***61 •**

Picture the Problem The impulse exerted by the ground on the meteorite equals the *change* in momentum of the meteorite and is also the product of the average force exerted by the ground on the meteorite and the time during which the average force acts.

Express the impulse exerted by the ground on the meteorite:

$$I = \Delta p_{\text{meteorite}} = p_f - p_i$$

Relate the kinetic energy of the meteorite to its initial momentum and solve for its initial momentum:

$$K_i = \frac{p_i^2}{2m} \Rightarrow p_i = \sqrt{2mK_i}$$

Express the ratio of the initial and final kinetic energies of the meteorite:

$$\frac{K_i}{K_f} = \frac{\frac{p_i^2}{2m}}{\frac{p_f^2}{2\text{m}}} = \frac{p_i^2}{p_f^2} = 2$$

Solve for p_f:

$$p_f = \frac{p_i}{\sqrt{2}}$$

Substitute in our expression for I and simplify:

$$I = \frac{p_i}{\sqrt{2}} - p_i = p_i\left(\frac{1}{\sqrt{2}} - 1\right)$$
$$= \sqrt{2mK_i}\left(\frac{1}{\sqrt{2}} - 1\right)$$

Because our interest is in its magnitude, evaluate $|I|$:

$$|I| = \left|\sqrt{2(30.8\times10^3\text{ kg})(617\times10^6\text{ J})}\left(\frac{1}{\sqrt{2}} - 1\right)\right| = \boxed{1.81\text{MN}\cdot\text{s}}$$

Express the impulse delivered to the meteorite as a function of the average force acting on it and solve for and evaluate F_{av}:

$$I = F_{av}\Delta t$$

and

$$F_{av} = \frac{I}{\Delta t} = \frac{1.81\text{MN}\cdot\text{s}}{3\text{s}} = \boxed{0.602\text{MN}}$$

62 ••

Picture the Problem The impulse exerted by the bat on the ball equals the *change* in momentum of the ball and is also the product of the average force exerted by the bat on the ball and the time during which the bat and ball were in contact.

Substitute numerical values and evaluate $v_{f,5}$:

$$v_{f,5} = \frac{(5\,\text{kg})(4\,\text{m/s}) - (10\,\text{kg})(3\,\text{m/s})}{5\,\text{kg}}$$

$$= \boxed{-2.00\,\text{m/s}}$$

where the minus sign means that the 5-kg object is moving to the left after the collision.

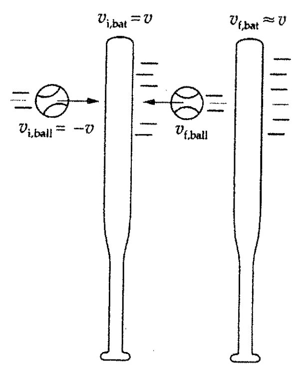
(*b*) Evaluate ΔK for the collision:

$$\Delta K = K_f - K_i = \tfrac{1}{2}(5\,\text{kg})(2\,\text{m/s})^2 - \left[\tfrac{1}{2}(5\,\text{kg})(4\,\text{m/s})^2 + \tfrac{1}{2}(10\,\text{kg})(3\,\text{m/s})^2\right] = -75.0\,\text{J}$$

$$\boxed{\text{Because } \Delta K \neq 0\text{, the collision was inelastic.}}$$

70 •

Picture the Problem The pictorial representation shows the ball and bat just before and just after their collision. Take the direction the bat is moving to be the positive direction. Because the collision is elastic, we can equate the speeds of recession and approach, with the approximation that $v_{i,bat} \approx v_{f,bat}$ to find $v_{f,ball}$.



Express the speed of approach of the bat and ball:

$$v_{f,bat} - v_{f,ball} = -(v_{i,bat} - v_{i,ball})$$

Because the mass of the bat is much greater than that of the ball:

$$v_{i,bat} \approx v_{f,bat}$$

Substitute to obtain:

$$v_{f,bat} - v_{f,ball} = -(v_{f,bat} - v_{i,ball})$$

Solve for and evaluate $v_{f,ball}$:

$$v_{f,ball} = v_{f,bat} + (v_{f,bat} - v_{i,ball})$$

$$= -v_{i,ball} + 2v_{f,bat} = v + 2v$$

$$= \boxed{3v}$$

***71** ••

Picture the Problem Let the direction the proton is moving before the collision be the positive x direction. We can use both conservation of momentum and conservation of mechanical energy to obtain an expression for velocity of the proton after the collision.

(*a*) Use the expression for the total momentum of a system to find v_{cm}:

$$\vec{P} = \sum_i m_i \vec{v}_i = M\vec{v}_{cm}$$

and

$$\vec{v}_{cm} = \frac{m\vec{v}_{p,i}}{m + 12m} = \tfrac{1}{13}(300\,\text{m/s})\hat{i}$$

$$= \boxed{(23.1\,\text{m/s})\hat{i}}$$

(*b*) Use conservation of momentum to obtain one relation for the final velocities:

$$m_p v_{p,i} = m_p v_{p,f} + m_{nuc} v_{nuc,f} \qquad (1)$$

Use conservation of mechanical energy to set the velocity of recession equal to the negative of the velocity of approach:

$$v_{nuc,f} - v_{p,f} = -(v_{nuc,i} - v_{p,i}) = v_{p,i} \quad (2)$$

To eliminate $v_{nuc,f}$, solve equation (2) for $v_{nuc,f}$, and substitute the result in equation (1):

$$v_{nuc,f} = v_{p,i} + v_{p,f}$$

$$m_p v_{p,i} = m_p v_{p,f} + m_{nuc}(v_{p,i} + v_{p,f})$$

Solve for and evaluate $v_{p,f}$:

$$v_{p,f} = \frac{m_p - m_{nuc}}{m_p + m_{nuc}} v_{p,i}$$

$$= \frac{m - 12m}{13m}(300\,\text{m/s}) = \boxed{-254\,\text{m/s}}$$

72 ••

Picture the Problem We can use conservation of momentum and the definition of an elastic collision to obtain two equations in v_{2f} and v_{3f} that we can solve simultaneously.

Use conservation of momentum to obtain one relation for the final velocities:

$$m_3 v_{3i} = m_3 v_{3f} + m_2 v_{2f} \qquad (1)$$

Use conservation of mechanical energy to set the velocity of recession equal to the negative of the velocity of approach:

$$v_{2f} - v_{3f} = -(v_{2i} - v_{3i}) = v_{3i} \qquad (2)$$

Use conservation of momentum to relate the speed of the bullet just before impact to the initial speed of the bob plus bullet:

$$mv_b = (m + M)V$$

Solve for the speed of the bullet:

$$v_b = \left(1 + \frac{M}{m}\right)V \quad (1)$$

Use conservation of energy to relate the initial kinetic energy of the bullet to the final potential energy of the system:

$$\Delta K + \Delta U = 0$$

or, because $K_f = U_i = 0$,

$$-K_i + U_f = 0$$

Substitute for K_i and U_f and solve for V:

$$-\tfrac{1}{2}(m + M)V^2 + (m + M)gL(1 - \cos\theta) = 0$$

and

$$V = \sqrt{2gL(1 - \cos\theta)}$$

Substitute for V in equation (1) to obtain:

$$v_b = \left(1 + \frac{M}{m}\right)\sqrt{2gL(1 - \cos\theta)}$$

Substitute numerical values and evaluate v_b:

$$v_b = \left(1 + \frac{1.5\,\text{kg}}{0.016\,\text{kg}}\right)\sqrt{2(9.81\,\text{m/s}^2)(2.3\,\text{m})(1 - \cos 60°)} = \boxed{450\,\text{m/s}}$$

***78 ••**

Picture the Problem We can apply conservation of momentum and the definition of an elastic collision to obtain equations relating the initial and final velocities of the colliding objects that we can solve for v_{1f} and v_{2f}.

Apply conservation of momentum to the elastic collision of the particles to obtain:

$$m_1v_{1f} + m_2v_{2f} = m_1v_{1i} + m_2v_{2i} \quad (1)$$

Relate the initial and final kinetic energies of the particles in an elastic collision:

$$\tfrac{1}{2}m_1v_{1f}^2 + \tfrac{1}{2}m_2v_{2f}^2 = \tfrac{1}{2}m_1v_{1i}^2 + \tfrac{1}{2}m_2v_{2i}^2$$

Rearrange this equation and factor to obtain:

$$m_2\left(v_{2f}^2 - v_{2i}^2\right) = m_1\left(v_{1i}^2 - v_{1f}^2\right)$$

or

$$m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) = m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) \quad (2)$$

Rearrange equation (1) to obtain:

$$m_2(v_{2f} - v_{2i}) = m_1(v_{1i} - v_{1f}) \qquad (3)$$

Divide equation (2) by equation (3) to obtain:

$$v_{2f} + v_{2i} = v_{1i} + v_{1f}$$

Rearrange this equation to obtain equation (4):

$$v_{1f} - v_{2f} = v_{2i} - v_{1i} \qquad (4)$$

Multiply equation (4) by m_2 and add it to equation (1) to obtain:

$$(m_1 + m_2)v_{1f} = (m_1 - m_2)v_{1i} + 2m_2v_{2i}$$

Solve for v_{1f} to obtain:

$$v_{1f} = \boxed{\frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i}}$$

Multiply equation (4) by m_1 and subtract it from equation (1) to obtain:

$$(m_1 + m_2)v_{2f} = (m_2 - m_1)v_{2i} + 2m_1v_{1i}$$

Solve for v_{2f} to obtain:

$$v_{2f} = \boxed{\frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i}}$$

Remarks: Note that the velocities satisfy the condition that $v_{2f} - v_{1f} = -(v_{2i} - v_{1i})$. This verifies that the speed of recession equals the speed of approach.

79 ••

Picture the Problem As in this problem, Problem 78 involves an elastic, one-dimensional collision between two objects. Both solutions involve using the conservation of momentum equation $m_1v_{1f} + m_2v_{2f} = m_1v_{1i} + m_2v_{2i}$ and the elastic collision equation $v_{1f} - v_{2f} = v_{2i} - v_{1i}$. In part (*a*) we can simply set the masses equal to each other and substitute in the equations in Problem 78 to show that the particles "swap" velocities. In part (*b*) we can divide the numerator and denominator of the equations in Problem 78 by m_2 and use the condition that $m_2 >> m_1$ to show that $v_{1f} \approx -v_{1i} + 2v_{2i}$ and $v_{2f} \approx v_{2i}$.

(*a*) From Problem 78 we have:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i} \qquad (1)$$

and

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i} \qquad (2)$$

Set $m_1 = m_2 = m$ to obtain:

$$v_{1f} = \frac{2m}{m + m}v_{2i} = \boxed{v_{2i}}$$

and

$$v_{2f} = \frac{2m}{m + m}v_{1i} = \boxed{v_{1i}}$$

Solve for v_1:

$$v_1 = \sqrt{2gL}$$

Substitute in equation (2) to obtain:

$$V = \frac{m_J}{m_{J+T}}\sqrt{2gL}$$

Substitute in equation (1) and simplify:

$$h = \frac{1}{2g}\left(\frac{m_J}{m_{J+T}}\right)^2 2gL = \left(\frac{m_J}{m_{J+T}}\right)^2 L$$

Substitute numerical values and evaluate h:

$$h = \left(\frac{54\,\text{kg}}{54\,\text{kg} + 82\,\text{kg}}\right)^2 (25\,\text{m}) = \boxed{3.94\,\text{m}}$$

Exploding Objects and Radioactive Decay

85 ••

Picture the Problem This nuclear reaction is ${}^4\text{Be} \rightarrow 2\alpha + 1.5\times10^{-14}$ J. In order to conserve momentum, the alpha particles will have move in opposite directions with the same velocities. We'll use conservation of energy to find their speeds.

Letting E represent the energy released in the reaction, express conservation of energy for this process:

$$2K_\alpha = 2\left(\tfrac{1}{2}m_\alpha v_\alpha^2\right) = E$$

Solve for v_α:

$$v_\alpha = \sqrt{\frac{E}{m_\alpha}}$$

Substitute numerical values and evaluate v_α:

$$v_\alpha = \sqrt{\frac{1.5\times10^{-14}\,\text{J}}{6.68\times10^{-27}\,\text{kg}}} = \boxed{1.50\times10^6\,\text{m/s}}$$

86 ••

Picture the Problem This nuclear reaction is ${}^5\text{Li} \rightarrow \alpha + \text{p} + 3.15 \times 10^{-13}$ J. To conserve momentum, the alpha particle and proton must move in opposite directions. We'll apply both conservation of energy and conservation of momentum to find the speeds of the proton and alpha particle.

Use conservation of momentum in this process to express the alpha particle's velocity in terms of the proton's:

$$p_i = p_f = 0$$

and

$$0 = m_p v_p - m_\alpha v_\alpha$$

Using its definition, relate the rocket's thrust to the relative speed of its exhaust gases:

$$F_{\text{th}} = \left|\frac{dm}{dt}\right| u_{\text{ex}}$$

Substitute numerical values and evaluate F_{th}:

$$F_{\text{th}} = (200\,\text{kg/s})(6\,\text{km/s}) = \boxed{1.20\,\text{MN}}$$

105 ••

Picture the Problem The thrust of a rocket F_{th} depends on the burn rate of its fuel *dm/dt* and the relative speed of its exhaust gases u_{ex} according to $F_{\text{th}} = |dm/dt|u_{\text{ex}}$. The final velocity v_{f} of a rocket depends on the relative speed of its exhaust gases u_{ex}, its payload to initial mass ratio m_{f}/m_0 and its burn time according to $v_{\text{f}} = -u_{\text{ex}} \ln(m_{\text{f}}/m_0) - gt_{\text{b}}$.

(*a*) Using its definition, relate the rocket's thrust to the relative speed of its exhaust gases:

$$F_{\text{th}} = \left|\frac{dm}{dt}\right| u_{\text{ex}}$$

Substitute numerical values and evaluate F_{th}:

$$F_{\text{th}} = (200\,\text{kg/s})(1.8\,\text{km/s}) = \boxed{360\,\text{kN}}$$

(*b*) Relate the time to burnout to the mass of the fuel and its burn rate:

$$t_{\text{b}} = \frac{m_{\text{fuel}}}{dm/dt} = \frac{0.8m_0}{dm/dt}$$

Substitute numerical values and evaluate t_{b}:

$$t_{\text{b}} = \frac{0.8(30{,}000\,\text{kg})}{200\,\text{kg/s}} = \boxed{120\,\text{s}}$$

(*c*) Relate the final velocity of a rocket to its initial mass, exhaust velocity, and burn time:

$$v_{\text{f}} = -u_{\text{ex}} \ln\left(\frac{m_{\text{f}}}{m_0}\right) - gt_{\text{b}}$$

Substitute numerical values and evaluate v_{f}:

$$v_{\text{f}} = -(1.8\,\text{km/s})\ln\left(\frac{1}{5}\right) - (9.81\,\text{m/s}^2)(120\,\text{s}) = \boxed{1.72\,\text{km/s}}$$

***106** ••

Picture the Problem We can use the dimensions of thrust, burn rate, and acceleration to show that the dimension of specific impulse is time. Combining the definitions of rocket thrust and specific impulse will lead us to $u_{\text{ex}} = gI_{\text{sp}}$.

Substitute numerical values and evaluate W_{ext}:

$$W_{ext} = (200\,\text{kg})(9.81\,\text{m/s}^2)(180\,\text{m}) + \tfrac{1}{2}(200\,\text{kg})\left[(80\,\text{m/s})^2 - (125\,\text{m/s})^2\right] = \boxed{-569\,\text{kJ}}$$

117 ••

Picture the Problem Because this is a perfectly inelastic collision, the velocity of the block after the collision is the same as the velocity of the center of mass before the collision. The distance the block travels before hitting the floor is the product of its velocity and the time required to fall 0.8 m; which we can find using a constant-acceleration equation.

Relate the distance D to the velocity of the center of mass and the time for the block to fall to the floor:

$$D = v_{cm}\Delta t$$

Relate the velocity of the center of mass to the total momentum of the system and solve for v_{cm}:

$$\vec{P} = \sum_i m_i \vec{v}_i = M\vec{v}_{cm}$$

and

$$v_{cm} = \frac{m_{bullet}v_{bullet} + m_{block}v_{block}}{m_{bullet} + m_{block}}$$

Substitute numerical values and evaluate v_{cm}:

$$v_{cm} = \frac{(0.015\,\text{kg})(500\,\text{m/s})}{0.015\,\text{kg} + 0.8\,\text{kg}} = 9.20\,\text{m/s}$$

Using a constant-acceleration equation, find the time for the block to fall to the floor:

$$\Delta y = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

Because $v_0 = 0$, $\Delta t = \sqrt{\dfrac{2\Delta y}{g}}$

Substitute to obtain:

$$D = v_{cm}\sqrt{\frac{2\Delta y}{g}}$$

Substitute numerical values and evaluate D:

$$D = (9.20\,\text{m/s})\sqrt{\frac{2(0.8\,\text{m})}{9.81\,\text{m/s}^2}} = \boxed{3.72\,\text{m}}$$

118 ••

Picture the Problem Let the direction the particle whose mass is m is moving initially be the positive x direction and the direction the particle whose mass is $4m$ is moving initially be the negative y direction. We can determine the impulse delivered by $\vec{F}$ and, hence, the change in the momentum of the system from the change in the momentum of the particle whose mass is m. Knowing $\Delta\vec{p}$, we can express the final momentum of the

Find the smaller value for p_m:

$$p_m = 1.418\,\text{kg}\cdot\text{m/s} - 0.158\,\text{kg}\cdot\text{m/s} = 1.260\,\text{kg}\cdot\text{m/s}$$

Substitute in equation (1) to determine the two values for m:

$$m = \frac{(1.576\,\text{kg}\cdot\text{m/s})^2}{2(2.47\,\text{J})} = \boxed{0.503\,\text{kg}}$$

or

$$m = \frac{(1.260\,\text{kg}\cdot\text{m/s})^2}{2(2.47\,\text{J})} = \boxed{0.321\,\text{kg}}$$

129 ••

Picture the Problem Choose the zero of gravitational potential energy at the location of the spring's maximum compression. Let the system include the spring, the blocks, and the earth. Then the net external force is zero as is work done against friction. We can use conservation of energy to relate the energy transformations taking place during the evolution of this system.

Apply conservation of energy:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

Because $\Delta K = 0$:

$$\Delta U_g + \Delta U_s = 0$$

Express the change in the gravitational potential energy:

$$\Delta U_g = -mg\Delta h - Mgx\sin\theta$$

Express the change in the potential energy of the spring:

$$\Delta U_s = \tfrac{1}{2}kx^2$$

Substitute to obtain:

$$-mg\Delta h - Mgx\sin\theta + \tfrac{1}{2}kx^2 = 0$$

Solve for M:

$$M = \frac{\frac{1}{2}kx^2 - mg\Delta h}{gx\sin 30°} = \frac{kx}{g} - \frac{2m\Delta h}{x}$$

Relate Δh to the initial and rebound positions of the block whose mass is m:

$$\Delta h = (4\,\text{m} - 2.56\,\text{m})\sin 30° = 0.720\,\text{m}$$

Substitute numerical values and evaluate M:

$$M = \frac{(11\times10^3\,\text{N/m})(0.04\,\text{m})}{9.81\,\text{m/s}^2} - \frac{2(1\,\text{kg})(0.72\,\text{m})}{0.04\,\text{m}} = \boxed{8.85\,\text{kg}}$$