(*b*) Letting primed quantities describe the indicated location, use the law of the conservation of mechanical energy to relate the speed of the bob at this point to θ:

$$K_f' - K_i + U_f' - U_i = 0$$
where $K_i = 0$.
$$\therefore K_f' + U_f' - U_i = 0$$

Express U_f':

$$U_f' = mgh' = mgL(1-\cos\theta)$$

Substitute for K_f', U_f' and U_i:

$$\tfrac{1}{2}m(v_f')^2 + mgL(1-\cos\theta) - mgL(1-\cos\theta_0) = 0$$

Solving for θ yields :

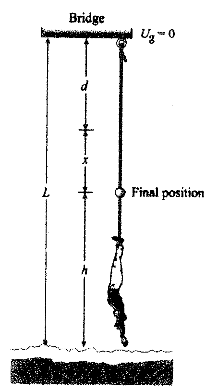
$$\theta = \cos^{-1}\left[\frac{(v_f')^2}{2gL} + \cos\theta_0\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \cos^{-1}\left[\frac{(1.4\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)(0.8\,\text{m})} + \cos 60°\right] = \boxed{51.3°}$$

***35 ••**

Picture the Problem Choose $U_g = 0$ at the bridge, and let the system be the earth, the jumper and the bungee cord. Then $W_{ext} = 0$. Use the conservation of mechanical energy to relate to relate her initial and final gravitational potential energies to the energy stored in the stretched bungee, U_s cord. In part (*b*), we'll use a similar strategy but include a kinetic energy term because we are interested in finding her maximum speed.



(*a*) Express her final height *h* above the water in terms of *L*, *d* and the distance *x* the bungee cord has stretched:

$$h = L - d - x \qquad (1)$$

Use the conservation of mechanical energy to relate her gravitational potential energy as she just touches the water to the energy stored in the stretched bungee cord:

$W_{ext} = \Delta K + \Delta U = 0$

Because $\Delta K = 0$ and $\Delta U = \Delta U_g + \Delta U_s$,

$-mgL + \frac{1}{2}kx^2 = 0,$

where x is the maximum distance the bungee cord has stretched.

Solve for k:

$$k = \frac{2mgL}{x^2}$$

Find the maximum distance the bungee cord stretches:

$x = 310\text{ m} - 50\text{ m} = 260\text{ m}.$

Substitute numerical values and evaluate k:

$$k = \frac{2(60\,\text{kg})(9.81\,\text{m/s}^2)(310\,\text{m})}{(260\,\text{m})^2} = 5.40\,\text{N/m}$$

Express the relationship between the forces acting on her when she has finally come to rest and solve for x:

$F_{net} = kx - mg = 0$ and $x = \dfrac{mg}{k}$

Substitute numerical values and evaluate x:

$$x = \frac{(60\,\text{kg})(9.81\,\text{m/s}^2)}{5.40\,\text{N/m}} = 109\,\text{m}$$

Substitute in equation (1) and evaluate h:

$$h = 310\,\text{m} - 50\,\text{m} - 109\,\text{m} = \boxed{151\,\text{m}}$$

(*b*) Using conservation of energy, express her total energy E:

$E = K + U_g + U_s = E_i = 0$

Because v is a maximum when K is a maximum, solve for K::

$$K = -U_g - U_s = mg(d + x) - \tfrac{1}{2}kx^2 \qquad (1)$$

Use the condition for an extreme value to obtain:

$\dfrac{dK}{dx} = mg - kx = 0$ for extreme values

Solve for and evaluate x:

$$x = \frac{mg}{k} = \frac{(60\,\text{kg})(9.81\,\text{m/s}^2)}{5.40\,\text{N/m}} = 109\,\text{m}$$

From equation (1) we have:

$$\tfrac{1}{2}mv^2 = mg(d + x) - \tfrac{1}{2}kx^2$$

Solve for v to obtain:

$$v=\sqrt{2g(d+x)-\frac{kx^2}{m}}$$

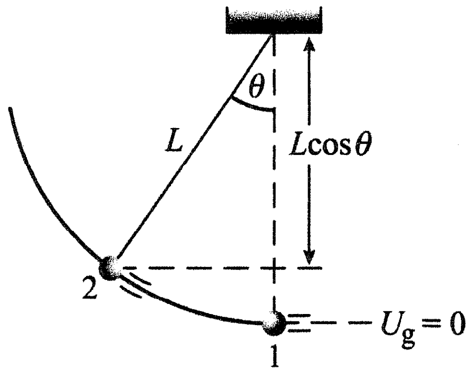
Substitute numerical values and evaluate v for $x = 109$ m:

$$v=\sqrt{2(9.81\,\text{m/s}^2)(50\,\text{m}+109\,\text{m})-\frac{(5.4\,\text{N/m})(109\,\text{m})^2}{60\,\text{kg}}}=\boxed{45.3\,\text{m/s}}$$

Because $\frac{d^2K}{dx^2}=-k<0$, $x = 109$ m corresponds to K_{max} and so v is a maximum.

36 ••

Picture the Problem Let the system be the earth and pendulum bob. Then $W_{ext} = 0$. Choose $U_g = 0$ at the low point of the bob's swing and apply the law of the conservation of mechanical energy to its motion. When the bob reaches the 30° position its energy will be partially kinetic and partially potential. When it reaches its maximum height, its energy will be entirely potential. Applying Newton's 2nd law will allow us to express the tension in the string as a function of the bob's speed and its angular position.



(*a*) Apply conservation of energy to relate the energies of the bob at points 1 and 2:

$$W_{ext}=\Delta K+\Delta U=0$$

or

$$K_2-K_1+U_2-U_1=0$$

Because $U_1 = 0$:

$$\tfrac{1}{2}mv_2^2-\tfrac{1}{2}mv_1^2+U_2=0$$

The potential energy of the system when the bob is at point 2 is given by:

$$U_2=mgL(1-\cos\theta)$$

Substitute for U_2 to obtain:

$$\tfrac{1}{2}mv_2^2-\tfrac{1}{2}mv_1^2+mgL(1-\cos\theta)=0$$

Solve for v_2:

$$v_2=\sqrt{v_1^2-2gL(1-\cos\theta)}$$

Substitute numerical values and evaluate v_2:

$$v_2=\sqrt{(4.5\,\text{m/s})^2-2(9.81\,\text{m/s}^2)(3\,\text{m})(1-\cos 30^\circ)}=\boxed{3.52\,\text{m/s}}$$

Finally, solve for v_2:

$$v_2 = \boxed{L\sqrt{2\frac{g}{L}(1-\cos\theta)+\frac{k}{m}\left(\sqrt{\tfrac{13}{4}-3\cos\theta}-\tfrac{1}{2}\right)^2}}$$

The Conservation of Energy

42 •

Picture the Problem The energy of the eruption is initially in the form of the kinetic energy of the material it thrusts into the air. This energy is then transformed into gravitational potential energy as the material rises.

(*a*) Express the energy of the eruption in terms of the height Δh to which the debris rises:

$$E = mg\Delta h$$

Relate the density of the material to its mass and volume:

$$\rho = \frac{m}{V}$$

Substitute for *m* to obtain:

$$E = \rho V g\Delta h$$

Substitute numerical values and evaluate *E*:

$$E = \left(1600\,\text{kg/m}^3\right)\left(4\,\text{km}^3\right)\left(9.81\,\text{m/s}^2\right)\left(500\,\text{m}\right) = \boxed{3.14\times10^{16}\,\text{J}}$$

(*b*) Convert 3.13×10^{16} J to megatons of TNT:

$$3.14\times10^{16}\,\text{J} = 3.14\times10^{16}\,\text{J}\times\frac{1\,\text{Mton TNT}}{4.2\times10^{15}\,\text{J}} = \boxed{7.48\,\text{Mton TNT}}$$

43 ••

Picture the Problem The work done by the student equals the change in his/her gravitational potential energy and is done as a result of the transformation of metabolic energy in the climber's muscles.

(*a*) The increase in gravitational potential energy is:

$$\begin{aligned}\Delta U &= mg\Delta h\\ &= \left(80\,\text{kg}\right)\left(9.81\,\text{m/s}^2\right)\left(120\,\text{m}\right)\\ &= \boxed{94.2\,\text{kJ}}\end{aligned}$$

(*b*) The energy required to do this work comes from chemical energy stored in the body.

(*c*) Relate the chemical energy expended by the student to the change in his/her potential energy and solve for E:

$0.2E = \Delta U$

and

$E = 5\Delta U = 5(94.2\,\text{kJ}) = \boxed{471\,\text{kJ}}$

Kinetic Friction

44 •

Picture the Problem Let the car and the earth be the system. As the car skids to a stop on a horizontal road, its kinetic energy is transformed into internal (i.e., thermal) energy. Knowing that energy is transformed into heat by friction, we can use the definition of the coefficient of kinetic friction to calculate its value.

(*a*) The energy dissipated by friction is given by:

$f\Delta s = \Delta E_{\text{therm}}$

Apply the work-energy theorem for problems with kinetic friction:

$W_{\text{ext}} = \Delta E_{\text{mech}} + \Delta E_{\text{therm}} = \Delta E_{\text{mech}} + f\Delta s$

or, because $\Delta E_{\text{mech}} = \Delta K = -K_{\text{i}}$ and $W_{\text{ext}} = 0$,

$0 = -\frac{1}{2}mv_{\text{i}}^2 + f\Delta s$

Solve for $f\Delta s$ to obtain:

$f\Delta s = \frac{1}{2}mv_{\text{i}}^2$

Substitute numerical values and evaluate $f\Delta s$:

$f\Delta s = \frac{1}{2}(2000\,\text{kg})(25\,\text{m/s})^2 = \boxed{625\,\text{kJ}}$

(*b*) Relate the kinetic friction force to the coefficient of kinetic friction and the weight of the car and solve for the coefficient of kinetic friction:

$f_{\text{k}} = \mu_{\text{k}}mg \Rightarrow \mu_{\text{k}} = \frac{f_{\text{k}}}{mg}$

Express the relationship between the energy dissipated by friction and the kinetic friction force and solve f_{k}:

$\Delta E_{\text{therm}} = f_{\text{k}}\Delta s \Rightarrow f_{\text{k}} = \frac{\Delta E_{\text{therm}}}{\Delta s}$

Substitute to obtain:

$\mu_{\text{k}} = \frac{\Delta E_{\text{therm}}}{mg\Delta s}$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{(611\,\text{J})\tan 20°}{(20\,\text{kg})(9.81\,\text{m/s}^2)(3.2\,\text{m})} = \boxed{0.354}$$

49 ••

Picture the Problem Let the system consist of the two blocks, the shelf, and the earth. Given this choice, there are no external forces doing work to change the energy of the system. Due to the friction between the 4-kg block and the surface on which it slides, not all of the energy transformed during the fall of the 2-kg block is realized in the form of kinetic energy. We can find the energy dissipated by friction and then use the work-energy theorem with kinetic friction to find the speed of either block when they have moved the given distance.

(*a*) The energy dissipated by friction when the 2-kg block falls a distance y is given by:

$$\Delta E_{\text{therm}} = f\Delta s = \mu_k m_1 g y$$

Substitute numerical values and evaluate ΔE_{therm}:

$$\Delta E_{\text{therm}} = (0.35)(4\,\text{kg})(9.81\,\text{m/s}^2)y = \boxed{(13.7\,\text{N})y}$$

(*b*) From the work-energy theorem with kinetic friction we have:

$$W_{\text{ext}} = \Delta E_{\text{mech}} + \Delta E_{\text{therm}}$$

or, because $W_{\text{ext}} = 0$,

$$\Delta E_{\text{mech}} = -\Delta E_{\text{therm}} = \boxed{-(13.7\,\text{N})y}$$

(*c*) Express the total mechanical energy of the system:

$$\tfrac{1}{2}(m_1 + m_2)v^2 - m_2 g y = -\Delta E_{\text{therm}}$$

Solve for v to obtain:

$$v = \sqrt{\frac{2(m_2 g y - \Delta E_{\text{therm}})}{m_1 + m_2}} \quad (1)$$

Substitute numerical values and evaluate v:

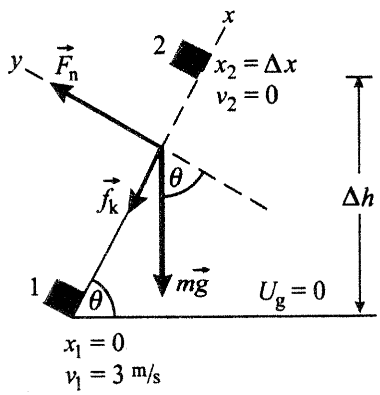
$$v = \sqrt{\frac{2[(2\,\text{kg})(9.81\,\text{m/s}^2)(2\,\text{m}) - (13.73\,\text{N})(2\,\text{m})]}{4\,\text{kg} + 2\,\text{kg}}} = \boxed{1.98\,\text{m/s}}$$

Substitute numerical values and evaluate P:

$$P = 80(75\,\text{kg})(9.81\,\text{m/s}^2)(2.5\,\text{m/s})[\sin 15° + (0.06)\cos 15°] = \boxed{46.6\,\text{kW}}$$

69 ••

Picture the Problem The free-body diagram for the box is superimposed on the pictorial representation shown to the right. The work done by friction slows and momentarily stops the box as it slides up the incline. The box's speed when it returns to bottom of the incline will be less than its speed when it started up the incline due to the energy dissipated by friction while it was in motion. Let the system include the box, the earth, and the incline. Then $W_{ext} = 0$. We can use the work-energy theorem with friction to solve the several parts of this problem.



(*a*) $\boxed{\text{From the FBD we can see that the forces acting on the box are the normal force exerted by the inclined plane, a kinetic friction force, and the gravitational force (the weight of the box) exerted by the earth.}}$

(*b*) Apply the work-energy theorem with friction to relate the distance Δx the box slides up the incline to its initial kinetic energy, its final potential energy, and the work done against friction:

$$-\tfrac{1}{2}mv_1^2 + mg\Delta h + \mu_k mg\Delta x\cos\theta = 0$$

Referring to the figure, relate Δh to Δx to obtain:

$$\Delta h = \Delta x \sin\theta$$

Substitute for Δh to obtain:

$$-\tfrac{1}{2}mv_1^2 + mg\Delta x\sin\theta + \mu_k mg\Delta x\cos\theta = 0$$

Solve for Δx to obtain:

$$\Delta x = \frac{v_1^2}{2g(\sin\theta + \mu_k\cos\theta)}$$

Substitute numerical values and evaluate Δx:

$$\Delta x = \frac{(3\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)[\sin 60° + (0.3)\cos 60°]} = \boxed{0.451\,\text{m}}$$

The energy dissipated by friction is:

$$\Delta E_{\text{therm}} = f_k \Delta x = \mu_k mg\Delta x \cos\theta$$

(*c*) Substitute numerical values and evaluate ΔE_{therm}:

$$\Delta E_{\text{therm}} = (0.3)(2\,\text{kg})(9.81\,\text{m/s}^2)(0.451\,\text{m})\cos 60° = \boxed{1.33\,\text{J}}$$

(*d*) Use the work-energy theorem with friction to obtain:

$$W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{therm}} = 0$$

or

$$K_1 - K_2 + U_1 - U_2 + \Delta E_{\text{therm}} = 0$$

Because $K_2 = U_1 = 0$ we have:

$$K_1 - U_2 + \Delta E_{\text{therm}} = 0$$

or

$$\tfrac{1}{2}mv_1^2 - mg\Delta x \sin\theta + \mu_k mg\Delta x \cos\theta = 0$$

Solve for v_1 to obtain:

$$v_1 = \sqrt{2g\Delta x(\sin\theta - \mu_k \cos\theta)}$$

Substitute numerical values and evaluate v_1:

$$v_1 = \sqrt{2(9.81\,\text{m/s}^2)(0.451\,\text{m})[\sin 60° - (0.3)\cos 60°]} = \boxed{2.52\,\text{m/s}}$$

***70 •**

Picture the Problem The power provided by a motor that is delivering sufficient energy to exert a force F on a load which it is moving at a speed v is Fv.

The power provided by the motor is given by:

$$P = Fv$$

Because the elevator is ascending with constant speed, the tension in the support cable(s) is:

$$F = (m_{\text{elev}} + m_{\text{load}})g$$

Substitute for F to obtain:

$$P = (m_{\text{elev}} + m_{\text{load}})gv$$

Apply conservation of energy during the dart's ascent:

$$W_{ext} = \Delta K + \Delta U + \Delta E_{therm} = 0$$

or, because $\Delta K = 0$,

$$U_{g,f} - U_{g,i} + U_{s,f} - U_{s,i} + \Delta E_{therm} = 0$$

Because $U_{g,i} = U_{s,f} = 0$:

$$U_{g,f} - U_{s,i} + \Delta E_{therm} = 0$$

Substitute for $U_{g,i}$ and $U_{s,f}$ and solve for ΔE_{therm}:

$$\Delta E_{therm} = U_{s,i} - U_{g,f} = \tfrac{1}{2}kx^2 - mgh$$

Substitute numerical values and evaluate ΔE_{therm}:

$$\Delta E_{therm} = \tfrac{1}{2}(5000\,\text{N/m})(0.03\,\text{m})^2 - (0.007\,\text{kg})(9.81\,\text{m/s}^2)(24\,\text{m}) = \boxed{0.602\,\text{J}}$$

Apply conservation of energy during the dart's descent:

$$W_{ext} = \Delta K + \Delta U + \Delta E_{therm} = 0$$

or, because $K_i = U_{g,f} = 0$,

$$K_f - U_{g,i} + \Delta E_{therm} = 0$$

Substitute for K_f and $U_{g,i}$ to obtain:

$$\tfrac{1}{2}mv_f^2 - mgh + \Delta E_{therm} = 0$$

Solve for v_f:

$$v_f = \sqrt{\frac{2(mgh - \Delta E_{therm})}{m}}$$

Substitute numerical values and evaluate v_f:

$$v_f = \sqrt{\frac{2[(0.007\,\text{kg})(9.81\,\text{m/s}^2)(24\,\text{m}) - 0.602\,\text{J}]}{0.007\,\text{kg}}} = \boxed{17.3\,\text{m/s}}$$

***73 ••**

Picture the Problem Let the system consist of the earth, rock and air. Given this choice, there are no external forces to do work on the system and $W_{ext} = 0$. Choose $U_g = 0$ to be where the rock begins its upward motion. The initial kinetic energy of the rock is partially transformed into potential energy and partially dissipated by air resistance as the rock ascends. During its descent, its potential energy is partially transformed into kinetic energy and partially dissipated by air resistance.

(*a*) Using the definition of kinetic energy, calculate the initial kinetic energy of the rock:

$$K_i = \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}(2\,\text{kg})(40\,\text{m/s})^2 = \boxed{1.60\,\text{kJ}}$$

(*b*) Apply the work-energy theorem with friction to relate the energies of the system as the rock ascends:

$$\Delta K + \Delta U + \Delta E_{\text{therm}} = 0$$

Because $K_f = 0$:

$$-K_i + \Delta U + \Delta E_{\text{therm}} = 0$$

and

$$\Delta E_{\text{therm}} = K_i - \Delta U$$

Substitute numerical values and evaluate ΔE_{therm}:

$$\Delta E_{\text{therm}} = 1600\,\text{J} - (2\,\text{kg})(9.81\,\text{m/s}^2)(50\,\text{m}) = \boxed{619\,\text{J}}$$

(*c*) Apply the work-energy theorem with friction to relate the energies of the system as the rock descends:

$$\Delta K + \Delta U + 0.7\Delta E_{\text{therm}} = 0$$

Because $K_i = U_f = 0$:

$$K_f - U_i + 0.7\Delta E_{\text{therm}} = 0$$

Substitute for the energies to obtain:

$$\tfrac{1}{2}mv_f^2 - mgh + 0.7\Delta E_{\text{therm}} = 0$$

Solve for v_f to obtain:

$$v_f = \sqrt{2gh - \frac{1.4\Delta E_{\text{therm}}}{m}}$$

Substitute numerical values and evaluate v_f:

$$v_f = \sqrt{2(9.81\,\text{m/s}^2)(50\,\text{m}) - \frac{1.4(619\,\text{J})}{2\,\text{kg}}} = \boxed{23.4\,\text{m/s}}$$

74 ••

Picture the Problem Let the distance the block slides before striking the spring be *L*. The pictorial representation shows the block at the top of the incline (1), just as it strikes the spring (2), and the block against the fully compressed spring (3). Let the block, spring, and the earth comprise the system. Then $W_{ext} = 0$. Let $U_g = 0$ where the spring is at maximum compression. We can apply the work-energy theorem to relate the energies of the system as it evolves from state 1 to state 3.

(*c*) Apply the work-energy theorem with friction to the upward trajectory of the flare:

$$\Delta K + \Delta U_g + \Delta E_{therm} = 0$$

Solve for ΔE_{therm}:

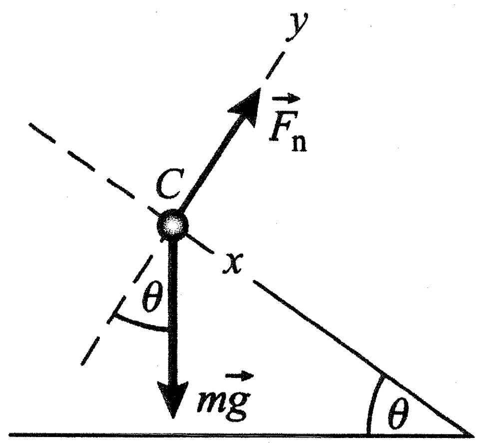
$$\Delta E_{therm} = -\Delta K - \Delta U_g = K_i - K_f + U_i - U_f$$

Because $K_f = U_i = 0$:

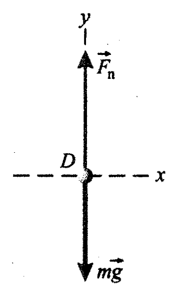
$$\Delta E_{therm} = \boxed{\tfrac{1}{2}mv_0^2 - mgh}$$

79 ••

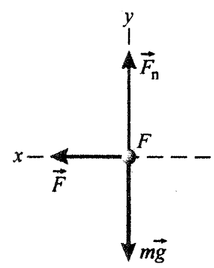
Picture the Problem Let $U_D = 0$. Choose the system to include the earth, the track, and the car. Then there are no external forces to do work on the system and change its energy and we can use Newton's 2nd law and the work-energy theorem to describe the system's energy transformations to point G … and then the work-energy theorem with friction to determine the braking force that brings the car to a stop. The free-body diagram for point C is shown to the right.



The free-body diagram for point D is shown to the right.



The free-body diagram for point F is shown to the right.



(*a*) Apply the work-energy theorem to the system's energy transformations between A and B:

$$\Delta K + \Delta U = 0$$

or

$$K_B - K_A + U_B - U_A = 0$$

If we assume that the car arrives at point B with $v_B = 0$, then:

$$-\tfrac{1}{2}mv_A^2 + mg\Delta h = 0$$

where Δh is the difference in elevation between A and B.

Solve for and evaluate Δh:

$$\Delta h = \frac{v_A^2}{2g} = \frac{(12\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = 7.34\,\text{m}$$

The height above the ground is:

$$h + \Delta h = 10\,\text{m} + 7.34\,\text{m} = \boxed{17.3\,\text{m}}$$

(*b*) If the car just makes it to point B; i.e., if it gets there with $v_B = 0$, then the force exerted by the track on the car will be the normal force:

$$F_{\text{track on car}} = F_n = mg = (500\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{4.91\,\text{kN}}$$

(*c*) Apply $\sum F_x = ma_x$ to the car at point C (see the FBD) and solve for a:

$$mg\sin\theta = ma$$

and

$$a = g\sin\theta$$

Substitute numerical values and evaluate a:

$$a = (9.81\,\text{m/s}^2)\sin 30° = \boxed{4.91\,\text{m/s}^2}$$

(*d*) Apply $\sum F_y = ma_y$ to the car at point D (see the FBD) and solve for F_n:

$$F_n - mg = m\frac{v_D^2}{R} \Rightarrow F_n = mg + m\frac{v_D^2}{R}$$

Apply the work-energy theorem to the system's energy transformations between B and D:

$$\Delta K + \Delta U = 0$$

or

$$K_D - K_B + U_D - U_B = 0$$

Because $K_B = U_D = 0$:

$$K_D - U_B = 0$$

Substitute to obtain:

$$\tfrac{1}{2}mv_D^2 - mg(h + \Delta h) = 0$$

Solving for v_D^2 yields:

$$v_D^2 = 2g(h + \Delta h)$$

Substitute for v_D^2 in the expression for F_n and simplify to obtain:

$$F_n = mg + m\frac{v_D^2}{R} = mg + m\frac{2g(h+\Delta h)}{R} = mg\left[1+\frac{2(h+\Delta h)}{R}\right]$$

Substitute numerical values and evaluate F_n:

$$F_n = (500\,\text{kg})(9.81\,\text{m/s}^2)\left[1+\frac{2(17.3\,\text{m})}{20\,\text{m}}\right] = \boxed{13.4\,\text{kN, directed upward.}}$$

(*e*) F has two components at point F; one horizontal (the inward force that the track exerts) and the other vertical (the normal force). Apply $\sum\vec{F} = m\vec{a}$ to the car at point F:

$$\sum F_y = F_n - mg = 0 \Rightarrow F_n = mg$$

and

$$\sum F_x = F_c = m\frac{v_F^2}{R}$$

Express the resultant of these two forces:

$$F = \sqrt{F_c^2 + F_n^2} = \sqrt{\left(m\frac{v_F^2}{R}\right)^2 + (mg)^2} = m\sqrt{\frac{v_F^4}{R^2} + g^2}$$

Substitute numerical values and evaluate F:

$$F = (500\,\text{kg})\sqrt{\frac{(12\,\text{m/s})^4}{(30\,\text{m})^2} + (9.81\,\text{m/s}^2)^2} = \boxed{5.46\,\text{kN}}$$

Find the angle the resultant makes with the x axis:

$$\theta = \tan^{-1}\left(\frac{F_n}{F_c}\right) = \tan^{-1}\left(\frac{gR}{v_F^2}\right) = \tan^{-1}\left[\frac{(9.81\,\text{m/s}^2)(30\,\text{m})}{(12\,\text{m/s})^2}\right] = \boxed{63.9^\circ}$$

(*f*) Apply the work-energy theorem with friction to the system's energy transformations between F and the car's stopping position:

$$-K_G + \Delta E_{\text{therm}} = 0$$

and

$$\Delta E_{\text{therm}} = K_G = \tfrac{1}{2}mv_G^2$$

The work done by friction is also given by:

$$\Delta E_{therm} = f\Delta s = F_{brake}d$$

where d is the stopping distance.

Equate the two expressions for ΔE_{therm} and solve for F_{brake}:

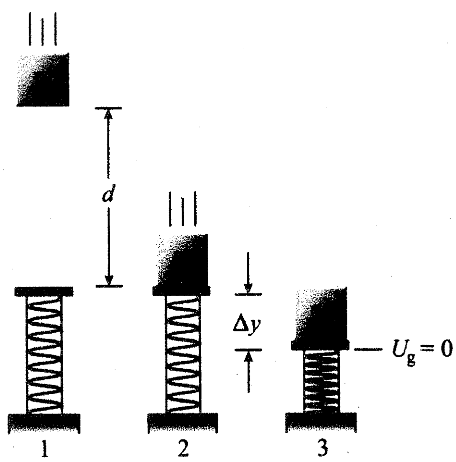
$$F_{brake} = \frac{mv_F^2}{2d}$$

Substitute numerical values and evaluate F_{brake}:

$$F_{brake} = \frac{(500\,\text{kg})(12\,\text{m/s})^2}{2(25\,\text{m})} = \boxed{1.44\,\text{kN}}$$

***80** •

Picture the Problem The rate of conversion of mechanical energy can be determined from $P = \vec{F}\cdot\vec{v}$. The pictorial representation shows the elevator moving downward just as it goes into freefall as state 1. In state 2 the elevator is moving faster and is about to strike the relaxed spring. The momentarily at rest elevator on the compressed spring is shown as state 3. Let $U_g = 0$ where the spring has its maximum compression and the system consist of the earth, the elevator, and the spring. Then $W_{ext} = 0$ and we can apply the conservation of mechanical energy to the analysis of the falling elevator and compressing spring.



(*a*) Express the rate of conversion of mechanical energy to thermal energy as a function of the speed of the elevator and braking force acting on it:

$$P = F_{braking}v_0$$

Because the elevator is moving with constant speed, the net force acting on it is zero and:

$$F_{braking} = Mg$$

Substitute for $F_{braking}$ to obtain:

$$P = Mgv_0$$

Substitute numerical values and evaluate P:

$$P = (2000\,\text{kg})(9.81\,\text{m/s}^2)(1.5\,\text{m/s}) = \boxed{29.4\,\text{kW}}$$

Express the trigonometric relationship between $a/2$, 30°, and y_{cm}:

$$\tan 30° = \frac{y_{cm}}{a/2}$$

Solve for y_{cm}:

$$y_{cm} = \tfrac{1}{2}a\tan 30° = 0.289a$$

The center of mass of an equilateral triangle oriented as shown above is at $\boxed{(0, 0.289a)}$.

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Picture the Problem Let the subscript 1 refer to the 3-m by 3-m sheet of plywood before the 2-m by 1-m piece has been cut from it. Let the subscript 2 refer to 2-m by 1-m piece that has been removed and let σ be the area density of the sheet. We can find the center-of-mass of these two regions; treating the missing region as though it had negative mass, and then finding the center-of-mass of the U-shaped region by applying its definition.

Express the coordinates of the center of mass of the sheet of plywood:

$$x_{cm} = \frac{m_1 x_{cm,1} - m_2 x_{cm,2}}{m_1 - m_2}$$

$$y_{cm} = \frac{m_1 y_{cm,1} - m_2 y_{cm,2}}{m_1 - m_2}$$

Use symmetry to find $x_{cm,1}$, $y_{cm,1}$, $x_{cm,2}$, and $y_{cm,2}$:

$$x_{cm,1} = 1.5\,\text{m},\ y_{cm,1} = 1.5\,\text{m}$$

and

$$x_{cm,2} = 1.5\,\text{m}, y_{cm,2} = 2.0\,\text{m}$$

Determine m_1 and m_2:

$$m_1 = \sigma A_1 = 9\sigma\,\text{kg}$$

and

$$m_2 = \sigma A_2 = 2\sigma\,kg$$

Substitute numerical values and evaluate x_{cm}:

$$x_{cm} = \frac{(9\sigma\,\text{kg})(1.5\,\text{m}) - (2\sigma\,\text{kg})(1.5\,\text{kg})}{9\sigma\,\text{kg} - 2\sigma\,\text{kg}}$$

$$= 1.50\,\text{m}$$

Substitute numerical values and evaluate y_{cm}:

$$y_{cm} = \frac{(9\sigma\,\text{kg})(1.5\,\text{m}) - (2\sigma\,\text{kg})(2\,\text{m})}{9\sigma\,\text{kg} - 2\sigma\,\text{kg}}$$

$$= 1.36\,\text{m}$$

The center of mass of the U-shaped sheet of plywood is at $\boxed{(1.50\,\text{m}, 1.36\,\text{m})}$.

Substitute and simplify to obtain:

$$F_{\text{net,ext}} = (M+m)g - (M+m)\left(\frac{mg}{M+m}\right)$$
$$= \boxed{Mg}$$

as expected, given our answer to part (*a*).

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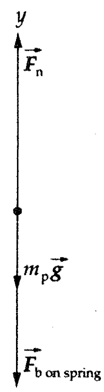
Picture the Problem The free-body diagram shows the forces acting on the platform when the spring is partially compressed. The scale reading is the force the scale exerts on the platform and is represented on the FBD by F_n. We can use Newton's 2nd law to determine the scale reading in part (*a*) and the work-energy theorem in conjunction with Newton's 2nd law in parts (*b*) and (*c*).



(*a*) Apply $\sum F_y = ma_y$ to the spring when it is compressed a distance *d*:

$$\sum F_y = F_n - m_p g - F_{\text{ball on spring}} = 0$$

Solve for F_n:

$$F_n = m_p g + F_{\text{ball on spring}}$$
$$= m_p g + kd = m_p g + k\left(\frac{m_b g}{k}\right)$$
$$= \boxed{m_p g + m_b g = (m_p + m_b)g}$$

(*b*) Use conservation of mechanical energy, with $U_g = 0$ at the position at which the spring is fully compressed, to relate the gravitational potential energy of the system to the energy stored in the fully compressed spring:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

Because $\Delta K = U_{g,f} = U_{s,i} = 0$,

$$U_{g,i} - U_{s,f} = 0$$

or

$$m_b g d - \tfrac{1}{2}kd^2 = 0$$

Solve for *d*:

$$d = \frac{2m_b g}{k}$$

Evaluate our force equation in (*a*) with $d = \frac{2m_b g}{k}$:

$$F_n = m_p g + F_{\text{ball on spring}}$$
$$= m_p g + kd = m_p g + k\left(\frac{2m_b g}{k}\right)$$
$$= \boxed{m_p g + 2m_b g = (m_p + 2m_b)g}$$

(*c*) When the ball is in its original position, the spring is relaxed and exerts no force on the ball. Therefore:

$$F_n = \text{scale reading}$$
$$= \boxed{m_p g}$$

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Picture the Problem Assume that the object whose mass is m_1 is moving downward and take that direction to be the positive direction. We'll use Newton's 2nd law for a system of particles to relate the acceleration of the center of mass to the acceleration of the individual particles.

(*a*) Relate the acceleration of the center of mass to m_1, m_2, m_c and their accelerations:

$$M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_c\vec{a}_c$$

Because m_1 and m_2 have a common acceleration a and $a_c = 0$:

$$a_{cm} = a\frac{m_1 - m_2}{m_1 + m_2 + m_c}$$

From Problem 4-81 we have:

$$a = g\frac{m_1 - m_2}{m_1 + m_2}$$

Substitute to obtain:

$$a_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}g\right)\left(\frac{m_1 - m_2}{m_1 + m_2 + m_c}\right)$$
$$= \boxed{\frac{(m_1 - m_2)^2}{(m_1 + m_2)(m_1 + m_2 + m_c)}g}$$

(*b*) Use Newton's 2nd law for a system of particles to obtain:

$$F - Mg = -Ma_{cm}$$

where $M = m_1 + m_2 + m_c$ and F is positive upwards.