

Kap. 13

Udempede svingninger

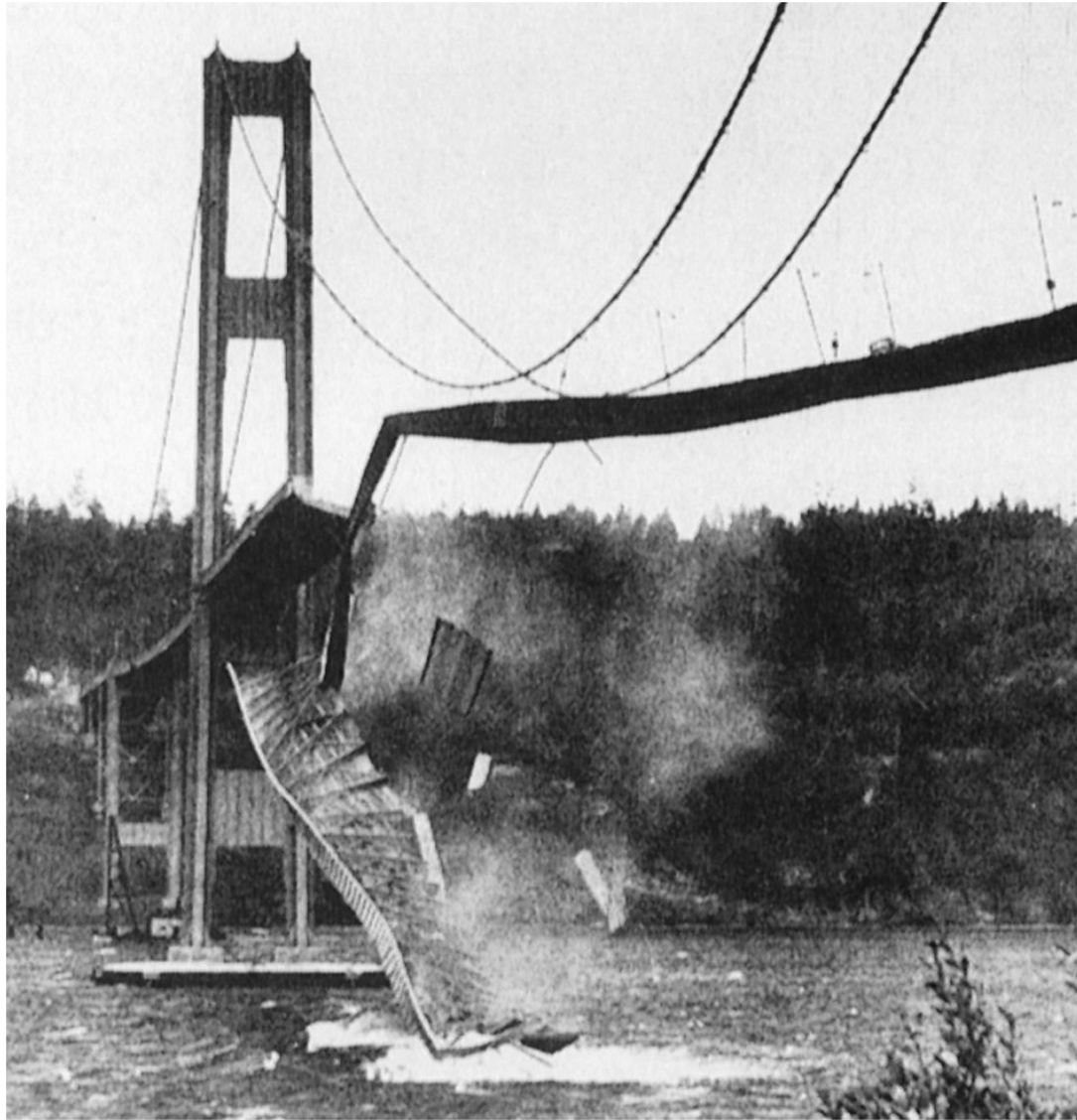
- **Udempet harmonisk oscillasjon**

$$x(t) = A \sin(\omega t + \delta)$$

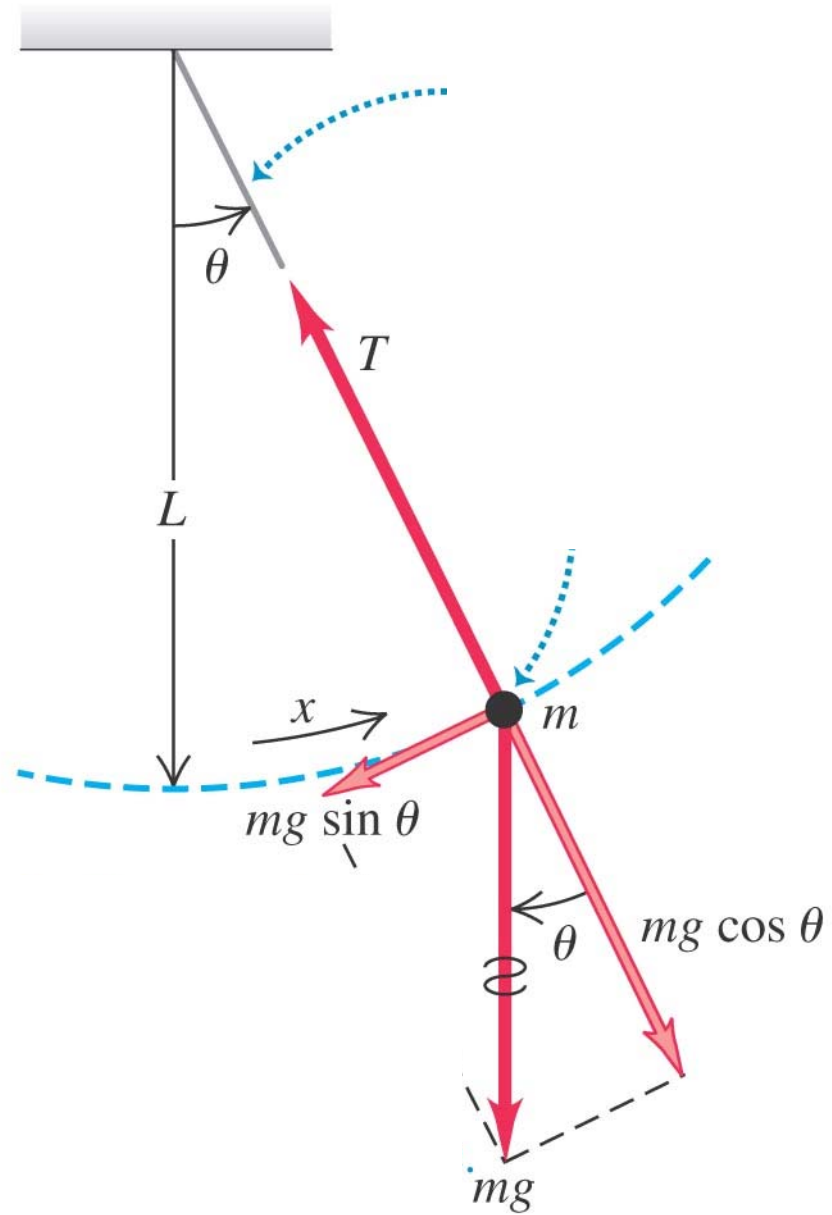
- Ekvivalens til roterende bevegelser

- Eksempler:

- Fjærpendel
- Matematisk pendel
- Fysisk pendel
- Torsjonspendel



Tacoma Narrows Bridge on the morning of Nov. 7, 1940. The bridge was an unusually light design, and, as engineers discovered, peculiarly sensitive to high winds. Rather than resist them, as most bridges do, the Tacoma Narrows tended to sway and vibrate. On November 7, in a 40-mile-per-hour wind, the center span began to sway, then twist. The combined force of the winds and internal stress was too great for the bridge, and it self-destructed. No one was killed, as the bridge had been closed because of previous swaying. This is one of the best-known and most closely studied engineering failures, thanks in large part to the film and photographs that recorded the collapse.



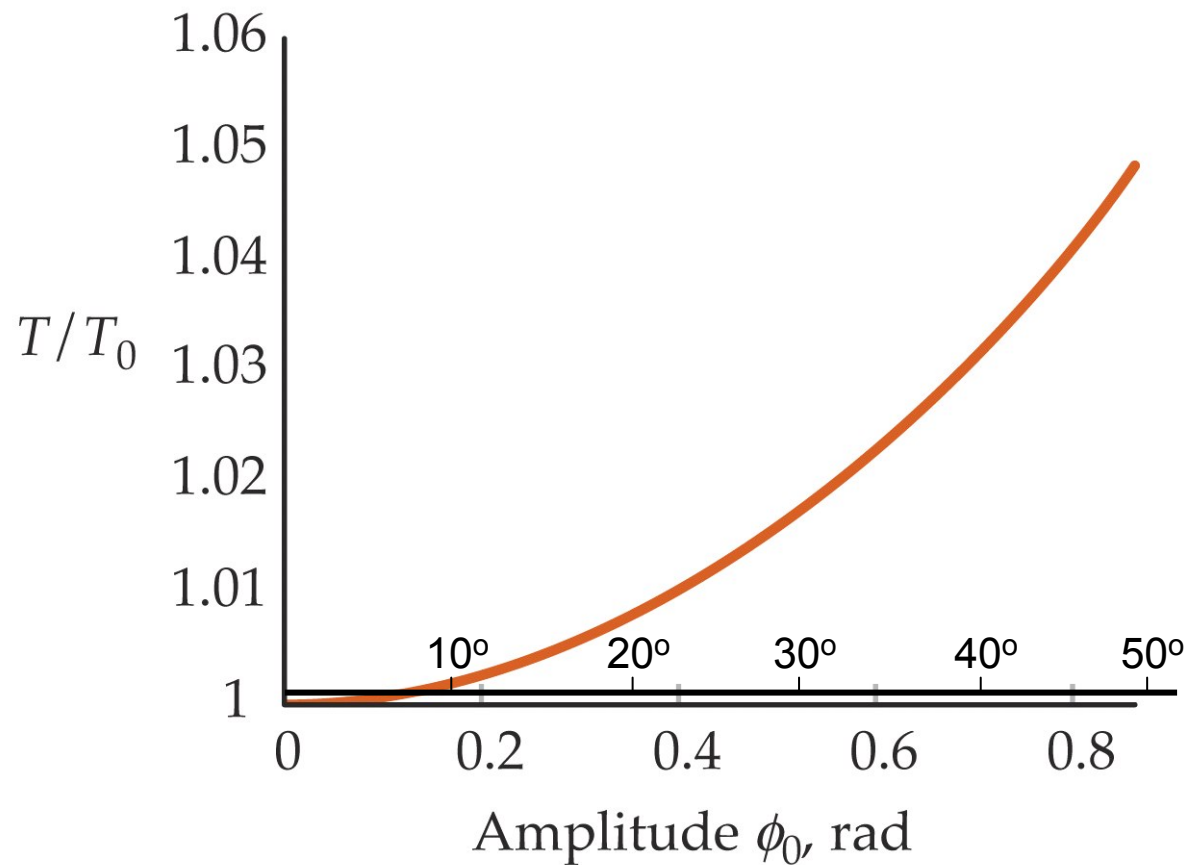
Matematisk pendel

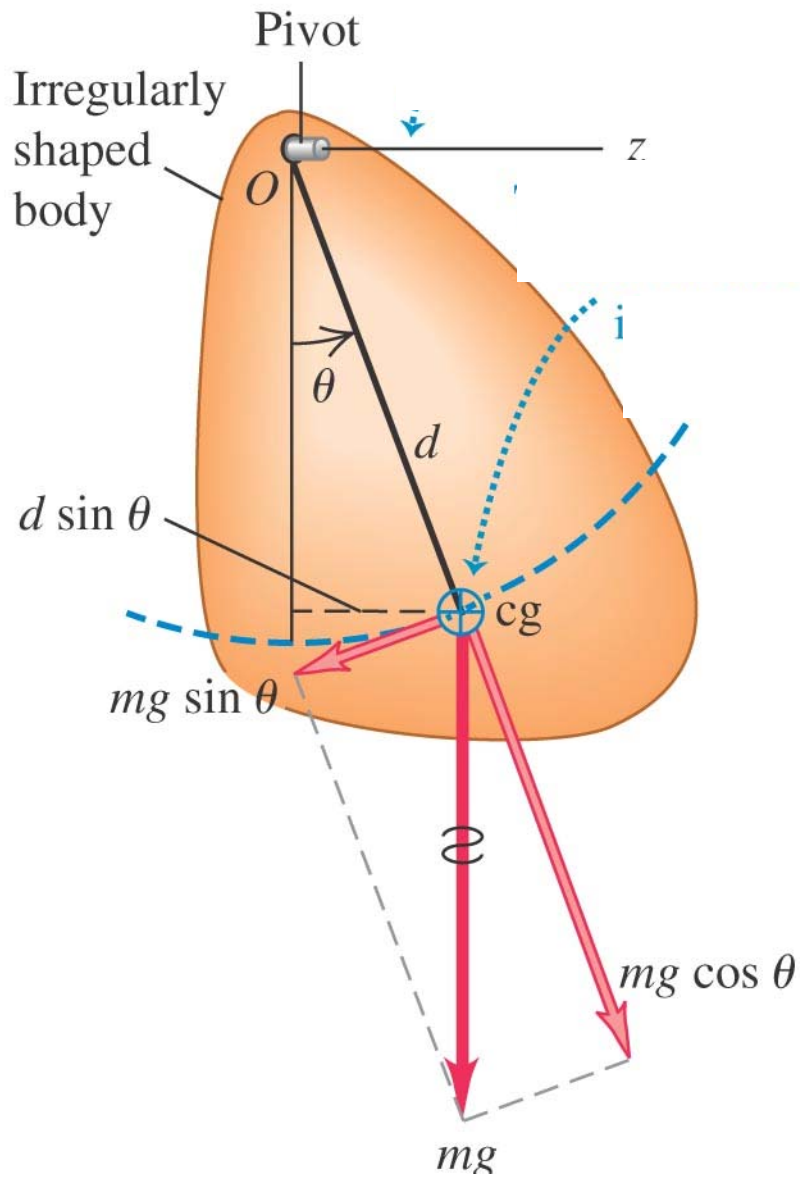
$$\omega^2 = g/L \quad T = 2\pi/\omega$$

Periode ved "store" vinkelamplituder Φ_0 :

$$T = T_0 \left[1 + \frac{1}{2^2} \sin^2 \frac{1}{2} \phi_0 + \frac{1}{2^2} \left(\frac{3}{4} \right)^2 \sin^4 \frac{1}{2} \phi_0 + \dots \right] \quad \approx (13.35)$$

$\Phi \rightarrow \theta$





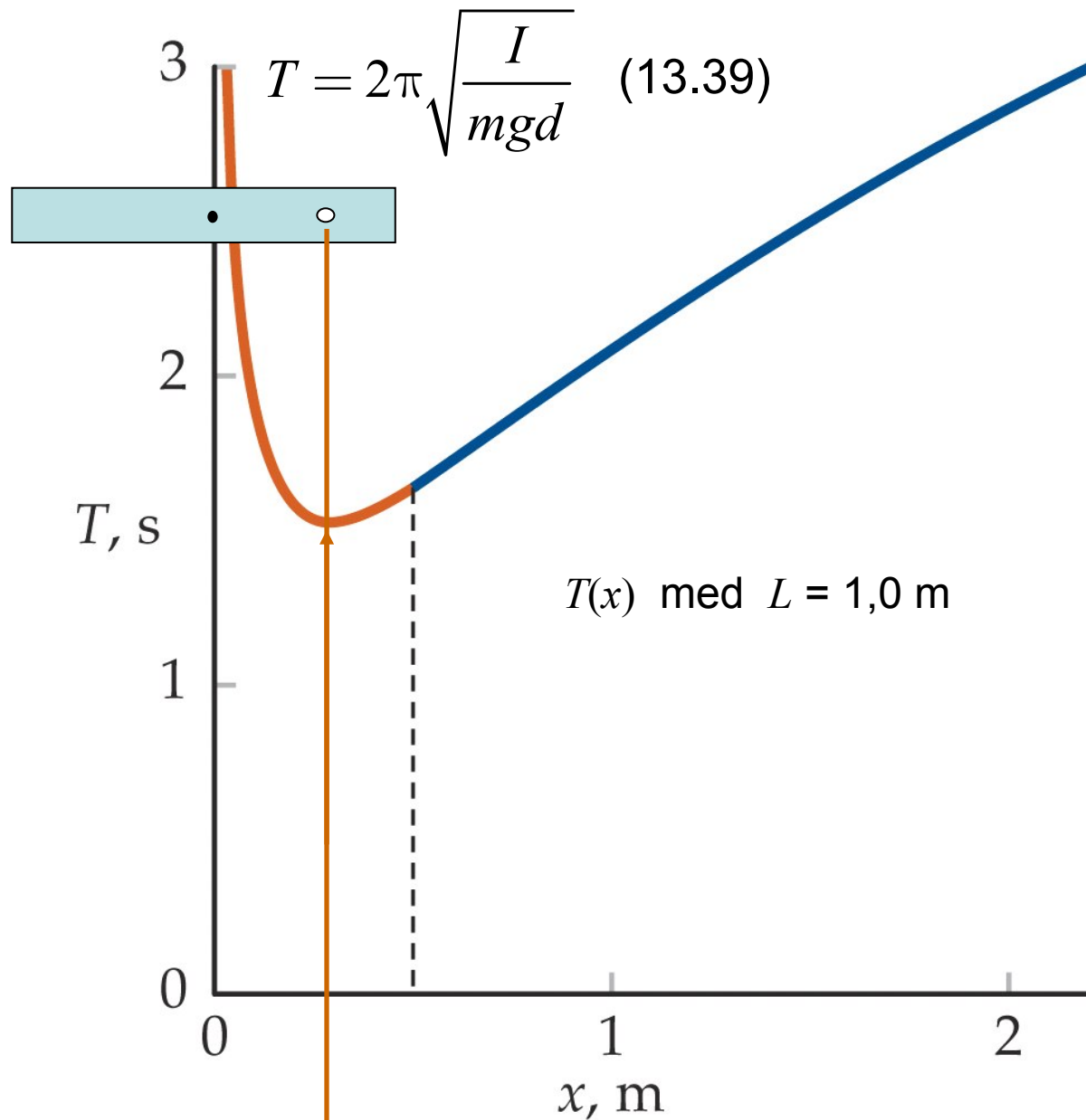
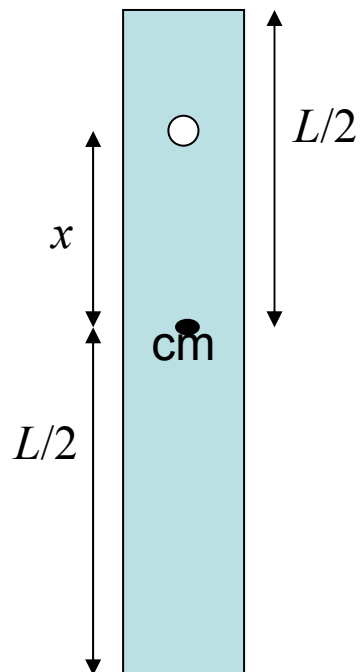
Fysisk pendel

$$\omega^2 = mgd/I$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{I}{mgd}}$$

(Y&F Fig. 13.23)

Svingende fjøl



Minimum ved $x = L/\sqrt{12} = R_G$

Torsjonssvingninger

(Y&F Ch.13.4: Angular SHM)

Dreiemoment = - (torsjonsstivhet) · (vinkel)

$$\tau = - \kappa \cdot \theta$$

+ spinnsatsen:

$$\tau = I \frac{d^2}{dt^2} \theta$$

= harmonisk oscillator:

$$\frac{d^2}{dt^2} \theta + \omega^2 \theta = 0$$

der

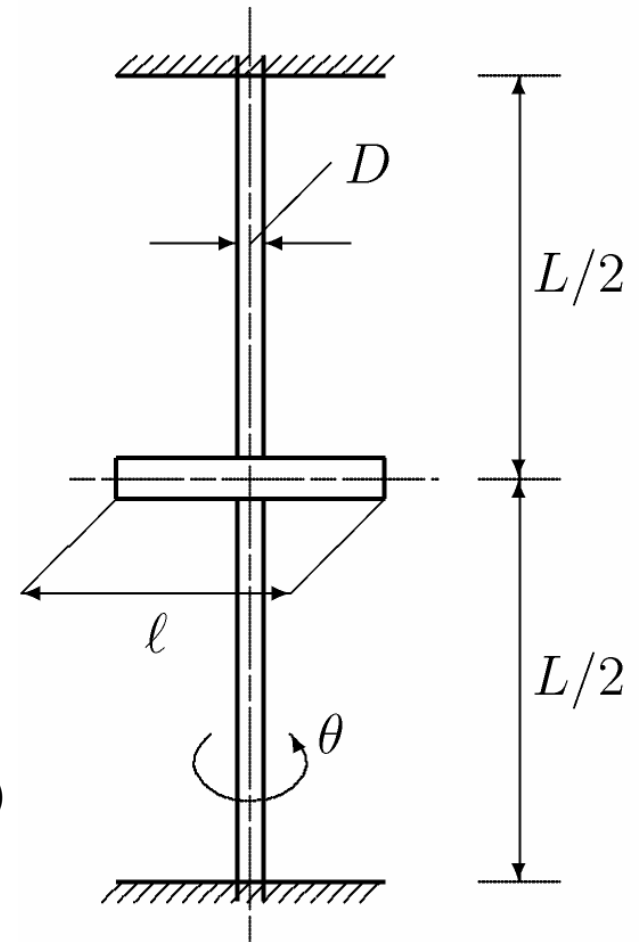
$$\omega^2 = \kappa / I$$

$$I = (1/12) M l^2$$

(tverrstavens treghetsmoment)

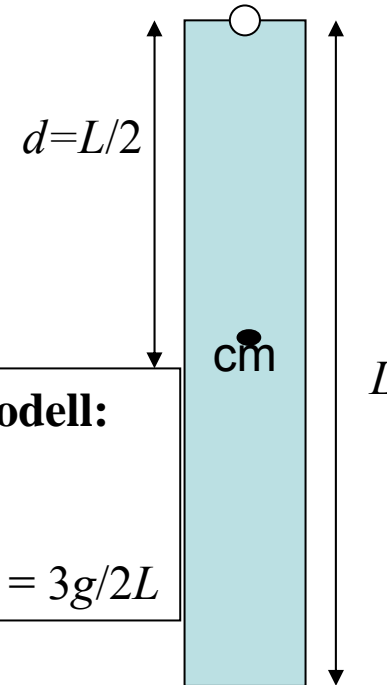
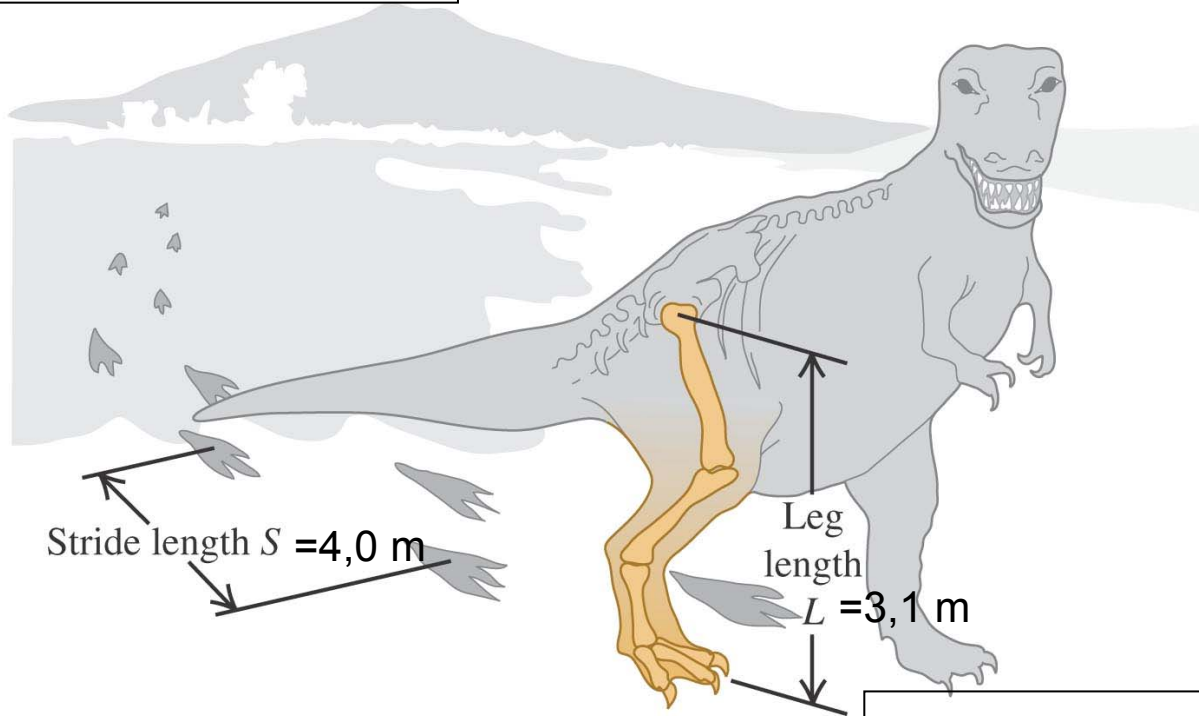
$$\kappa = 2 \cdot [\pi/32 \cdot \mu \cdot D^4 / (L/2)]$$

(trådenes torsjonsstivhet)



Y&F: Ex. 13.10

Fotpendling



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Enkel modell:

$$I = (1/3) ML^2$$

$$d = L/2$$

$$\Rightarrow \omega^2 = Mgd/I = 3g/2L$$

$$T = 2\pi/\omega \approx 2,9 \text{ s}$$

$$v = S/T = 5,0 \text{ km/h}$$

$$T \text{ prop. med } \sqrt{L}, \quad S \text{ prop. med } L \Rightarrow v = S/T \text{ prop. med } \sqrt{L}$$

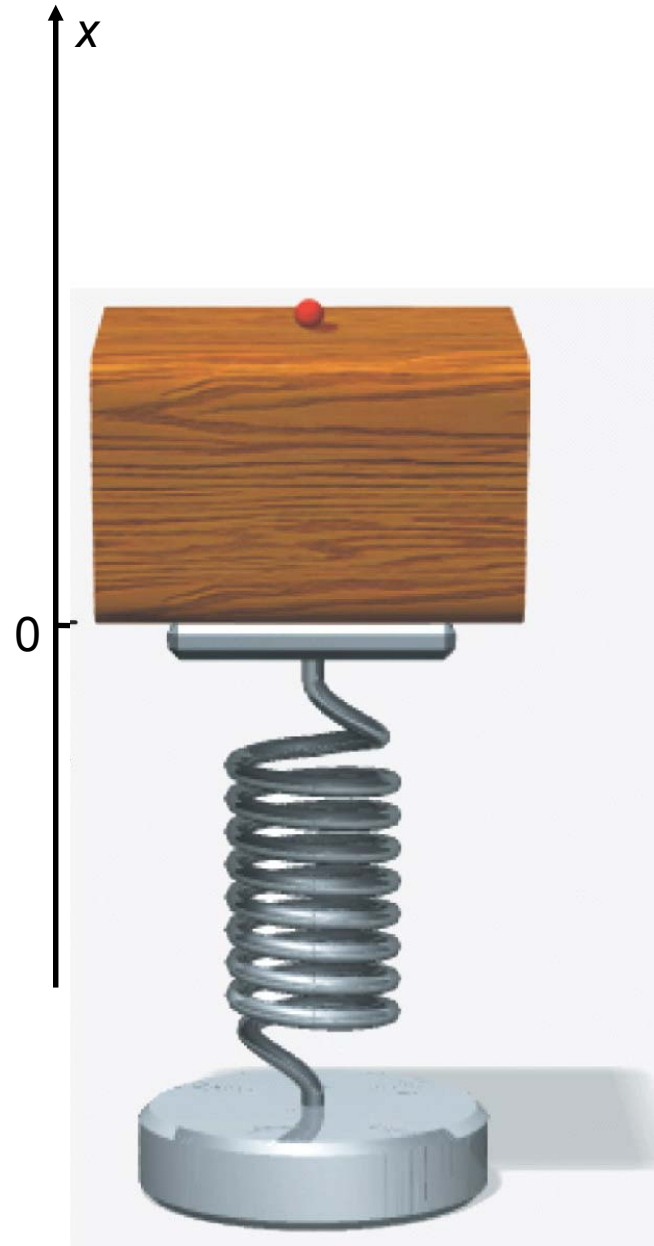
Eksempel:

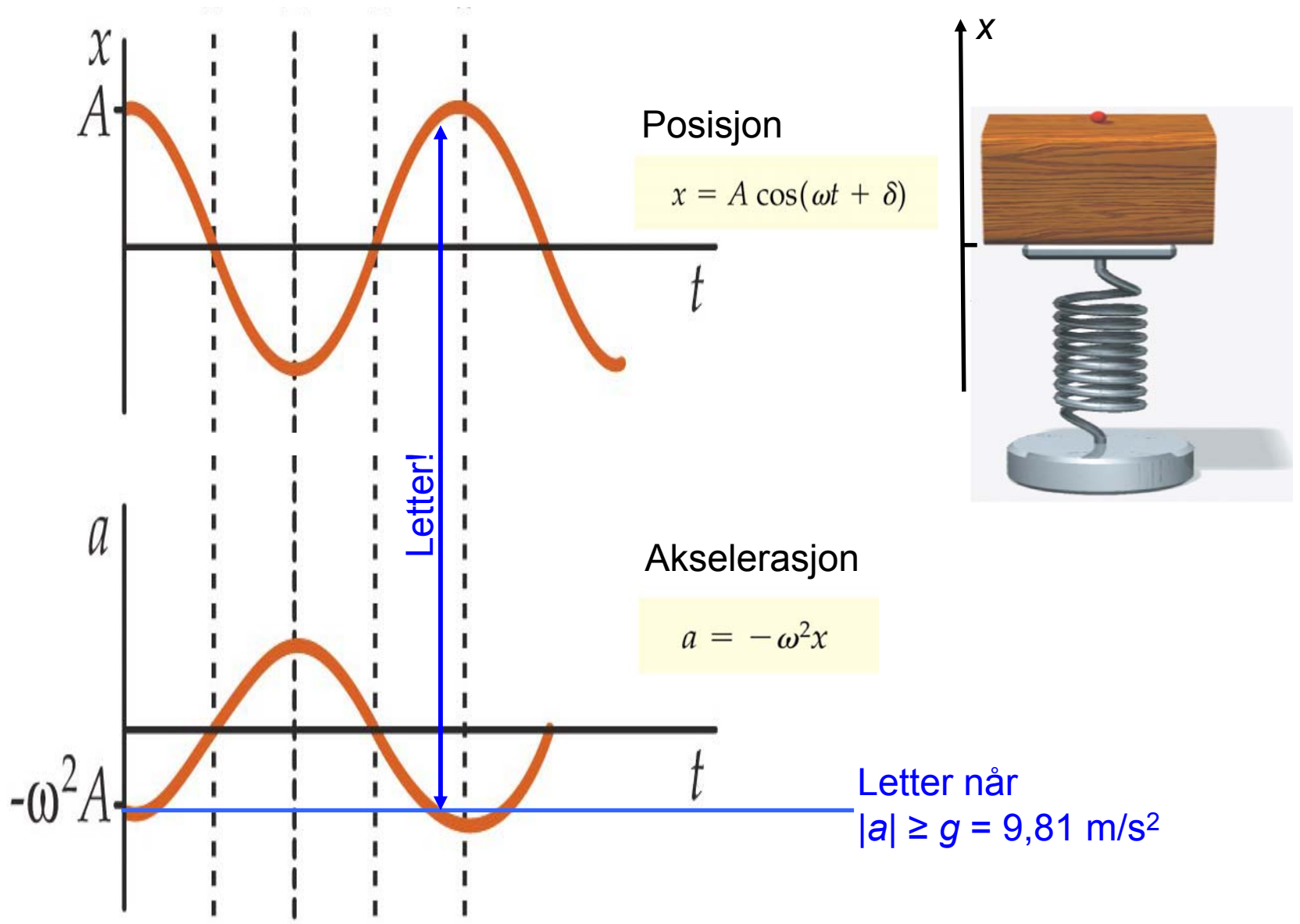
Svingning:

$$f = 1,0 \text{ Hz}$$

Ved hvilken
amplitude x
letter kula?

Letter når klossens
akselerasjon **nedover**
er større enn $9,81 \text{ m/s}^2$





Kap. 13 Udempede svingninger

- **Udempet harmonisk oscillasjon**

$$d^2/dt^2 x + \omega^2 x = 0$$

x -komponent av roterende bevegelse med vinkelhastighet ω :

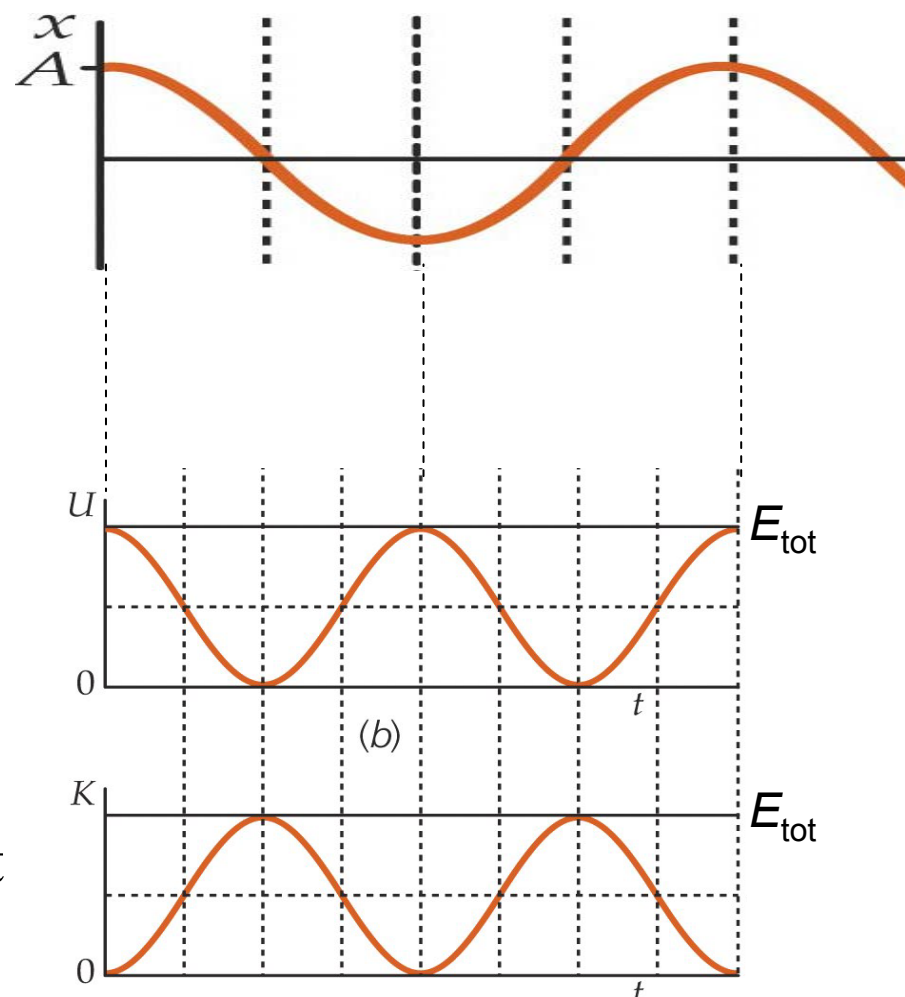
$$x(t) = A \cos(\omega t + \varphi)$$

- **Eksempler:**

- Fjærpendel $\omega^2 = k / m$
- Matematisk pendel $\omega^2 = g / l$
- Fysisk pendel $\omega^2 = mgd / I$
- Torsjonspendel $\omega^2 = \kappa / I$

- **Energi:**

- $E_p(t) = \frac{1}{2} k A^2 \cos^2(\omega t + \varphi)$
- $E_k(t) = \frac{1}{2} m v^2 \sin^2(\omega t + \varphi)$
- $E_{\text{tot}} = E_k(t) + E_p(t)$
 $= \frac{1}{2} k A^2 = \frac{1}{2} m \omega^2 A^2 = \text{konst}$



Kap. 13 Udempede svingninger

Kriterium for harmonisk oscillasjon (SHM):

Krafta som trekker mot likevekt er prop. med avstand x (eks. $F = -kx$)

Dette gir:

1. $d^2/dt^2 x + \omega^2 x = 0$ - fra (N2)

2. $E_p(t)$ prop. med x^2

Fjærpendel: $E_p(t) = \frac{1}{2} k x^2$

Tyngdependel $E_p(t) = mgh$
 $= mgL(1 - \cos\theta)$
 $\approx mgL/2 \cdot \theta^2$

Totalenergien $E_{\text{tot}} = E_k(t) + E_p(t)$ er konstant og svinger mellom $E_k(t)$ og $E_p(t)$