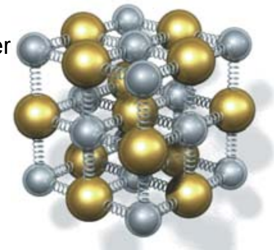
Kap. 13
Udempede svingninger

- **Mye svingning i dagliglivet:**
  - Pendler
  - Musikkinstrument
  - Elektriske og magnetiske svingninger
  - Klokker
  - Termiske vibrasjoner (= temper
  - Måner og planeter
  - Historien og økonomien
  - m.m.
  - Farlige svingninger:



Tacoma Narrows Bridge on the morning of Nov. 7, 1940. The bridge was an unusually light design, and, as engineers discovered, peculiarly sensitive to high winds. Rather than resist them, as most bridges do, the Tacoma Narrows tended to sway and vibrate. On November 7, in a 40-mile-per-hour wind, the center span began to sway, then twist. The combined force of the winds and internal stress was too great for the bridge, and it self-destructed. No one was killed, as the bridge had been closed because of previous swaying. This is one of the best-known and most closely studied engineering failures, thanks in large part to the film and photographs that recorded the collapse.

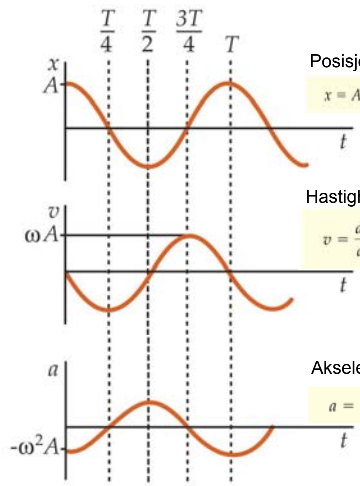
Kap. 13
Udempede svingninger

- **Enkel harmonisk oscillasjon (SHM):**

  $d^2/dt^2\, x + \omega^2 x = 0$ (13.4)

  med løsning: $x(t) = A \cos(\omega t + \varphi)$ (13.13)
- Beskrivelse med roterende vektor
- Energi for SHM
- Eksempler:
  - Fjærpendel $\omega^2 = k/m$
  - Matematisk pendel
  - Fysisk pendel
  - Torsjonspendel

I dag



Posisjon

$x = A\cos(\omega t + \delta)$ (13.13)

δ = fasevinkel, φ hos oss
$\delta = 0$ i figuren

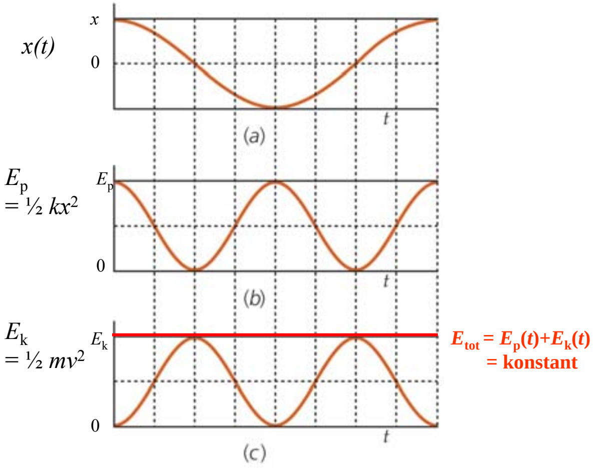
Hastighet

$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \delta)$ (13.15)

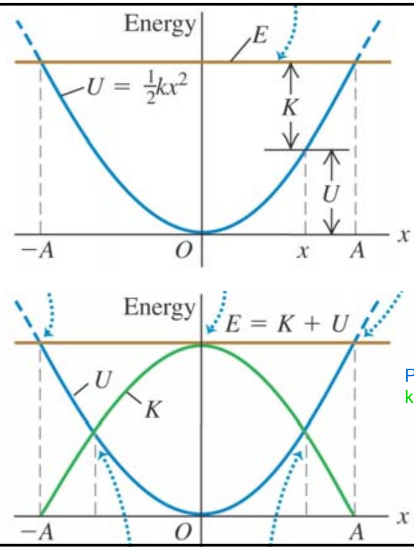
Akselerasjon

$a = -\omega^2 x$ (13.16)

(Fjærpendel: $\omega^2 = k/m$)



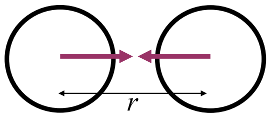
Energi i SHM (Simple Harmonic Motion)

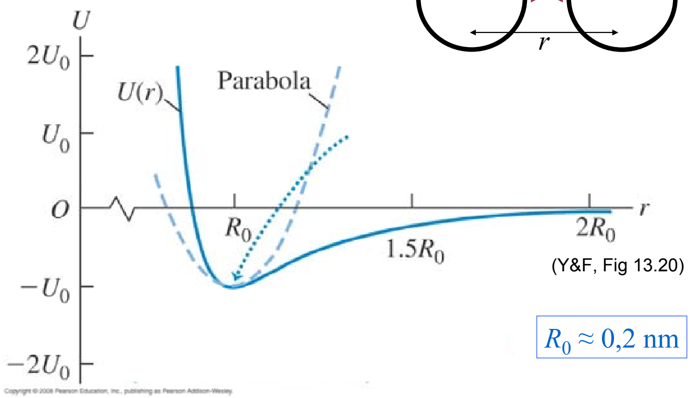


Pot. energi

Pot. energi og kinetisk energi

Energifunksjonen $U(x) = E_p(x)$ kan ofte bare være bare ***tilnærmet*** parabolsk.
Eks 1: van der Waals tiltrekning mellom to atomer :

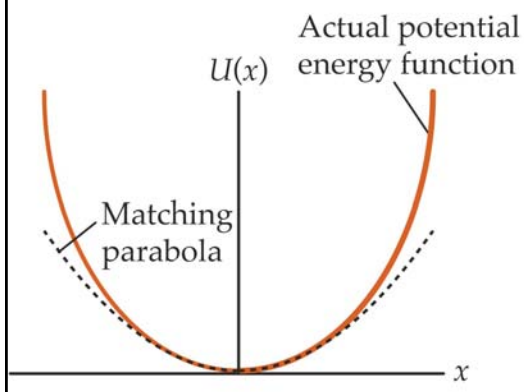


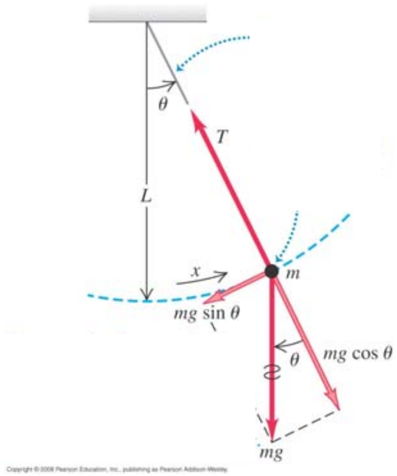


(Y&F, Fig 13.20)

$R_0 \approx 0{,}2$ nm

Energifunksjonen $U(x) = E_p(x)$ kan ofte bare være ***tilnærmet*** parabolsk.
Eks.2: Trille kule inni halvkuleformet bolle:





Matematisk pendel

$\omega^2 = g/L \qquad T = 2\pi/\omega$

Feil ved f.eks. 30°:

sin 30° = sin π/6 = 0,500
π/6 = 0,524

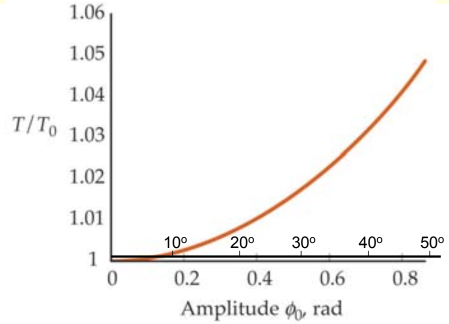
sin θ = θ feil med 5 %

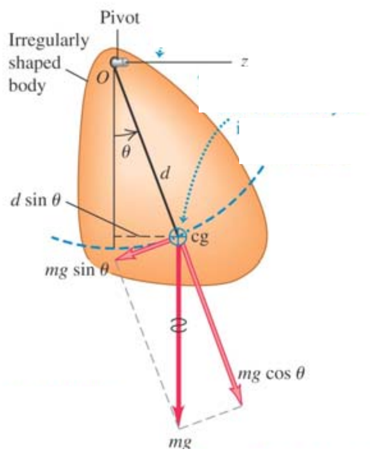
Matematisk pendel $T_0 = 2\pi\sqrt{g/L}$

Periode ved "store" vinkelamplituder Φ_0:

$$T = T_0\left[1 + \frac{1}{2^2}\sin^2\frac{1}{2}\phi_0 + \frac{1}{2^2}\left(\frac{3}{4}\right)^2\sin^4\frac{1}{2}\phi_0 + \cdots\right]$$

≈ (13.35)
($\Phi \to \theta$)





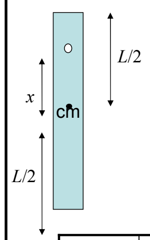
Fysisk pendel

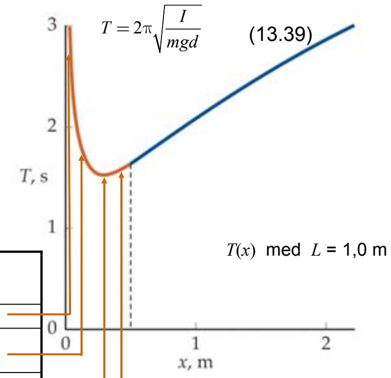
$\omega^2 = mgd/I$

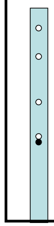
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}}$$

(Y&F Fig. 13.23)

Svingende fjøl







hull	x/mm	beregnet T/s	målt T/s
≈sentrum	15	4,40	4,3
nedre	153	1,60	1,60
midtre	272 = R_G	1,48	1,50
øvre	428	1,56	1,56

Torsjonssvingninger

(Y&F Ch.13.4: Angular SHM)

Dreiemoment = - (torsjonsstivhet) ·(vinkel)

$\tau = -\kappa \cdot \theta$

+ spinnsatsen:

$\tau = I\, d^2/dt^2\, \theta$

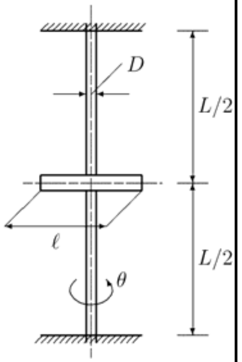
= harmonisk oscillator:

$d^2/dt^2\theta + \omega^2\theta = 0$

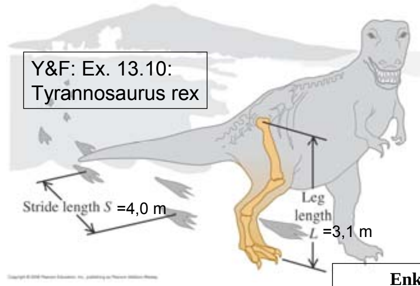
der $\omega^2 = \kappa / I$

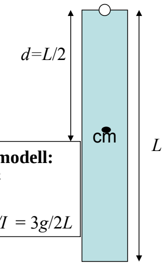
$I = (1/12)\ M\ell^2$ (tverrstavens treghetsmoment)

$\kappa = 2 \cdot [\pi/32 \cdot \mu \cdot D^4/(L/2)]$ (trådenes torsjonsstivhet)



Fotpendling





Enkel modell:

$I = (1/3)\ ML^2$

$d = L/2$

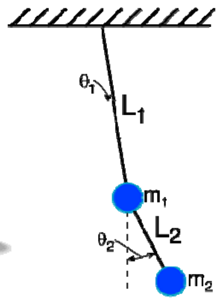
=> $\omega^2 = Mgd/I = 3g/2L$

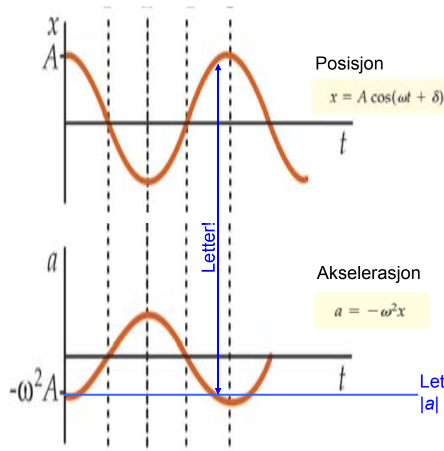
$T = 2\pi/\omega \approx 2{,}9$ s

$v = S/T = 5{,}0$ km/h

T prop. med $\sqrt{L}$, S prop. med L => $v = S/T$ prop.med $\sqrt{L}$

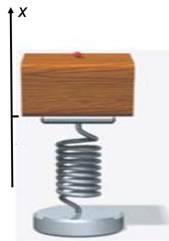
Føttene er en dobbel-pendel





Posisjon

$x = A\cos(\omega t + \delta)$



Akselerasjon

$a = -\omega^2 x$

Letter når $|a| \geq g = 9{,}81$ m/s^2

Kap. 13 Udempede svingninger

- **Udempet harmonisk oscillasjon**

  $d^2/dt^2\ x\ + \omega^2 x\ = 0$

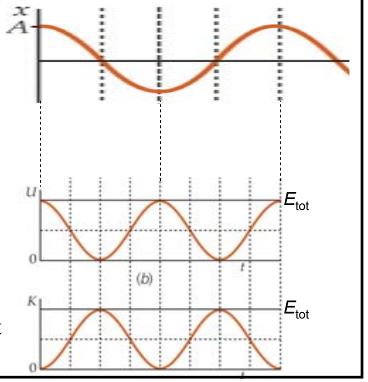
  x-komponent av roterende bevegelse med vinkelhastighet ω :

  $x(t) = A\ \cos(\omega t + \varphi\ )$

- **Eksempler:**
  - Fjærpendel $\omega^2 = k\,/\,m$
  - Matematisk pendel $\omega^2 = g\,/\,l$
  - Fysisk pendel $\omega^2 = mgd\,/\,I$
  - Torsjonspendel $\omega^2 = \kappa\,/\,I$

- **Energi:**
  - $E_p(t) = ½\ k\ A^2 \cos^2(\omega t + \varphi\ )$
  - $E_k(t) = ½\ mv^2\ \sin^2(\omega t + \varphi\ )$
  - $E_{tot} = E_k(t) + E_p(t)$

    $= ½\ k\ A^2 = ½\ m\ \omega^2\ A^2 = \text{konst}$



Kap. 13 Udempede svingninger

Kriterium for harmonisk oscillasjon (SHM):

Krafta som trekker mot likevekt

er prop. med avstand x (eks. $F = -\,kx$)

Dette gir:

1. $d^2/dt^2\ x\ + \omega^2 x\ = 0$ - fra (N2)
2. $E_p(t)$ prop. med x^2

   Fjærpendel: $E_p(t)\ = ½\ k\ x^2$

   Torsjonspendel: $E_p(t)\ = ½\ \kappa\ \theta^2$

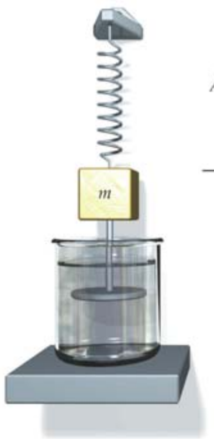
   Tyngdependel $E_p(t)\ = mgh$

   $= mgL(1 - \cos\theta)$

   $\approx mgL/2 \cdot \theta^2$

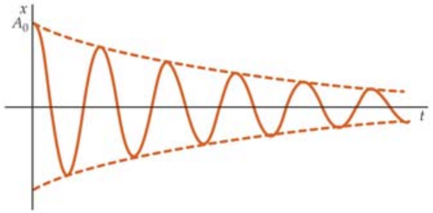
Totalenergien $E_{tot} = E_k(t) + E_p(t)$ er konstant og svinger mellom $E_k(t)$ og $E_p(t)$

Dempet svingning



Svingelikning:

$d^2/dt^2\ x\ + 2\delta\ d/dt\ x\ + \omega_0^2\ x\ = 0$



$x(t) = A\ \ e^{-\delta t}\ \cos(\omega_d\ t + \theta_0)$

(svakt dempet)

Dempet frekvens: $\omega_d^2\ = \omega_0^2\ - \delta^2$