}{m_1 + m_2} F \end{aligned}$$

(c) Apply  $\sum \vec{F} = m\vec{a}$  to the block, substitute the acceleration of the system obtained in (a), and simplify to obtain:

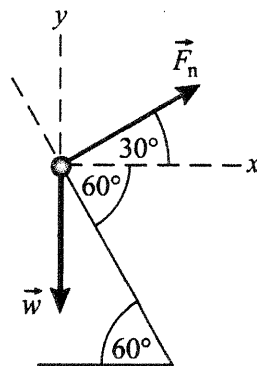
$$\begin{aligned} T &= m_1 a = m_1 \left( \frac{F}{m_1 + m_2} \right) \\ &= \frac{m_1}{m_1 + m_2} F \end{aligned}$$

(d) The rope sags and so  $\vec{F}$  has both vertical and horizontal components; with its horizontal component being less than  $\vec{F}$ . Consequently,  $a$  will be somewhat smaller.



\*98 ..

**Picture the Problem** The free-body diagram shows the forces acting on the block. Choose the coordinate system shown on the diagram. Because the surface of the wedge is frictionless, the force it exerts on the block must be normal to its surface.



Use this acceleration in equation (1) or equation (2) to obtain:

$$T = \frac{40}{9}mg$$

Express the difference between  $T_0$  and  $T$  and solve for  $m$ :

$$T_0 - T = 5mg - \frac{40}{9}mg = 0.3 \text{ N}$$

and

$$m = \boxed{0.0550 \text{ kg} = 55.0 \text{ g}}$$

(b) Proceed as in (a) to obtain:

$$T - 3mg = 3ma$$

and

$$5mg - T = 5ma$$

Eliminate  $T$  and solve for  $a$ :

$$a = \frac{1}{4}g = \frac{1}{4}(9.81 \text{ m/s}^2) = \boxed{2.45 \text{ m/s}^2}$$

Eliminate  $a$  in either of the motion equations and solve for  $T$  to obtain:

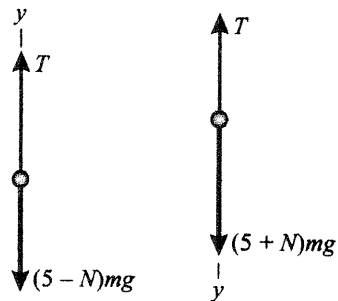
$$T = \frac{15}{4}mg$$

Substitute numerical values and evaluate  $T$ :

$$\begin{aligned} T &= \frac{15}{4}(0.0550 \text{ kg})(9.81 \text{ m/s}^2) \\ &= \boxed{2.03 \text{ N}} \end{aligned}$$

## 100 ••

**Picture the Problem** The free-body diagram represents the Atwood's machine with  $N$  washers moved from the left side to the right side. Application of Newton's 2<sup>nd</sup> law to each collection of washers will result in two equations that can be solved simultaneously to relate  $N$ ,  $a$ , and  $g$ . The acceleration can then be found from the given data.



Apply  $\sum F_y = ma_y$  to the rising washers:

$$T - (5 - N)mg = (5 - N)ma$$

Apply  $\sum F_y = ma_y$  to the descending washers:

$$(5 + N)mg - T = (5 + N)ma$$

Add these equations to eliminate  $T$ :

$$\begin{aligned} (5 + N)mg - (5 - N)mg \\ = (5 - N)ma + (5 + N)ma \end{aligned}$$

Simplify to obtain:

$$2Nmg = 10ma$$

Solve for  $N$ :

$$N = 5a/g$$

Using a constant-acceleration equation, relate the distance the washers fell to their time of fall:

$$\Delta y = v_0 \Delta t + \frac{1}{2} a (\Delta t)^2$$

or, because  $v_0 = 0$ ,

$$\Delta y = \frac{1}{2} a (\Delta t)^2$$

Solve for the acceleration:

$$a = \frac{2\Delta y}{(\Delta t)^2}$$

Substitute numerical values and evaluate  $a$ :

$$a = \frac{2(0.471 \text{ m})}{(0.40 \text{ s})^2} = 5.89 \text{ m/s}^2$$

Substitute in the expression for  $N$ :

$$N = 5 \left( \frac{5.89 \text{ m/s}^2}{9.81 \text{ m/s}^2} \right) = \boxed{3}$$

### 101 ••

**Picture the Problem** Draw the free-body diagram for the block of mass  $m$  and apply Newton's 2<sup>nd</sup> law to obtain the acceleration of the system and then the tension in the rope connecting the two blocks.



(a) Letting  $T$  be the tension in the connecting string, apply

$\sum F_x = ma_x$  to the block of mass  $m$ :

$$T - F_1 = ma$$

Apply  $\sum F_x = ma_x$  to both blocks to determine the acceleration of the system:

$$F_2 - F_1 = (m + 2m)a = (3m)a$$

Substitute and solve for  $a$ :

$$a = (F_2 - F_1)/3m$$

Substitute for  $a$  in the first equation and solve for  $T$ :

$$T = \boxed{\frac{1}{3}(F_2 + 2F_1)}$$

(b) Substitute for  $F_1$  and  $F_2$  in the equation derived in part (a):

$$T = (2Ct + 2Ct)/3 = 4Ct/3$$

Evaluate this expression for  $T = T_0$  and  $t = t_0$  and solve for  $t_0$ :

$$t_0 = \boxed{\frac{3T_0}{4C}}$$