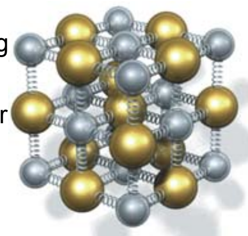
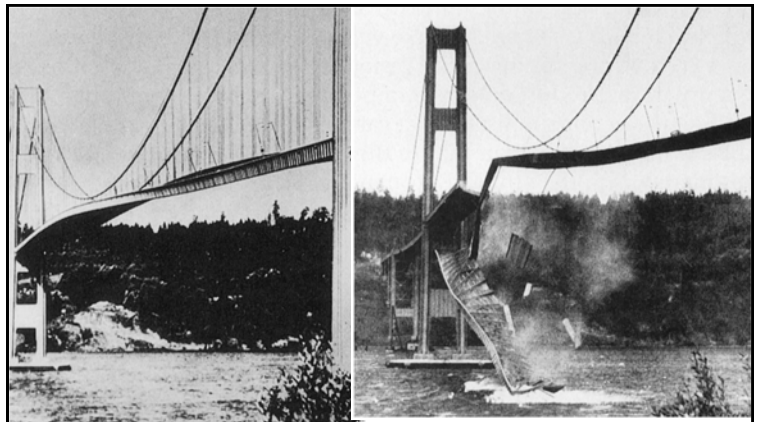
Kap. 14
Mekaniske svingninger

- **Mye som svinger i dagliglivet:**
  - Pendler
  - Musikkinstrument
  - Elektriske og magnetiske svingning
  - Klokker
  - Termiske vibrasjoner (= temperatur
  - Måner og planeter
  - Historien og økonomien
  - m.m.
  - Farlige svingninger:





Tacoma Narrows Bridge on the morning of Nov. 7, 1940. The bridge was an unusually light design, and, as engineers discovered, peculiarly sensitive to high winds. Rather than resist them, as most bridges do, the Tacoma Narrows tended to sway and vibrate. On November 7, in a 40-mile-per-hour wind, the center span began to sway, then twist. The combined force of the winds and internal stress was too great for the bridge, and it self-destructed. No one was killed, as the bridge had been closed because of previous swaying. This is one of the best-known and most closely studied engineering failures, thanks in large part to the film and photographs that recorded the collapse. Full video: http://www.youtube.com/watch?v=j-zczJXSxnw

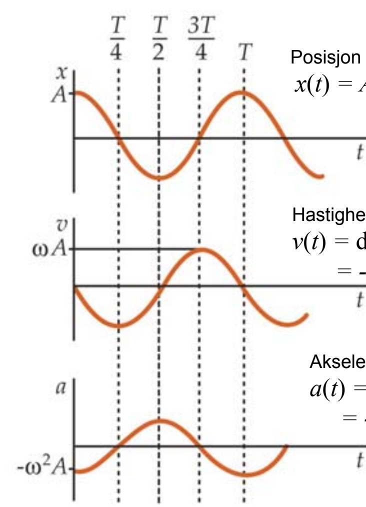
14. Mekaniske svingninger

= Svingning mellom potensiell energi og kinetisk energi.

- Vi skal se på:
- 14.1-6. Udempet harmonisk svingning

  $x(t) = A \cos(\omega_0 t + \varphi)$
- 14.7. Dempet svingning

  $x(t) = A e^{-\gamma t} \cdot \cos(\omega_d t + \varphi)$
- 14.8. Tvungen svingning (resonans)
- Eksempler:
  - Fjærpendel
  - Matematisk pendel
  - Fysisk pendel
- Y & F: Kap. 14
- L & L: Kap 9 (ikke 9.5, 9.8, 9.10, 9.11)



Posisjon

$x(t) = A \cos(\omega_0 t + \varphi)$ (14.13)

φ = fasevinkel, (= 0 i grafen)

Hastighet

$v(t) = \mathrm{d}x/\mathrm{d}t$

$= - A\omega_0 \sin(\omega_0 t + \varphi)$ (14.15)

Akselerasjon

$a(t) = \mathrm{d}v/\mathrm{d}t$

$= - A\omega_0^2 \cos(\omega_0 t + \varphi) = -\omega_0^2 x$ (14.16)

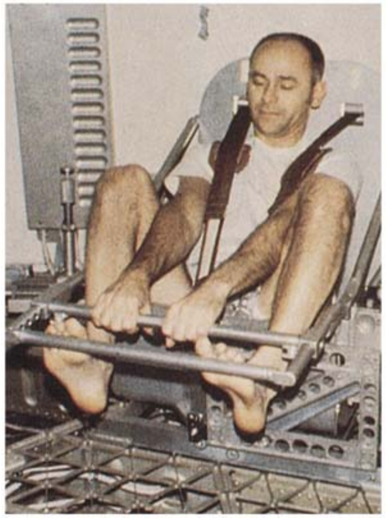
(Fjærpendel: $\omega_0^2 = k/m$)

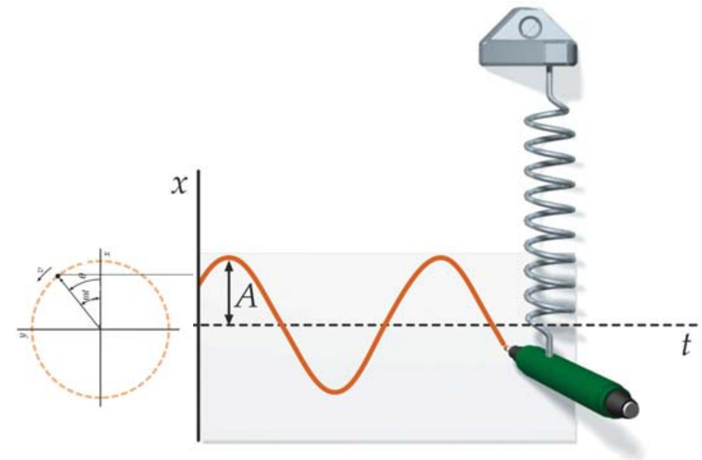
I romfartøy:
Tyngden mangler.

Måling av masse ved SHM:

$m = k / \omega_0^2$

Kjent k
Måler ω_0

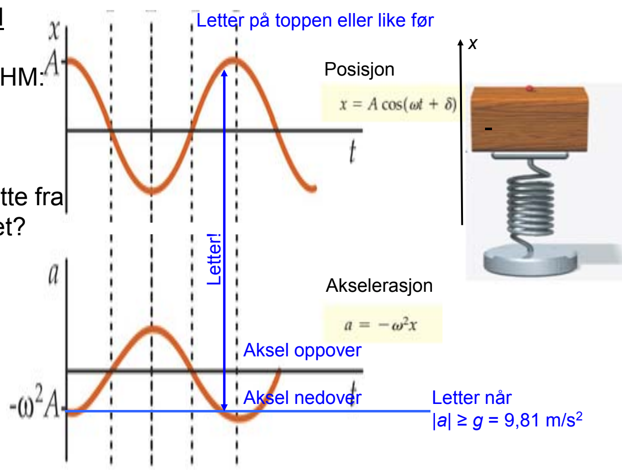




Eksempel

Vertikal SHM:

Vil kula lette fra underlaget?



Letter når klossens akselerasjon **nedover** er større enn 9,81 m/s^2

Fra en eksamen (Fysmat des. 2009) (flervalgsoppgave)

- Ei pakke vaskemiddel står oppå en vaskemaskin som er i ferd med å sentrifugere på 1200 omdreininger per minutt. Vaskemaskinen vibrerer dermed vertikalt med en amplitude på 1,0 mm. Vil vaskemiddelpakka på noe tidspunkt miste kontakten med underlaget? Hvorfor, evt. hvorfor ikke?

Mulige svar

A. Ja, fordi vaskemaskinens maksimale akselerasjon overstiger 9,8 m/s^2.
B. Ja, fordi vaskemaskinens maksimale hastighet overstiger 9,8 m/s.
C. Nei, fordi vaskemaskinens maksimale akselerasjon aldri overstiger 9,8 m/s^2.
D. Nei, fordi vaskemaskinens maksimale hastighet aldri overstiger 9,8 m/s.
E. Nei, fordi vaskemaskinens maksimale vertikale utsving aldri overstiger 9,8 mm.

SHM:
$\omega = 2\pi f = 2\pi \cdot 1200/60\ 1/\text{s} = 40\pi\ 1/\text{s}$

$x = A\cos(\omega t) \Rightarrow a = d^2x/dt^2 = -\omega^2 A\cos(\omega t)$

$a_{max} = \omega^2 A = (40\pi)^2 \cdot 0{,}001\ \text{m} = 15{,}8\ \text{m/s}^2 > g \Rightarrow$ Alt. A

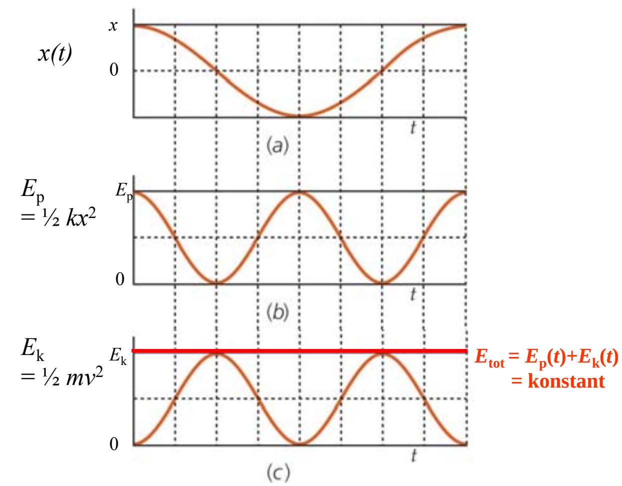
Fra eksamen (Fysmat des. 2003) Horisontal svingning.

Oppgave 4
En pakke med masse m er plassert på en horisontal plattform som svinger harmonisk langs bakken med periode T. Friksjonskoeffisienten mellom pakken og plattformen er μ og tyngdens akselerasjon er g. Svingeamplituden A økes nå langsomt (med konstant T).
Ved hvilken amplitude A_0 begynner pakken å skli? (Forsøk med en mynt på et papirark.)

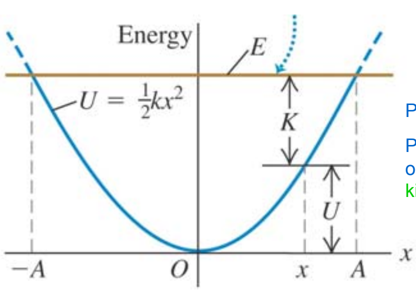
Friksjonsbegrenset

Pakken akselereres av friksjonskrafta som er max $F_f = \mu mg$,
dvs. dens maksimale akselerasjon den kan følge er
$a_{max} = F_f / m = \mu g$.

Underlagets akselerasjon er
$a = -\omega^2 A \cos(\omega t)$, dvs. amplitudeverdi $\omega^2 A = (2\pi/T)^2 A$
Sammenholdt: $\mu g = (2\pi/T)^2 A$
=> $A_0 = \omega^2 A = \mu g\, (T/2\pi)^2$



Energi i SHM (Simple Harmonic Motion)



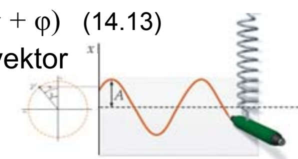
Pot. energi $U = E_p$

Pot. energi $U = E_p$
og
kinetisk energi $K = E_k$

14.1-6 Udempede svingninger

- **Enkel harmonisk oscillasjon (SHM):**
  $d^2/dt^2\, x + \omega_0^2\, x = 0$ (14.4)
  med løsning: $x(t) = A \cos(\omega_0 t + \varphi)$ (14.13)
- Beskrivelse med roterende vektor
- Energi for SHM
  $E_{tot} = E_p(t)+E_k(t)$ = konstant
- Eksempler:
  - Fjærpendel $\omega_0^2 = k / m$
  - Matematisk pendel $\omega_0^2 = g / l$
  - Fysisk pendel $\omega_0^2 = mgd / I$

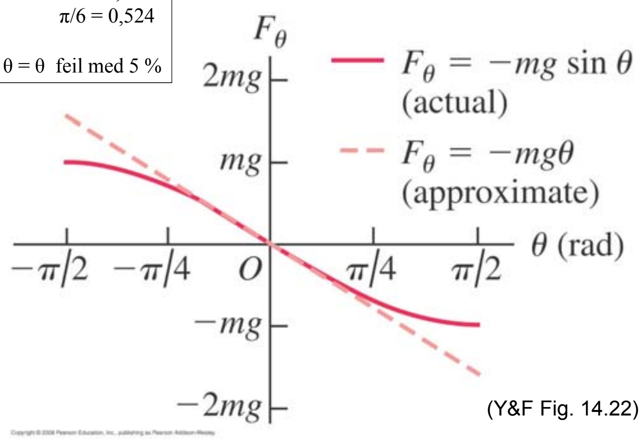
I dag



Feil ved f.eks. 30°:

$\sin 30^{\circ} = \sin \pi/6 = 0{,}500$
$\pi/6 = 0{,}524$

=> $\sin\theta = \theta$ feil med 5 %



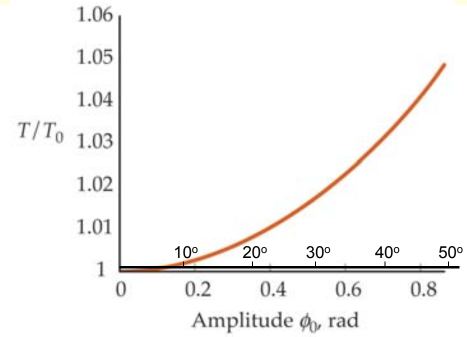
(Y&F Fig. 14.22)

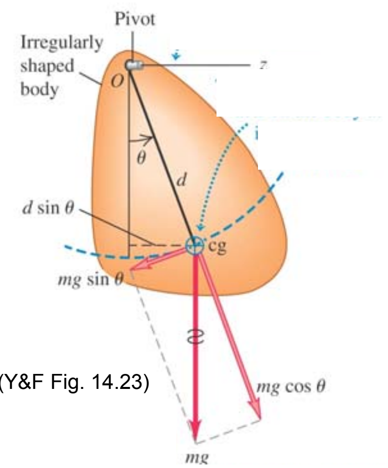
Matematisk pendel $T_0 = 2\pi\sqrt{(L/g)}$

Periode ved "store" vinkelamplituder Φ_0:

$$T = T_0\left[1 + \frac{1}{2^2}\sin^2\frac{1}{2}\phi_0 + \frac{1}{2^2}\left(\frac{3}{4}\right)^2\sin^4\frac{1}{2}\phi_0 + \cdots\right]$$

≈ (14.35)
($\Phi \to \theta$)





(Y&F Fig. 14.23)

Fysisk pendel.

N2-rot gir:

$$d^2/dt^2\,\theta + \omega_0^2\,\theta = 0$$

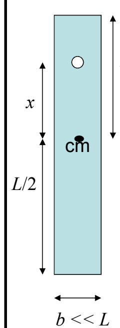
der $\omega_0^2 = mgd/I$

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{I}{mgd}}$$

Y&F kap. 14.6,
LL Eks. 9.2.

Anbefalte opg:
YF 14.57, LL 9.3

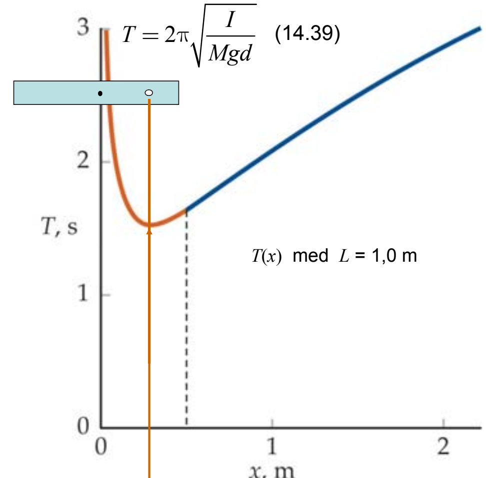
Svingende fjøl



$I = I_{cm} + Mx^2$

$I_{cm} = 1/12\, M\,(L^2+b^2)$
$\approx 1/12\, M\, L^2$

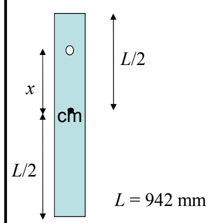
$$T = 2\pi\sqrt{\frac{I}{Mgd}} \quad (14.39)$$

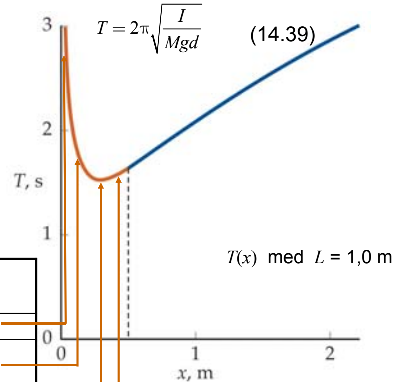


Minimum ved $x = L/\sqrt{12} = R_G$

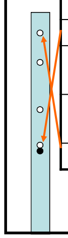
Svingende fjøl

Øving 13

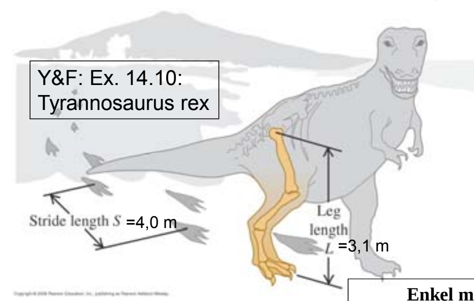


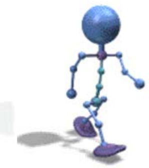


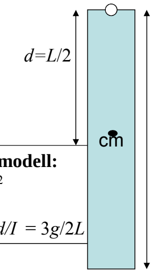
hull	x/mm	beregnet T/s	målt T/s
≈sentrum	15	4,40	4,0
nedre	153	1,60	1,54
midtre	272 = R_G	1,48	1,46
øvre	428	1,56	1,56



Fotpendling







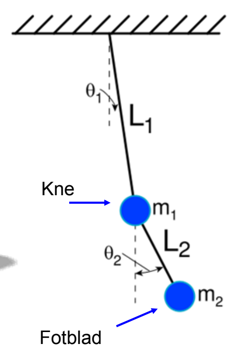
Enkel modell:
$I = (1/3)\, ML^2$
$d = L/2$
$\Rightarrow \omega_0^2 = Mgd/I = 3g/2L$

$T = 2\pi/\omega_0 \approx 2{,}9$ s

$v = S/T = 5{,}0$ km/h

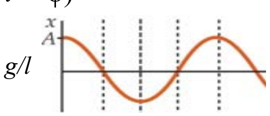
T prop. med $\sqrt{L}$, S prop. med L => $v = S/T$ prop.med $\sqrt{L}$

Føttene er en dobbel-pendel

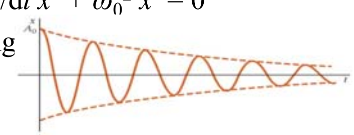


14 Mekaniske svingninger

- **14.1-6 Udempet harmonisk oscillasjon (SHM)**
  $d^2/dt^2\, x + \omega_0^2\, x = 0$
  med løsning: $x(t) = A \cos(\omega_0 t + \varphi)$
  - masse/fjær: $\omega_0^2 = k/m$
  - tyngependel (matematisk): $\omega_0^2 = g/l$
  - fysisk pendel: $\omega_0^2 = mgd/I$



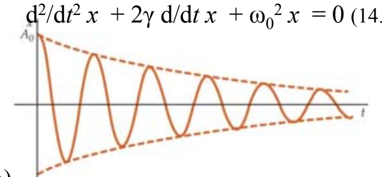
- **14.7 Dempet harmonisk oscillasjon**
  $d^2/dt^2\, x + 2\gamma\, d/dt\, x + \omega_0^2\, x = 0$
  med løsning



- **14.8 Tvungen svingning (resonans)**

14.7. Dempet svingning

Svingelikning: $d^2/dt^2\, x + 2\gamma\, d/dt\, x + \omega_0^2\, x = 0$ (14.41)



$\gamma < \omega_0$ svak dempet:

$$x(t) = Ae^{-\gamma t}\cos(\omega_d t + \varphi)$$

$$\omega_d = \sqrt{\omega_0^2 - \gamma^2}$$

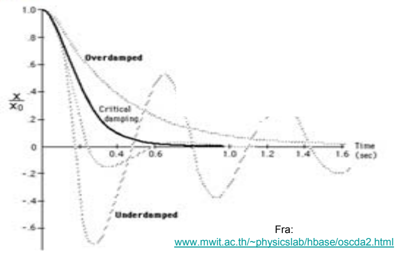
$\gamma = \omega_0$ kritisk dempet:

$$x(t) = (A_1 + tA_2)e^{-\gamma t}$$

$\gamma > \omega_0$ overkritisk dempet:

$$x(t) = A_1 e^{-\alpha_1 t} + A_2 e^{-\alpha_2 t}$$

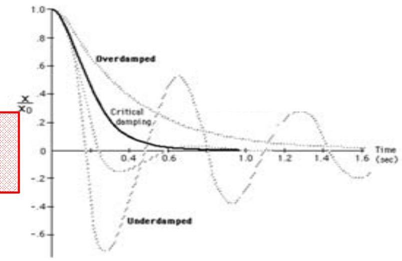
$$\alpha_1 = \gamma + \sqrt{\gamma^2 - \omega_0^2} \qquad \alpha_2 = \gamma - \sqrt{\gamma^2 - \omega_0^2}$$



Fra: www.mwit.ac.th/~physicslab/hbase/oscda2.html

14.7. Dempet svingning

Svingelikning: $d^2/dt^2\, x + 2\gamma\, d/dt\, x + \omega_0^2\, x = 0$ (14.41)



$\gamma < \omega_0$ svak dempet:

$$\boxed{x(t) = Ae^{-\gamma t}\cos(\omega_d t + \varphi) \qquad \omega_d = \sqrt{\omega_0^2 - \gamma^2}}$$

$\gamma = \omega_0$ kritisk dempet:

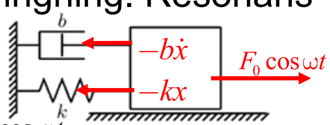
$$x(t) = (A_1 + tA_2)e^{-\gamma t}$$

$\gamma > \omega_0$ overkritisk dempet, alternative uttrykk:

$$x(t) = A_1 e^{-\gamma t}e^{-\beta t} + A_2 e^{-\gamma t}e^{\beta t} \qquad \beta = \sqrt{\gamma^2 - \omega_0^2}$$

$$\boxed{x(t) = Ae^{-\gamma t}\cosh(\beta t + \varphi) \qquad \beta = \sqrt{\gamma^2 - \omega_0^2}}$$

14.8. Tvungen svingning. Resonans



Svingelikning:

$$d^2/dt^2\, x + 2\gamma\, d/dt\, x + \omega_0^2\, x = f_0 \cdot \cos \omega t$$

Etter kort tid bestemmer pådraget frekvensen:

$$x(t) = A_0 \cos(\omega t - \delta)$$

Amplitude $A_0(\omega)$ og fase $\delta(\omega)$:

$$A_0 = \frac{f_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\gamma\omega)^2}} \qquad (14.46)$$

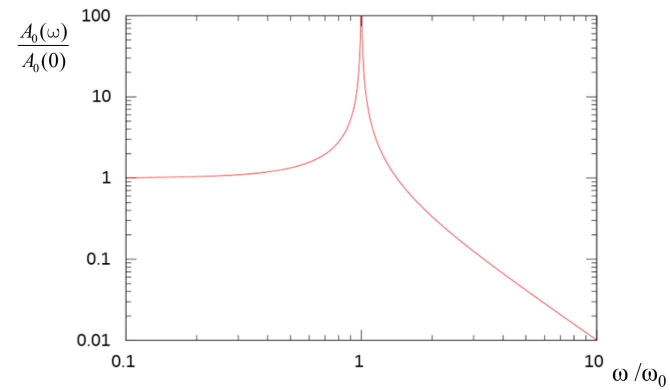
$$\tan\delta = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$

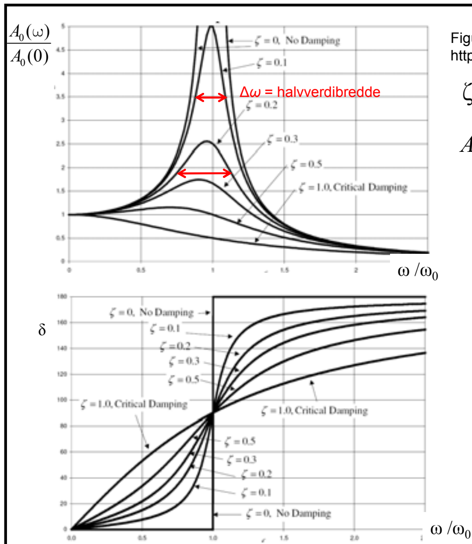
Resonans:

$$A_0(\max) = \frac{f_0}{2\gamma\sqrt{\omega_0^2 - \gamma^2}} \approx \frac{f_0}{2\gamma\omega_0} \quad \text{når} \quad \omega = \sqrt{\omega_0^2 - 2\gamma^2} \approx \omega_0$$

$$A_0 = \frac{f_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\gamma\omega)^2}} \quad \text{i log-log-plot}$$

med $\zeta = \gamma/\omega_0 = 1/200$ (svært svak demping)





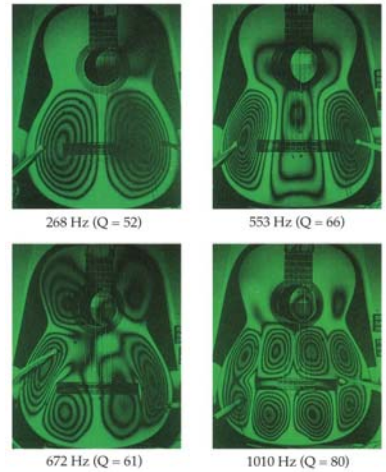
Figurer:
http://en.wikipedia.org/wiki/Vibration

$$\zeta = \gamma/\omega_0$$

$$A_0 = \frac{f_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\gamma\omega)^2}}$$

$$\tan\delta = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$

Svingninger (resonanser) i ei gitarkasse

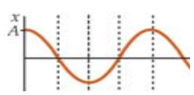


Q-faktor ved svingninger:
$Q = ½ \cdot 1/\zeta = ½ \cdot \omega_0/\gamma$

(stor Q = liten demping)

14. Mekaniske svingninger. Oppsummering 1

- **Udempet harmonisk oscillasjon (SHM)**
  Kriterium SHM: **Krafta som trekker mot likevekt er prop. med avstand x (eks. $F = -kx$)**



Dette gir fra Newton 2: $d^2/dt^2\, x + \omega_0^2\, x = 0$
med løsning: $x(t) = A \cos(\omega_0 t + \varphi)$

- masse/fjær: $\omega_0^2 = k/m$
- tyngependel (matematisk): $\omega_0^2 = g/l$
- fysisk pendel: $\omega_0^2 = mgd/I$

- **Dempet harmonisk oscillasjon**

$d^2/dt^2\, x + 2\gamma\, d/dt\, x + \omega_0^2\, x = 0$
med løsning: $x(t) = A\, e^{-\gamma t} \cdot \cos(\omega_d t + \varphi)$
(svak dempning $\gamma < \omega_0$) $\omega_d^2 = \omega_0^2 - \gamma^2$

14. Mekaniske svingninger. Oppsummering 2

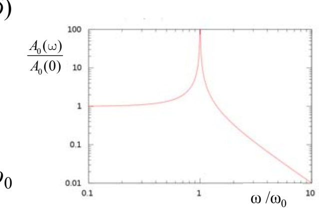
Tvungen svingning (resonans)

- $d^2/dt^2\, x + 2\gamma\, d/dt\, x + \omega_0^2\, x = F_0/m \cdot \cos\omega t$
  med løsning $x(t) = A_0 \cos(\omega t - \delta)$

$$A_0 = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\gamma\omega)^2}}$$

$$\tan\delta = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$

Resonans (stor A_0) når $\omega = \omega_0$



Energi SHM (udempet):

- Totalenergien $E_{tot} = E_k(t) + E_p(t)$ er konstant og svinger mellom $E_k(t)$ og $E_p(t)$
- $E_p(x)$ prop. med x^2 for alle svingninger
  Eks. fjærpendel: $E_p(t) = ½\, k\, x^2$
  Tyngdependel: $E_p(t) = mgh = mgL(1 - \cos\theta) \approx mgL/2 \cdot \theta^2$