*6

Determine the Concept If we define the system to include the falling body and the earth, then no work is done by an external agent and $\Delta K + \Delta U_{\rm g} + \Delta E_{\rm therm} = 0$. Solving for the change in the gravitational potential energy we find $\Delta U_{\rm g} = -(\Delta K + {\rm friction~energy})$.

(b) is correct.

7 ••

Picture the Problem Because the constant friction force is responsible for a constant acceleration, we can apply the constant-acceleration equations to the analysis of these statements. We can also apply the work-energy theorem with friction to obtain expressions for the kinetic energy of the car and the rate at which it is changing. Choose the system to include the earth and car and assume that the car is moving on a horizontal surface so that $\Delta U = 0$.

(a) A constant frictional force causes a constant acceleration. The stopping distance of the car is related to its speed before the brakes were applied through a constant-acceleration equation.

 $v^2 = v_0^2 + 2a\Delta s$ where v = 0. $\therefore \Delta s = \frac{-v_0^2}{2a} \text{ where } a < 0.$

Thus, $\Delta s \propto v_0^2$ and statement (a) is false.

(b) Apply the work-energy theorem with friction to obtain:

 $\Delta K = -W_{\rm f} = -\mu_{\rm k} mg \Delta s$

Express the rate at which K is dissipated:

 $\frac{\Delta K}{\Delta t} = -\mu_{\rm k} mg \frac{\Delta s}{\Delta t}$

Thus, $\frac{\Delta K}{\Delta t} \propto v$ and therefore not constant.

Statement (b) is false.

(c) In part (b) we saw that:

 $K \propto \Delta s$

Because $\Delta s \propto \Delta t$:

 $K \propto \Delta t$ and statement (c) is false.

Because none of the above are correct:

(d) is correct.

8

Picture the Problem We'll let the zero of potential energy be at the bottom of each ramp and the mass of the block be m. We can use conservation of energy to predict the speed of the block at the foot of each ramp. We'll consider the distance the block travels on each ramp, as well as its speed at the foot of the ramp, in deciding its descent times.

Use conservation of energy to find the speed of the blocks at the bottom of each ramp:

Because
$$K_{\text{top}} = U_{\text{bot}} = 0$$
:

Substitute to obtain:

Solve for
$$v_{bot}$$
:

$$\Delta K + \Delta U = 0$$
 or

$$K_{\text{bot}} - K_{\text{top}} + U_{\text{bot}} - U_{\text{top}} = 0$$

$$K_{\text{bot}} - U_{\text{top}} = 0$$

$$\frac{1}{2}mv_{\text{bot}}^2 - mgH = 0$$

$$v_{\text{bot}} = \sqrt{2gH}$$
 independently of the shape of the ramp.

Because the block sliding down the circular arc travels a greater distance (an arc length is greater than the length of the chord it defines) but arrives at the bottom of the ramp with the same speed that it had at the bottom of the inclined plane, it will require more time to arrive at the bottom of the arc. (b) is correct.

9 ••

Determine the Concept No. From the work-kinetic energy theorem, no total work is being done on the rock, as its kinetic energy is constant. However, the rod must exert a tangential force on the rock to keep the speed constant. The effect of this force is to cancel the component of the force of gravity that is tangential to the trajectory of the rock.

Estimation and Approximation

*10 ••

Picture the Problem We'll use the data for the "typical male" described above and assume that he spends 8 hours per day sleeping, 2 hours walking, 8 hours sitting, 1 hour in aerobic exercise, and 5 hours doing moderate physical activity. We can approximate his energy utilization using $E_{\rm activity} = A P_{\rm activity} \Delta t_{\rm activity}$, where A is the surface area of his body, $P_{\rm activity}$ is the rate of energy consumption in a given activity, and $\Delta t_{\rm activity}$ is the time spent in the given activity. His total energy consumption will be the sum of the five terms corresponding to his daily activities.

(a) Express the energy consumption of the hypothetical male:

$$\begin{split} E = E_{\text{sleeping}} + E_{\text{walking}} + E_{\text{sitting}} \\ + E_{\text{mod. act.}} + E_{\text{aerobic act.}} \end{split}$$

Evaluate E_{sleeping} :

$$E_{\text{sleeping}} = AP_{\text{sleeping}} \Delta t_{\text{sleeping}}$$
$$= (2 \text{ m}^2)(40 \text{ W/m}^2)(8 \text{ h})(3600 \text{ s/h})$$
$$= 2.30 \times 10^6 \text{ J}$$

Evaluate E_{walking} :

$$E_{\text{walking}} = AP_{\text{walking}} \Delta t_{\text{walking}}$$
$$= (2 \text{ m}^2)(160 \text{ W/m}^2)(2 \text{ h})(3600 \text{ s/h})$$
$$= 2.30 \times 10^6 \text{ J}$$

Substitute in equation (1) and evaluate dV/dt:

$$\frac{dV}{dt} = \frac{4.57 \times 10^8 \text{ W}}{0.2(1 \text{kg/L})(9.81 \text{m/s}^2)(211 \text{m})}$$
$$= \boxed{1.10 \times 10^6 \text{ L/s}}$$

The Conservation of Mechanical Energy

16

Picture the Problem The work done in compressing the spring is stored in the spring as potential energy. When the block is released, the energy stored in the spring is transformed into the kinetic energy of the block. Equating these energies will give us a relationship between the compressions of the spring and the speeds of the blocks.

Let the numeral 1 refer to the first case and the numeral 2 to the second case. Relate the compression of the spring in the second case to its potential energy, which equals its initial kinetic energy when released:

$$\frac{1}{2}kx_2^2 = \frac{1}{2}m_2v_2^2
= \frac{1}{2}(4m_1)(3v_1)^2
= 18m_1v_1^2$$

Relate the compression of the spring in the first case to its potential energy, which equals its initial kinetic energy when released:

$$\frac{1}{2}kx_1^2 = \frac{1}{2}m_1v_1^2$$
or
$$m_1v_1^2 = kx_1^2$$

Substitute to obtain:

$$\frac{1}{2}kx_2^2 = 18kx_1^2$$

Solve for x_2 :

$$x_2 = \boxed{6x_1}$$

17

Picture the Problem Choose the zero of gravitational potential energy to be at the foot of the hill. Then the kinetic energy of the woman on her bicycle at the foot of the hill is equal to her gravitational potential energy when she has reached her highest point on the hill.

Equate the kinetic energy of the rider at the foot of the incline and her gravitational potential energy when she has reached her highest point on the hill and solve for h:

$$\frac{1}{2}mv^2 = mgh \Rightarrow h = \frac{v^2}{2g}$$

Relate her displacement along the incline d to h and the angle of the incline:

$$d = h/\sin\theta$$

Substitute for h to obtain:

$$d\sin\theta = \frac{v^2}{2g}$$

Solve for d:

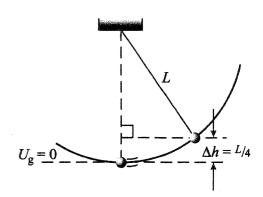
$$d = \frac{v^2}{2g\sin\theta}$$

Substitute numerical values and evaluate *d*:

$$d = \frac{(10 \,\text{m/s})^2}{2(9.81 \,\text{m/s}^2) \sin 3^\circ} = 97.4 \,\text{m}$$
and (c) is correct.

*18 •

Picture the Problem The diagram shows the pendulum bob in its initial position. Let the zero of gravitational potential energy be at the low point of the pendulum's swing, the equilibrium position. We can find the speed of the bob at it passes through the equilibrium position by equating its initial potential energy to its kinetic energy as it passes through its lowest point.



Equate the initial gravitational potential energy and the kinetic energy of the bob as it passes through its lowest point and solve for v:

$$mg\Delta h = \frac{1}{2}mv^2$$

and
 $v = \sqrt{2g\Delta h}$

Express Δh in terms of the length L of the pendulum:

$$\Delta h = \frac{L}{4}$$

Substitute and simplify:

$$v = \sqrt{\frac{gL}{2}}$$

19

Picture the Problem Choose the zero of gravitational potential energy to be at the foot of the ramp. Let the system consist of the block, the earth, and the ramp. Then there are no external forces acting on the system to change its energy and the kinetic energy of the block at the foot of the ramp is equal to its gravitational potential energy when it has reached its highest point.

Relate the gravitational potential energy of the block when it has reached h, its highest point on the ramp, to its kinetic energy at the foot of the ramp:

$$mgh = \frac{1}{2}mv^2$$

Solve for *h*:

$$h = \frac{v^2}{2g}$$

Relate the displacement d of the block along the ramp to h and the angle the ramp makes with the horizontal:

$$d = h/\sin\theta$$

Substitute for *h*:

$$d\sin\theta = \frac{v^2}{2g}$$

Solve for *d*:

$$d = \frac{v^2}{2g\sin\theta}$$

Substitute numerical values and evaluate d:

$$d = \frac{(7 \,\text{m/s})^2}{2(9.81 \,\text{m/s}^2) \sin 40^\circ} = \boxed{3.89 \,\text{m}}$$

20

Picture the Problem Let the system consist of the earth, the block, and the spring. With this choice there are no external forces doing work to change the energy of the system. Let $U_{\rm g}=0$ at the elevation of the spring. Then the initial gravitational potential energy of the 3-kg object is transformed into kinetic energy as it slides down the ramp and then, as it compresses the spring, into potential energy stored in the spring.

(a) Apply conservation of energy to relate the distance the spring is compressed to the initial potential energy of the block:

$$W_{\text{ext}} = \Delta K + \Delta U = 0$$

and, because $\Delta K = 0$,
$$-mgh + \frac{1}{2}kx^2 = 0$$

Apply conservation of energy:

Substitute for ΔK and solve for $v_{\mathcal{E}}$ noting that m represents the sum of the masses of the objects as they are both moving in the final state:

Express and evaluate $\Delta U_{\rm g}$:

Substitute for ΔU_g in equation (1) and evaluate v_f :

25 .

Picture the Problem The free-body diagram shows the forces acting on the block when it is about to move. $F_{\rm sp}$ is the force exerted by the spring and, because the block is on the verge of sliding, $f_{\rm s} = f_{\rm s,max}$. We can use Newton's $2^{\rm nd}$ law, under equilibrium conditions, to express the elongation of the spring as a function of m, k and θ and then substitute in the expression for the potential energy stored in a stretched or compressed spring.

Express the potential energy of the spring when the block is about to move:

Apply $\sum \vec{F} = m\vec{a}$, under equilibrium conditions, to the block:

$$W_{\rm ext} = \Delta K + \Delta U_{\rm g} = 0$$

or, because $W_{\rm ext} = 0$,
 $\Delta K = -\Delta U_{\rm g}$

$$\frac{1}{2}mv_{\rm f}^2 - \frac{1}{2}mv_{\rm i}^2 = -\Delta U_{\rm g}$$
or, because $v_{\rm i} = 0$,
$$v_{\rm f} = \sqrt{\frac{-2\Delta U_{\rm g}}{m}}$$
 (1)

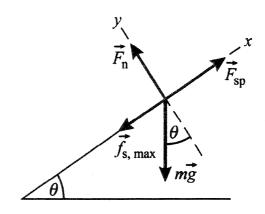
$$\Delta U_{g} = U_{g,f} - U_{g,i}$$

$$= 0 - (3 \text{ kg} - 2 \text{ kg})(0.5 \text{ m})$$

$$\times (9.81 \text{ m/s}^{2})$$

$$= -4.91 \text{ J}$$

$$v_{\rm f} = \sqrt{\frac{-2(-4.91\text{J})}{5\text{kg}}} = \boxed{1.40\text{ m/s}}$$



$$U = \frac{1}{2} k x^2$$

$$\sum F_x = F_{sp} - f_{s,max} - mg \sin \theta = 0$$
and
$$\sum F_y = F_p - mg \cos \theta = 0$$

*27 ••

Picture the Problem The diagram represents the ball traveling in a circular path with constant energy. $U_{\rm g}$ has been chosen to be zero at the lowest point on the circle and the superimposed free-body diagrams show the forces acting on the ball at the top and bottom of the circular path. We'll apply Newton's $2^{\rm nd}$ law to the ball at the top and bottom of its path to obtain a relationship between $T_{\rm T}$ and $T_{\rm B}$ and the conservation of mechanical energy to relate the speeds of the ball at these two locations.

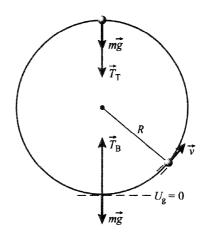
Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to the ball at the bottom of the circle and solve for T_{B} :

Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to the ball at the top of the circle and solve for T_{T} :

Subtract equation (2) from equation (1) to obtain:

Using conservation of energy, relate the mechanical energy of the ball at the bottom of its path to its mechanical energy at the top of the circle and solve for $m\frac{v_{\rm B}^2}{R}-m\frac{v_{\rm T}^2}{R}$:

Substitute in equation (3) to obtain:



$$T_{\rm B} - mg = m\frac{v_{\rm B}^2}{R}$$
 and
$$T_{\rm B} = mg + m\frac{v_{\rm B}^2}{R}$$
 (1)

$$T_{\rm T} + mg = m \frac{v_{\rm T}^2}{R}$$
 and
$$v_{\rm T}^2$$

$$T_{\rm T} = -mg + m\frac{v_{\rm T}^2}{R} \tag{2}$$

$$T_{\rm B} - T_{\rm T} = mg + m\frac{v_{\rm B}^2}{R}$$

$$-\left(-mg + m\frac{v_{\rm T}^2}{R}\right)$$

$$= m\frac{v_{\rm B}^2}{R} - m\frac{v_{\rm T}^2}{R} + 2mg \quad (3)$$

$$\frac{1}{2}mv_{\rm B}^2 = \frac{1}{2}mv_{\rm T}^2 + mg(2R)$$

$$m\frac{v_{\rm B}^2}{R} - m\frac{v_{\rm T}^2}{R} = 4mg$$

$$T_{\rm B} - T_{\rm T} = \boxed{6mg}$$