Using the work-kinetic energy theorem, relate the work done on the particle to its *change* in kinetic energy and solve for the particle's kinetic energy at $x = 4$ m:

$$W_{0\to4} = K_4 - K_0$$
$$K_4 = K_0 + W_{0\to4} = 6.00\,\text{J} + 12.0\,\text{J} = 18.0\,\text{J}$$

Substitute numerical values in equation (1) and evaluate v_4:

$$v_4 = \sqrt{\frac{2(18.0\,\text{J})}{3\,\text{kg}}} = \boxed{3.46\,\text{m/s}}$$

***26 ••**

Picture the Problem The work done by this force as it displaces the particle is the area under the curve of F as a function of x. Note that the constant C has units of N/m^3.

Because F varies with position non-linearly, express the work it does as an integral and evaluate the integral between the limits $x = 1.5$ m and $x = 3$ m:

$$W = \left(C\,\text{N/m}^3\right)\int_{1.5\,\text{m}}^{3\,\text{m}} x'^3\,dx'$$
$$= \left(C\,\text{N/m}^3\right)\left[\tfrac{1}{4}x'^4\right]_{1.5\,\text{m}}^{3\,\text{m}}$$
$$= \frac{\left(C\,\text{N/m}^3\right)}{4}\left[(3\,\text{m})^4 - (1.5\,\text{m})^4\right]$$
$$= \boxed{19C\,\text{J}}$$

27 ••

Picture the Problem The work done on the dog by the leash as it stretches is the area under the curve of F as a function of x. We can find this area (the work Lou does holding the leash) by integrating the force function.

Because F varies with position non-linearly, express the work it does as an integral and evaluate the integral between the limits $x = 0$ and $x = x_1$:

$$W = \int_0^{x_1}\left(-kx' - ax'^2\right)dx'$$
$$= \left[-\tfrac{1}{2}kx'^2 - \tfrac{1}{3}ax'^3\right]_0^{x_1}$$
$$= \boxed{-\tfrac{1}{2}kx_1^2 - \tfrac{1}{3}ax_1^3}$$

28 ••

Picture the Problem The work done on an object can be determined by finding the area bounded by its graph of F_x as a function of x and the x axis. We can find the kinetic energy and the speed of the particle at any point by using the work-kinetic energy theorem.

Relate the speed of the block when it has moved a distance Δx down the incline to its kinetic energy at that location:

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2mg\Delta x \sin 60°}{m}} = \sqrt{2g\Delta x \sin 60°}$$

Determine this speed when $\Delta x = 1.5$ m:

$$v = \sqrt{2(9.81\,\text{m/s}^2)(1.5\,\text{m})\sin 60°} = \boxed{5.05\,\text{m/s}}$$

(*d*) As in part (*c*), express the change in the kinetic energy of the block in terms of the distance, Δx, it has moved down the incline and solve for K_f:

$$\Delta K = K_f - K_i = W = (mg \sin 60°)\Delta x$$

and

$$K_f = (mg \sin 60°)\Delta x + K_i$$

Substitute for the kinetic energy terms and solve for v_f to obtain:

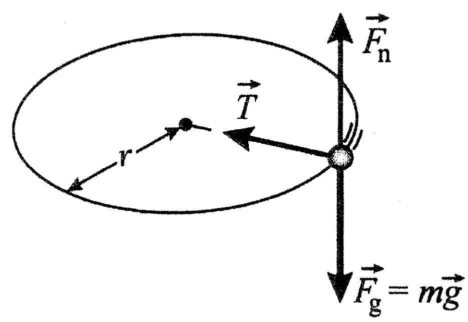
$$v_f = \sqrt{2g \sin 60° \Delta x + v_i^2}$$

Substitute numerical values and evaluate v_f:

$$v_f = \sqrt{2(9.81\,\text{m/s}^2)(1.5\,\text{m})\sin 60° + (2\,\text{m/s})^2} = \boxed{5.43\,\text{m/s}}$$

31 •

Picture the Problem The free-body diagram shows the forces acting on the object as in moves along its circular path on a frictionless horizontal surface. We can use Newton's 2nd law to obtain an expression for the tension in the string and the definition of work to determine the amount of work done by each force during one revolution.



(*a*) Apply $\sum F_r = ma_r$ to the 2-kg object to obtain:

$$T = m\frac{v^2}{r}$$

Substitute numerical values and evaluate T:

$$T = (2\,\text{kg})\frac{(2.5\,\text{m/s})^2}{3\,\text{m}} = \boxed{4.17\,\text{N}}$$

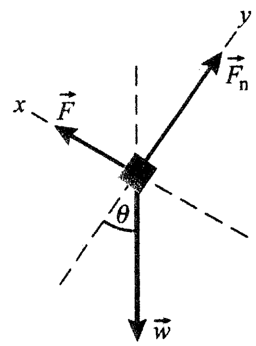
(*b*) From the FBD we can see that the forces acting on the object are:

$$\boxed{\vec{T}, \vec{F}_g, \text{and } \vec{F}_n}$$

Because all of these forces act perpendicularly to the direction of motion of the object, none of them do any work.

***32 •**

Picture the Problem The free-body diagram, with $\vec{F}$ representing the force required to move the block at constant speed, shows the forces acting on the block. We can apply Newton's 2nd law to the block to relate F to its weight w and then use the definition of the mechanical advantage of an inclined plane. In the second part of the problem we'll use the definition of work.



(*a*) Express the mechanical advantage M of the inclined plane:

$$M = \frac{w}{F}$$

Apply $\sum F_x = ma_x$ to the block:

$F - w\sin\theta = 0$ because $a_x = 0$.

Solve for F and substitute to obtain:

$$M = \frac{w}{w\sin\theta} = \frac{1}{\sin\theta}$$

Refer to the figure to obtain:

$$\sin\theta = \frac{H}{L}$$

Substitute to obtain:

$$M = \boxed{\frac{1}{\sin\theta} = \frac{L}{H}}$$

(*b*) Express the work done pushing the block up the ramp:

$$W_{\text{ramp}} = FL = mgL\sin\theta$$

Express the work done lifting the block into the truck:

$$W_{\text{lifting}} = mgH = mgL\sin\theta$$

and

$$\boxed{W_{\text{ramp}} = W_{\text{lifting}}}$$

33 •

Picture the Problem We can find the work done per revolution in lifting the weight and the work done in each revolution of the handle and then use the definition of mechanical advantage.

Express the mechanical advantage of the jack:

$$M = \frac{W}{F}$$

Because all possible vectors $\vec{B}$ lie in a plane, the resultant $\vec{r}$ must lie in a plane as well, as is shown above.

*45 ••

Picture the Problem The rules for the differentiation of vectors are the same as those for the differentiation of scalars and scalar multiplication is commutative.

(*a*) Differentiate $\vec{r}\cdot\vec{r} = r^2 = \text{constant}$:

$$\frac{d}{dt}(\vec{r}\cdot\vec{r}) = \vec{r}\cdot\frac{d\vec{r}}{dt} + \frac{d\vec{r}}{dt}\cdot\vec{r} = 2\vec{v}\cdot\vec{r} = \frac{d}{dt}(\text{constant}) = 0$$

Because $\vec{v}\cdot\vec{r} = 0$:

$$\boxed{\vec{v}\perp\vec{r}}$$

(*b*) Differentiate $\vec{v}\cdot\vec{v} = v^2 = \text{constant}$ with respect to time:

$$\frac{d}{dt}(\vec{v}\cdot\vec{v}) = \vec{v}\cdot\frac{d\vec{v}}{dt} + \frac{d\vec{v}}{dt}\cdot\vec{v} = 2\vec{a}\cdot\vec{v} = \frac{d}{dt}(\text{constant}) = 0$$

Because $\vec{a}\cdot\vec{v} = 0$:

$$\boxed{\vec{a}\perp\vec{v}}$$

The results of (*a*) and (*b*) tell us that $\vec{a}$ is perpendicular to $\vec{r}$ and and parallel (or antiparallel) to $\vec{r}$.

(*c*) Differentiate $\vec{v}\cdot\vec{r} = 0$ with respect to time:

$$\frac{d}{dt}(\vec{v}\cdot\vec{r}) = \vec{v}\cdot\frac{d\vec{r}}{dt} + \vec{r}\cdot\frac{d\vec{v}}{dt} = v^2 + \vec{r}\cdot\vec{a} = \frac{d}{dt}(0) = 0$$

Because $v^2 + \vec{r}\cdot\vec{a} = 0$:

$$\boxed{\vec{r}\cdot\vec{a} = -v^2} \quad (1)$$

Express a_r in terms of θ, where θ is the angle between $\vec{r}$ and $\vec{a}$:

$$a_r = a\cos\theta$$

Express $\vec{r}\cdot\vec{a}$:

$$\vec{r}\cdot\vec{a} = ra\cos\theta = ra_r$$

Substitute in equation (1) to obtain:

$$ra_r = -v^2$$

Solve for a_r:

$$a_r = \boxed{-\frac{v^2}{r}}$$

Power

46 ••

Picture the Problem The power delivered by a force is defined as the rate at which the force does work; i.e., $P = \frac{dW}{dt}$.

Calculate the rate at which force *A* does work:

$$P_A = \frac{5\,\text{J}}{10\,\text{s}} = 0.5\,\text{W}$$

Calculate the rate at which force *B* does work:

$$P_B = \frac{3\,\text{J}}{5\,\text{s}} = 0.6\,\text{W and } \boxed{P_B > P_A}$$

47 •

Picture the Problem The power delivered by a force is defined as the rate at which the force does work; i.e., $P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$.

(*a*) If the box moves upward with a constant velocity, the net force acting it must be zero and the force that is doing work on the box is:

$$F = mg$$

The power input of the force is:

$$P = Fv = mgv$$

Substitute numerical values and evaluate *P*:

$$P = (5\,\text{kg})(9.81\,\text{m/s}^2)(2\,\text{m/s}) = \boxed{98.1\,\text{W}}$$

(*b*) Express the work done by the force in terms of the rate at which energy is delivered:

$$W = Pt = (98.1\text{ W})(4\text{ s}) = \boxed{392\,\text{J}}$$

48 •

Picture the Problem The power delivered by a force is defined as the rate at which the force does work; i.e., $P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$.

(*b*) For $\vec{F} = 6\,\text{N}\,\hat{i} - 5\,\text{N}\,\hat{j}$ and $\vec{v} = -5\text{ m/s }\hat{i} + 4\text{ m/s }\hat{j}$:

$$P = \vec{F}\cdot\vec{v} = \left(6\,\text{N}\,\hat{i} - 5\,\text{N}\,\hat{j}\right)\cdot\left(-5\,\text{m/s}\,\hat{i} + 4\,\text{m/s}\,\hat{j}\right) = \boxed{-50.0\,\text{W}}$$

(*c*) For $\vec{F} = 3\,\text{N}\,\hat{i} + 6\,\text{N}\,\hat{j}$ and $\vec{v} = 2\text{ m/s }\hat{i} + 3\text{ m/s }\hat{j}$:

$$P = \vec{F}\cdot\vec{v} = \left(3\,\text{N}\,\hat{i} + 6\,\text{N}\,\hat{j}\right)\cdot\left(2\,\text{m/s}\,\hat{i} + 3\,\text{m/s}\,\hat{j}\right) = \boxed{24.0\,\text{W}}$$

***51 •**

Picture the Problem Choose a coordinate system in which upward is the positive y direction. We can find P_{in} from the given information that $P_{\text{out}} = 0.27P_{\text{in}}$. We can express P_{out} as the product of the tension in the cable T and the constant speed v of the dumbwaiter. We can apply Newton's 2nd law to the dumbwaiter to express T in terms of its mass m and the gravitational field g.

Express the relationship between the motor's input and output power:

$$P_{\text{out}} = 0.27P_{\text{in}}$$

or

$$P_{\text{in}} = 3.7P_{\text{out}}$$

Express the power required to move the dumbwaiter at a constant speed v:

$$P_{\text{out}} = Tv$$

Apply $\sum F_y = ma_y$ to the dumbwaiter:

$$T - mg = ma_y$$

or, because $a_y = 0$,

$$T = mg$$

Substitute to obtain:

$$P_{\text{in}} = 3.7Tv = 3.7mgv$$

Substitute numerical values and evaluate P_{in}:

$$P_{\text{in}} = 3.7(35\,\text{kg})(9.81\,\text{m/s}^2)(0.35\,\text{m/s}) = \boxed{445\,\text{W}}$$

52 ••

Picture the Problem Choose a coordinate system in which upward is the positive y direction. We can express P_{drag} as the product of the drag force F_{drag} acting on the skydiver and her terminal velocity v_t. We can apply Newton's 2nd law to the skydiver to express F_{drag} in terms of her mass m and the gravitational field g.

(*a*) Express the power due to drag force acting on the skydiver as she falls at her terminal velocity v_t:

$$\vec{P}_{\text{drag}} = \vec{F}_{\text{drag}}\cdot\vec{v}_{\text{t}}$$

or, because $\vec{F}_{\text{drag}}$ and $\vec{v}_{\text{t}}$ are antiparallel,

$$P_{\text{drag}} = -F_{\text{drag}}v_{\text{t}}$$

Apply $\sum F_y = ma_y$ to the skydiver:

$$F_{\text{drag}} - mg = ma_y$$

or, because $a_y = 0$,

$$F_{\text{drag}} = mg$$

Substitute to obtain, for the magnitude of P_{drag}:

$$P_{\text{drag}} = \left| -mgv_{\text{t}} \right| \quad (1)$$

Substitute numerical values and evaluate P:

$$P_{\text{drag}} = \left| -(55\,\text{kg})(9.81\,\text{m/s}^2)(120\frac{\text{mi}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}} \times \frac{1.609\,\text{km}}{\text{mi}}) \right| = \boxed{2.89 \times 10^4\ \text{W}}$$

(*b*) Evaluate equation (1) with $v = 15$ mi/h:

$$P_{\text{drag}} = \left| -(55\,\text{kg})(9.81\,\text{m/s}^2)\left(15\frac{\text{mi}}{\text{h}}\right) \times \frac{1\,\text{h}}{3600\,\text{s}} \times \frac{1.609\,\text{km}}{\text{mi}}) \right| = \boxed{3.62\,\text{kW}}$$

***53 ••**

Picture the Problem Because, in the absence of air resistance, the acceleration of the cannonball is constant, we can use a constant-acceleration equation to relate its velocity to the time it has been in flight. We can apply Newton's 2nd law to the cannonball to find the net force acting on it and then form the dot product of $\vec{F}$ and $\vec{v}$ to express the rate at which the gravitational field does work on the cannonball. Integrating this expression over the time-of-flight T of the ball will yield the desired result.

Express the velocity of the cannonball as a function of time while it is in the air:

$$\vec{v}(t) = 0\hat{i} + (v_0 - gt)\hat{j}$$

Apply $\sum \vec{F} = m\vec{a}$ to the cannonball to express the force acting on it while it is in the air:

$$\vec{F} = -mg\,\hat{j}$$

Evaluate $\vec{F} \cdot \vec{v}$:

$$\vec{F} \cdot \vec{v} = -mg\,\hat{j} \cdot (v_0 - gt)\hat{j} = -mgv_0 + mg^2 t$$

Relate $\vec{F} \cdot \vec{v}$ to the rate at which work is being done on the cannonball:

$$\frac{dW}{dt} = \vec{F} \cdot \vec{v} = -mgv_0 + mg^2 t$$

Separate the variables and integrate over the time T that the cannonball is in the air:

$$W = \int_0^T \left(-mgv_0 + mg^2 t\right) dt = \tfrac{1}{2} mg^2 T^2 - mgv_0 T \quad (1)$$

$\theta = \pi/2$ is to plot its graph and note that, in the interval of interest, U is concave downward with its maximum value at $\theta = \pi/2$.

Force, Potential Energy, and Equilibrium

62 •

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to x, that is, $F_x = -dU/dx$. Consequently, given U as a function of x, we can find F_x by differentiating U with respect to x.

(*a*) Evaluate $F_x = -\frac{dU}{dx}$:

$$F_x = -\frac{d}{dx}\left(Ax^4\right) = \boxed{-4Ax^3}$$

(*b*) Set $F_x = 0$ and solve for x:

$$F_x = 0 \Rightarrow \boxed{x = 0}$$

63 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to x, that is $F_x = -dU/dx$. Consequently, given U as a function of x, we can find F_x by differentiating U with respect to x.

(*a*) Evaluate $F_x = -\frac{dU}{dx}$:

$$F_x = -\frac{d}{dx}\left(\frac{C}{x}\right) = \boxed{\frac{C}{x^2}}$$

(*b*) Because $C > 0$, F_x is positive for $x \neq 0$ and therefore $\vec{F}$ is directed away from the origin.

(*c*) Because U is inversely proportional to x and $C > 0$, $U(x)$ decreases with increasing x.

(*d*) When $C < 0$, F_x is negative for $x \neq 0$ and therefore $\vec{F}$ is directed toward the origin.

Because U is inversely proportional to x and $C < 0$, $U(x)$ becomes less negative as x increases and $U(x)$ increases with increasing x.

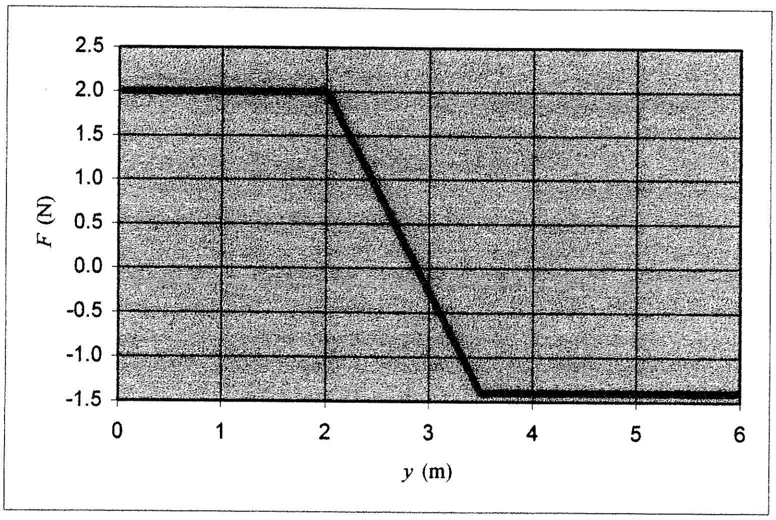
***64 ••**

Picture the Problem F_y is defined to be the negative of the derivative of the potential function with respect to y, i.e. $F_y = -dU/dy$. Consequently, we can obtain F_y by examining the slopes of the graph of U as a function of y.

The table to the right summarizes the information we can obtain from Figure 6-40:

Interval	Slope (N)	F_y (N)
$A \to B$	−2	2
$B \to C$	transitional	$-2 \to 1.4$
$C \to D$	1.4	−1.4

The following graph shows F as a function of y:



65 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to x, i.e. $F_x = -dU/dx$. Consequently, given F as a function of x, we can find U by integrating F_x with respect to x.

Evaluate the integral of F_x with respect to x:

$$U(x) = -\int F(x)dx = -\int \frac{a}{x^2}dx$$

$$= \boxed{\frac{a}{x} + U_0}$$

where U_0 is a constant determined by whatever conditions apply to U.

66 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to x, that is, $F_x = -dU/dx$. Consequently, given U as a function of x, we can find F_x by differentiating U with respect to x. To determine whether the object is in stable or unstable equilibrium at a given point, we'll evaluate d^2U/dx^2 at the point of interest.

Express the gravitational potential energy of the car when it is at a distance h above the ground:

$$U = mgh$$

Express the kinetic energy of the car when it is about to hit the ground:

$$K = \tfrac{1}{2}mv^2$$

Equate these two expressions (because at impact, all the potential energy has been converted to kinetic energy) and solve for h:

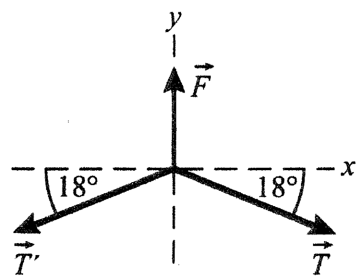
$$h = \frac{v^2}{2g}$$

Substitute numerical values and evaluate h:

$$h = \frac{[(100\,\text{km/h})(1\,\text{h}/3600\,\text{s})]^2}{2(9.81\,\text{m/s}^2)} = \boxed{39.3\,\text{m}}$$

77 •••

Picture the Problem The free-body diagram shows the forces acting on one of the strings at the bridge. The force whose magnitude is F is one-fourth of the force (103 N) the bridge exerts on the strings. We can apply the condition for equilibrium in the y direction to find the tension in each string. Repeating this procedure at the site of the plucking will yield the restoring force acting on the string. We can find the work done on the string as it returns to equilibrium from the product of the average force acting on it and its displacement.



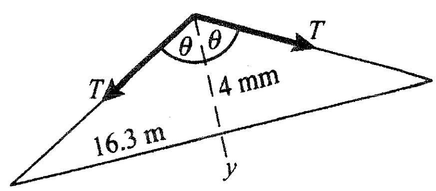
(a) Noting that, due to symmetry, $T' = T$, apply $\sum F_y = 0$ to the string at the point of contact with the bridge:

$$F - 2T\sin 18° = 0$$

Solve for and evaluate T:

$$T = \frac{F}{2\sin 18°} = \frac{\frac{1}{4}(103\,\text{N})}{2\sin 18°} = \boxed{41.7\,\text{N}}$$

(b) A free-body diagram showing the forces restoring the string to its equilibrium position just after it has been plucked is shown to the right:



Express the net force acting on the string immediately after it is released:

$$F_{net} = 2T\cos\theta$$

Use trigonometry to find θ:

$$\theta = \tan^{-1}\left(\frac{16.3\,\text{cm}}{4\,\text{mm}} \times \frac{10\,\text{mm}}{\text{cm}}\right) = 88.6°$$

Substitute and evaluate F_{net}:

$$F_{net} = 2(34.4\,\text{N})\cos 88.6° = \boxed{1.68\,\text{N}}$$

(*c*) Express the work done on the string in displacing it a distance dx':

$$dW = Fdx'$$

If we pull the string out a distance x', the magnitude of the force pulling it down is approximately:

$$F = (2T)\frac{x'}{L/2} = \frac{4T}{L}x'$$

Substitute to obtain:

$$dW = \frac{4T}{L}x'\,dx'$$

Integrate to obtain:

$$W = \frac{4T}{L}\int_0^x x'dx' = \frac{2T}{L}x^2$$

where x is the final displacement of the string.

Substitute numerical values and evaluate W:

$$W = \frac{2(41.7\,\text{N})}{32.6\times10^{-2}\,\text{m}}\left(4\times10^{-3}\,\text{m}\right)^2$$

$$= \boxed{4.09\,\text{mJ}}$$

78 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to x, that is $F_x = -dU/dx$. Consequently, given F as a function of x, we can find U by integrating F_x with respect to x.

Evaluate the integral of F_x with respect to x:

$$U(x) = -\int F(x)dx = -\int\left(-ax^2\right)dx$$

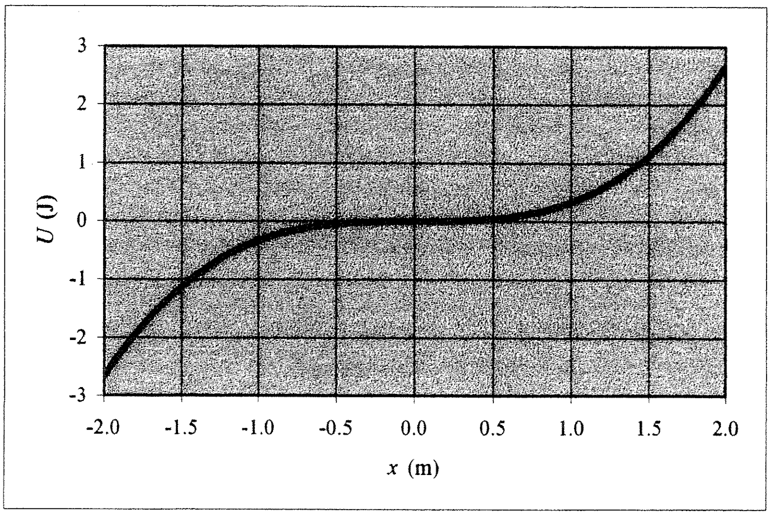
$$= \tfrac{1}{3}ax^3 + U_0$$

Apply the condition that $U(0) = 0$ to determine U_0:

$$U(0) = 0 + U_0 = 0 \Rightarrow U_0 = 0$$

$$\therefore U(x) = \boxed{\tfrac{1}{3}ax^3}$$

The graph of $U(x)$ follows:



***79 ••**

Picture the Problem We can use the definition of work to obtain an expression for the position-dependent force acting on the cart. The work done on the cart can be calculated from its change in kinetic energy.

(*a*) Express the force acting on the cart in terms of the work done on it:

$$F(x)=\frac{dW}{dx}$$

Because U is constant:

$$F(x)=\frac{d}{dx}\left(\tfrac{1}{2}mv^2\right)=\frac{d}{dx}\left[\tfrac{1}{2}m(Cx)^2\right]$$
$$=\boxed{mC^2x}$$

(*b*) The work done by this force changes the kinetic energy of the cart:

$$W=\Delta K=\tfrac{1}{2}mv_1^2-\tfrac{1}{2}mv_0^2$$
$$=\tfrac{1}{2}mv_1^2-0=\tfrac{1}{2}m(Cx_1)^2$$
$$=\boxed{\tfrac{1}{2}mC^2x_1^2}$$

80 ••

Picture the Problem The work done by $\vec{F}$ depends on whether it causes a displacement in the direction it acts.

(*a*) Because $\vec{F}$ is along x-axis and the displacement is along y-axis:

$$W=\int\vec{F}\cdot d\vec{s}=\boxed{0}$$

(*b*) Calculate the work done by $\vec{F}$ during the displacement from $x = 2$ m to 5 m:

$$W = \int \vec{F} \cdot d\vec{s} = \int_{2\,\text{m}}^{5\,\text{m}} (2\,\text{N/m}^2)x^2 dx$$

$$= (2\,\text{N/m}^2)\left[\frac{x^3}{3}\right]_{2\,\text{m}}^{5\,\text{m}} = \boxed{78.0\,\text{J}}$$

81 ••

Picture the Problem The velocity and acceleration of the particle can be found by differentiation. The power delivered to the particle can be expressed as the product of its velocity and the net force acting on it, and the work done by the force and can be found from the change in kinetic energy this work causes.

In the following, if t is in seconds and m is in kilograms, then v is in m/s, a is in m/s^2, P is in W, and W is in J.

(*a*) The velocity of the particle is given by:

$$v = \frac{dx}{dt} = \frac{d}{dt}(2t^3 - 4t^2)$$

$$= \boxed{(6t^2 - 8t)}$$

The acceleration of the particle is given by:

$$a = \frac{dv}{dt} = \frac{d}{dt}(6t^2 - 8t)$$

$$= \boxed{(12t - 8)}$$

(*b*) Express and evaluate the rate at which energy is delivered to this particle as it accelerates:

$$P = Fv = mav$$

$$= m(12t - 8)(6t^2 - 8t)$$

$$= \boxed{8mt(9t^2 - 18t + 8)}$$

(*c*) Because the particle is moving in such a way that its potential energy is not changing, the work done by the force acting on the particle equals the change in its kinetic energy:

$$W = \Delta K = K_1 - K_0$$

$$= \tfrac{1}{2}m\left[(v(t_1))^2 - (v(0))^2\right]$$

$$= \tfrac{1}{2}m\left[(6t_1^2 - 8t_1)\right]^2 - 0$$

$$= \boxed{2mt_1^2(3t_1 - 4)^2}$$

Remarks: We could also find *W* by integrating *P*(*t*) with respect to time.

82 ••

Picture the Problem We can calculate the work done by the given force from its definition. The power can be determined from $P = \vec{F}\cdot\vec{v}$ and v from the change in kinetic energy of the particle produced by the work done on it.

(*a*) Calculate the work done from its definition:

$$W = \int \vec{F}\cdot d\vec{s} = \int_0^{3\,\text{m}} \left(6 + 4x - 3x^2\right)dx$$

$$= \left[6x + \frac{4x^2}{2} - \frac{3x^3}{3}\right]_0^{3\,\text{m}} = \boxed{9.00\,\text{J}}$$

(*b*) Express the power delivered to the particle in terms of $F_{x=3\,\text{m}}$ and its velocity:

$$P = \vec{F}\cdot\vec{v} = F_{x=3\,\text{m}}v$$

Relate the work done on the particle to its kinetic energy and solve for its velocity:

$$W = \Delta K = K_{\text{final}} = \tfrac{1}{2}mv^2 \text{ since } v_0 = 0$$

Solve for v to obtain:

$$v = \sqrt{\frac{2K}{m}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{2(9\,\text{J})}{3\,\text{kg}}} = 2.45\,\text{m/s}$$

Evaluate $F_{x=3\,\text{m}}$:

$$F_{x=3\,\text{m}} = 6 + 4(3) - 3(3)^2 = -9\,\text{N}$$

Substitute for $F_{x=3\,\text{m}}$ and v and evaluate P:

$$P = (-9\,\text{N})(2.45\,\text{m/s}) = \boxed{-22.1\,\text{W}}$$

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Picture the Problem We'll assume that the firing height is negligible and that the bullet lands at the same elevation from which it was fired. We can use the equation $R = \left(v_0^2/g\right)\sin 2\theta$ to find the range of the bullet and constant-acceleration equations to find its maximum height. The bullet's initial speed can be determined from its initial kinetic energy.

Express the range of the bullet as a function of its firing speed and angle of firing:

$$R = \frac{v_0^2}{g}\sin 2\theta \qquad (1)$$

Rewrite the range equation using the trigonometric identity $\sin 2\theta = 2\sin\theta\cos\theta$:

$$R = \frac{v_0^2 \sin 2\theta}{g} = \frac{2v_0^2 \sin\theta\cos\theta}{g} \qquad (2)$$

Express the position coordinates of the projectile along its flight path in terms of the parameter t:

$$x = (v_0 \cos\theta)t$$

and

$$y = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2$$

Eliminate the parameter t and make use of the fact that the maximum height occurs when the projectile is at half the range to obtain:

$$h = \frac{(v_0 \sin\theta)^2}{2g}$$

Using equation (2), equate R and h to obtain:

$$\frac{(v_0 \sin\theta)^2}{2g} = \frac{2v_0^2 \sin\theta\cos\theta}{g}$$

or

$$\tan\theta = 4$$

Solving for θ yields:

$$\theta = \tan^{-1}(4) = 76.0°$$

Relate the bullet's kinetic energy to its mass and speed and solve for the square of its speed:

$$K = \tfrac{1}{2}mv_0^2 \Rightarrow v_0^2 = \frac{2K}{m}$$

Substitute for v_0^2 in equation (1) to obtain:

$$R = \frac{2K}{mg}\sin 2\theta$$

Substitute numerical values and evaluate R:

$$R = \frac{2(1200\,\text{J})}{(0.02\,\text{kg})(9.81\,\text{m/s}^2)}\sin 2(76°) = \boxed{5.74\,\text{km}}$$

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Picture the Problem The work done on the particle is the area under the force-versus-displacement curve. Note that for negative displacements, F is positive, so W is negative for $x < 0$.