

Formelliste

TFY4155/FY1003 Elektrisitet og magnetisme

Formlenes gyldighetsområde og de ulike symbolenes betydning antas å være kjent. Symbolbruk som i forelesningene.
 Siste revisjon: 1.4.14(komplekse størrelser) 5.5.14(kraftmoment) (2 sider).

Q, ρ og σ uten indeks viser til *frie* ladninger. Q_i, ρ_i og σ_i er induisert ladning.

I og \vec{J} uten indeks er ledningsstrøm (conducting current), I_d og \vec{J}_d er forskyvningsstrøm (displacement current).

Coulombs lov: $\vec{F}_{12} = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}_{12}$ $\vec{E} = \frac{1}{4\pi\epsilon} \frac{q}{r^2} \hat{\mathbf{r}}$

Gauss' lov integralform: $\oiint \vec{D} \cdot d\vec{A} = Q$ $\oiint \vec{E} \cdot d\vec{A} = Q/\epsilon$ $\oiint \vec{P} \cdot d\vec{A} = -Q_i$ $\oiint \vec{B} \cdot d\vec{A} = 0$

Gauss' lov differensialform: $\text{div} \vec{D} = \rho$ $\text{div} \vec{E} = \rho/\epsilon$ $\text{div} \vec{P} = -\rho_i$ $\text{div} \vec{B} = 0$

Fluks: $\Phi_E = \iint \vec{E} \cdot d\vec{A}$ $\Phi = \iint \vec{D} \cdot d\vec{A} = \epsilon \Phi_E$ $\Phi_B = \iint \vec{B} \cdot d\vec{A}$

Amperes lov: $\oint \vec{B} \cdot d\vec{s} = \mu \left(I + \epsilon \frac{\partial \Phi_E}{\partial t} \right)$ $\oint \vec{H} \cdot d\vec{s} = I + \frac{\partial \Phi}{\partial t}$ $\text{curl} \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

Faradays lov: $\mathcal{E} = -\frac{\partial \Phi_B}{\partial t} = -L \frac{dI}{dt}$ $\oint \vec{E} \cdot d\vec{s} = -\frac{\partial \Phi_B}{\partial t}$ $\text{curl} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Maxwells likninger: $\text{div} \vec{D} = \rho$ $\text{div} \vec{B} = 0$ $\text{curl} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\text{curl} \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

Forskyvningsstrøm: $I_d = \frac{\partial \Phi}{\partial t}$, $\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$

Elektrisk dipolmoment: $\vec{p} = q\vec{d}$ (fra - til +) Polarisering: $\vec{P} = \frac{\sum \vec{p}}{\text{volum}}$

Magnetisk (dipol)moment: $\vec{\mu} = \vec{m} = I\vec{A}$ Magnetisering: $\vec{M} = \frac{\sum \vec{\mu}}{\text{volum}}$

Kraftmoment: $\vec{\tau} = \vec{p} \times \vec{E}$ $\vec{\tau} = \vec{\mu} \times \vec{B}$

$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon \vec{E} = \epsilon_r \epsilon_0 \vec{E}$ $\vec{P} = \chi_e \epsilon_0 \vec{E}$ $\epsilon_r = 1 + \chi_e$

$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} = \mu \vec{H} = \mu_r \mu_0 \vec{H}$ $\vec{M} = \chi_m \vec{H}$ $\mu_r = 1 + \chi_m$

Elektrisk potensial: $V_a - V_b = -\int_b^a \vec{E} \cdot d\vec{s}$, $\vec{E} = -\vec{\nabla} V$, Relativt ∞ : $V(r) = \int \frac{dq}{4\pi\epsilon r}$

Energi og energitetthet: $U = \frac{1}{2} \iiint V dq$ Elektrisk: $u = \frac{1}{2} \vec{D} \cdot \vec{E}$ Magnetisk: $u = \frac{1}{2} \vec{B} \cdot \vec{H}$

Kondensatorer: $C = \frac{Q}{V}$ Kulekondensator: $C = 4\pi\epsilon_0 R$ Energi: $U = \frac{1}{2} QV = \frac{1}{2} CV^2$

Platekondensator: $C = \epsilon \frac{A}{d}$ Parallellkopling: $C = \sum_i C_i$ Seriekopling: $\frac{1}{C} = \sum_i \frac{1}{C_i}$

Kraft på strømførende leder: $d\vec{F} = Id\vec{s} \times \vec{B}$ Lorentzkrafta: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Biot-Savarts lov: $\vec{B} = \frac{\mu_0}{4\pi} \frac{q \vec{v} \times \hat{\mathbf{r}}}{r^2}$ $d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{s} \times \hat{\mathbf{r}}}{r^2}$

H -felt rundt ∞ lang leder: $H_\theta = \frac{I}{2\pi r}$ H -felt i lang, tynn solenoide: $H = I \cdot n = I \cdot \frac{N}{\ell}$

Ohms lov: $V = RI$, $R = \rho \frac{\ell}{A} = \frac{1}{\sigma} \frac{\ell}{A}$; $P = VI$

$\sigma \vec{E} = \vec{J}$, der strømtetthet $= \vec{J} = nq\vec{v}_d$ og $\vec{v}_d = \mu \vec{E} =$ driftsfart.

Induktans: $\mathcal{E} = -L \frac{dI}{dt}$ $\mathcal{E}_2 = -M_{21} \frac{dI_1}{dt}$, $M_{21} = M_{12}$ Spoler: $L = N \frac{\Phi_B}{I}$ $U = \frac{1}{2} LI^2$

Lenz lov: En indusert strøm er alltid slik at den forsøker å motvirke forandringen i den magnetiske fluks som er årsak til strømmen.

Kompleks AC-signal: $V(t) = V_0 e^{i\omega t} = |V_0| e^{i\alpha} e^{i\omega t}$ $I(t) = I_0 e^{i\omega t} = |I_0| e^{i\beta} e^{i\omega t}$

$Z = \frac{V(t)}{I(t)} = \frac{V_0}{I_0} = |Z| e^{i\phi}$ $Z_R = R$ $Z_L = i\omega L$ $Z_C = \frac{1}{i\omega C}$

Nablaoperatoren:

Kartesiske koordinater (x, y, z) , med enhetsvektorer henholdsvis \hat{i} , \hat{j} og \hat{k} :

$$\begin{aligned} \text{grad}V &= \vec{\nabla}V = \hat{i} \frac{\partial V}{\partial x} + \hat{j} \frac{\partial V}{\partial y} + \hat{k} \frac{\partial V}{\partial z} \\ \text{div}\vec{D} &= \vec{\nabla} \cdot \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \\ \vec{\nabla}^2 V &= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \\ \text{curl}\vec{D} &= \vec{\nabla} \times \vec{D} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ D_x & D_y & D_z \end{vmatrix} \end{aligned}$$

Sylinderkoordinater (r, ϕ, z) , med enhetsvektorer henholdsvis \hat{r} , $\hat{\phi}$ og \hat{k} :

$$\begin{aligned} \vec{\nabla}V &= \hat{r} \frac{\partial V}{\partial r} + \hat{\phi} \frac{1}{r} \frac{\partial V}{\partial \phi} + \hat{k} \frac{\partial V}{\partial z} \\ \vec{\nabla} \cdot \vec{D} &= \frac{1}{r} \frac{\partial}{\partial r} (r D_r) + \frac{1}{r} \frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z} \\ \vec{\nabla}^2 V &= \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \end{aligned}$$

Kulekoordinater (r, θ, ϕ) , med enhetsvektorer henholdsvis \hat{r} , $\hat{\theta}$, $\hat{\phi}$:

$$\begin{aligned} \vec{\nabla}V &= \hat{r} \frac{\partial V}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial V}{\partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \\ \vec{\nabla} \cdot \vec{D} &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (D_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial D_\phi}{\partial \phi} \\ \vec{\nabla}^2 V &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2} \end{aligned}$$

Divergensteoremet og Stokes' teorem for et tilfeldig vektorfelt \vec{F} :

$$\begin{aligned} \oint \vec{F} \cdot d\vec{A} &= \iiint \vec{\nabla} \cdot \vec{F} \, d\tau \\ \oint \vec{F} \cdot d\vec{s} &= \iint (\vec{\nabla} \times \vec{F}) \cdot d\vec{A} \end{aligned}$$

Infinitesimale volumelement:

$$\begin{aligned} d\tau &= dx \, dy \, dz \\ d\tau &= r^2 \, dr \, \sin \theta \, d\theta \, d\phi \xrightarrow{\text{kulesymmetri}} 4\pi r^2 \, dr \\ d\tau &= r \, dr \, d\phi \, dz \xrightarrow{\text{syl.symmetri}} 2\pi r \, dr \, \ell \end{aligned}$$