Institutt for fysikk 2015

Formelliste
TFY4155/FY1003 Elektrisitet og magnetisme

Formlenes gyldighetsområde og de ulike symbolenes betydning antas å være kjent. Symbolbruk som i forelesningene.
Siste revisjon: 13.4.15(bølger) (2 sider).

Q, ρ og σ uten indeks viser til *frie* ladninger. Q_i, ρ_i og σ_i er indusert ladning.
I og $\vec{J}$ uten indeks er ledningsstrøm (conducting current), I_d og $\vec{J}_\mathrm{d}$ er forskyvningsstrøm (displacement current).

Coulombs lov: $\vec{F}_{12} = \dfrac{1}{4\pi\epsilon}\dfrac{q_1 q_2}{r^2}\,\hat{\mathbf{r}}_{12}$ $\qquad \vec{E} = \dfrac{1}{4\pi\epsilon}\dfrac{q}{r^2}\,\hat{\mathbf{r}}$

Gauss' lov integralform: $\oiint \vec{D}\cdot\mathrm{d}\vec{A} = Q$ $\qquad \oiint \vec{E}\cdot\mathrm{d}\vec{A} = Q/\epsilon$ $\qquad \oiint \vec{P}\cdot\mathrm{d}\vec{A} = -Q_\mathrm{i}$ $\qquad \oiint \vec{B}\cdot\mathrm{d}\vec{A} = 0$

Gauss' lov differensialform: $\mathrm{div}\vec{D} = \rho$ $\qquad \mathrm{div}\vec{E} = \rho/\epsilon$ $\qquad \mathrm{div}\,\vec{P} = -\rho_\mathrm{i}$ $\qquad \mathrm{div}\vec{B} = 0$

Fluks: $\Phi_E = \iint \vec{E}\cdot\mathrm{d}\vec{A}$ $\qquad \Phi = \iint \vec{D}\cdot\mathrm{d}\vec{A} = \epsilon\,\Phi_E$ $\qquad \Phi_B = \iint \vec{B}\cdot\mathrm{d}\vec{A}$

Amperes lov: $\oint \vec{B}\cdot\mathrm{d}\vec{s} = \mu\left(I + \epsilon\dfrac{\partial\Phi_E}{\partial t}\right)$ $\qquad \oint \vec{H}\cdot\mathrm{d}\vec{s} = I + \dfrac{\partial\Phi}{\partial t}$ $\qquad \mathrm{curl}\,\vec{H} = \vec{J} + \dfrac{\partial\vec{D}}{\partial t}$

Faradays lov: $\mathcal{E} = -\dfrac{\partial\Phi_B}{\partial t} = -L\dfrac{\mathrm{d}I}{\mathrm{d}t}$ $\qquad \oint \vec{E}\cdot\mathrm{d}\vec{s} = -\dfrac{\partial\Phi_B}{\partial t}$ $\qquad \mathrm{curl}\,\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$

Maxwells likninger: $\mathrm{div}\vec{D} = \rho$ $\qquad \mathrm{div}\vec{B} = 0$ $\qquad \mathrm{curl}\,\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$ $\qquad \mathrm{curl}\,\vec{H} = \vec{J} + \dfrac{\partial\vec{D}}{\partial t}$

Forskyvningsstrøm: $I_\mathrm{d} = \dfrac{\partial\Phi}{\partial t}\,,\quad \vec{J}_\mathrm{d} = \dfrac{\partial\vec{D}}{\partial t}$

Elektrisk dipolmoment: $\vec{p} = q\vec{d}$ (fra – til +) $\qquad$ Polarisering: $\vec{P} = \dfrac{\sum\vec{p}}{\text{volum}}$

Magnetisk (dipol)moment: $\vec{\mu} = \vec{m} = I\vec{A}$ $\qquad$ Magnetisering: $\vec{M} = \dfrac{\sum\vec{\mu}}{\text{volum}}$

Kraftmoment: $\vec{\tau} = \vec{p}\times\vec{E}$ $\qquad \vec{\tau} = \vec{\mu}\times\vec{B}$

$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E} = \epsilon_\mathrm{r}\epsilon_0\vec{E}$ $\qquad \vec{P} = \chi_\mathrm{e}\epsilon_0\vec{E}$ $\qquad \epsilon_\mathrm{r} = 1 + \chi_\mathrm{e}$

$\vec{B} = \mu_0\vec{H} + \mu_0\vec{M} = \mu\vec{H} = \mu_\mathrm{r}\mu_0\vec{H}$ $\qquad \vec{M} = \chi_\mathrm{m}\vec{H}$ $\qquad \mu_\mathrm{r} = 1 + \chi_\mathrm{m}$

Elektrisk potensial: $V_a - V_b = -\displaystyle\int_b^a \vec{E}\cdot\mathrm{d}\vec{s}\,,$ $\qquad \vec{E} = -\vec{\nabla}V\,,$ $\qquad$ Relativt ∞: $V(r) = \displaystyle\int \frac{\mathrm{d}q}{4\pi\epsilon r}$

Energi og energitetthet: $U = \dfrac{1}{2}\displaystyle\iiint V\,\mathrm{d}q$ $\qquad$ Elektrisk: $u = \dfrac{1}{2}\vec{D}\cdot\vec{E}$ $\qquad$ Magnetisk: $u = \dfrac{1}{2}\vec{B}\cdot\vec{H}$

Kondensatorer: $C = \dfrac{Q}{V}$ $\qquad$ Kulekondensator: $C = 4\pi\epsilon_0 R$ $\qquad$ Energi: $U = \dfrac{1}{2}QV = \dfrac{1}{2}CV^2$

Platekondensator: $C = \epsilon\dfrac{A}{d}$ $\qquad$ Parallellkopling: $C = \displaystyle\sum_i C_i$ $\qquad$ Seriekopling: $\dfrac{1}{C} = \displaystyle\sum_i \frac{1}{C_i}$

Kraft på strømførende leder: $\mathrm{d}\vec{F} = I\mathrm{d}\vec{s}\times\vec{B}$ $\qquad$ Lorentzkrafta: $\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$

Biot-Savarts lov: $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\,\vec{v}\times\hat{\mathbf{r}}}{r^2}$ $\qquad \mathrm{d}\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\mathrm{d}\vec{s}\times\hat{\mathbf{r}}}{r^2}$

H-felt rundt ∞ lang leder: $H_\theta = \dfrac{I}{2\pi r}$ $\qquad$ H-felt i lang, tynn solenoide: $H = I\cdot n = I\cdot\dfrac{N}{\ell}$

Ohms lov: $V = RI\,,\quad R = \rho\dfrac{\ell}{A} = \dfrac{1}{\sigma}\dfrac{\ell}{A}\,;\quad P = VI$

$\sigma\vec{E} = \vec{J}\,,$ der strømtetthet $= \vec{J} = nq\vec{v}_\mathrm{d}$ og $\vec{v}_\mathrm{d} = \mu\vec{E}$ = driftsfart.

Induktans: $\mathcal{E} = -L\frac{\mathrm{d}I}{\mathrm{d}t}$ $\quad \mathcal{E}_2 = -M_{21}\frac{\mathrm{d}I_1}{\mathrm{d}t}, \quad M_{21} = M_{12}$ $\quad$ Spoler: $L = N\frac{\Phi_B}{I}$ $\quad U = \frac{1}{2}LI^2$

Lenz lov: En indusert strøm er alltid slik at den forsøker å motvirke forandringen i den magnetiske fluks som er årsak til strømmen.

Kompleks AC-signal: $V(t) = V_0 e^{i\omega t} = |V_0|e^{i\alpha}e^{i\omega t}$ $\quad I(t) = I_0 e^{i\omega t} = |I_0|e^{i\beta}e^{i\omega t}$

$$Z = \frac{V(t)}{I(t)} = \frac{V_0}{I_0} = |Z|e^{i\phi} \qquad Z_R = R \qquad Z_L = i\omega L \qquad Z_C = \frac{1}{i\omega C}$$

Elektromagnetiske bølger:

Bølgelikningen for $\vec{E}$: $\nabla^2\vec{E} = \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$ der $\nabla^2\vec{E} = \frac{\partial^2 E_x}{\partial x^2}\hat{\mathbf{i}} + \frac{\partial^2 E_y}{\partial y^2}\hat{\mathbf{j}} + \frac{\partial^2 E_z}{\partial z^2}\hat{\mathbf{k}}$ og $\frac{1}{c^2} = \mu\epsilon$

Bølge i $\pm x$-retning med $\vec{E}$ planpolarisert i y-retning: $\vec{E}(x,t) = E_0\,\hat{\mathbf{j}}\,\cos(\omega t \mp kx)$, $\quad \vec{B}(x,t) = B_0\,\hat{\mathbf{k}}\,\cos(\omega t \mp kx)$

med $\omega = 2\pi f$ $\quad k = \frac{2\pi}{\lambda}$ $\quad |c| = \frac{\omega}{k} = \sqrt{\frac{1}{\mu\epsilon}}$ $\quad B_0 = \pm\frac{E_0}{c}$ $\quad$ Bølge(vandre)retning som $\vec{E}\times\vec{B}$

Nablaoperatoren:

Kartesiske koordinater (x,y,z), med enhetsvektorer henholdsvis $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ og $\hat{\mathbf{k}}$:

$$\begin{aligned}
\mathrm{grad}V = \vec{\nabla}V &= \hat{\mathbf{i}}\frac{\partial V}{\partial x} + \hat{\mathbf{j}}\frac{\partial V}{\partial y} + \hat{\mathbf{k}}\frac{\partial V}{\partial z} \\
\mathrm{div}\vec{D} = \vec{\nabla}\cdot\vec{D} &= \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \\
\vec{\nabla}^2 V &= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \\
\mathrm{curl}\,\vec{D} = \vec{\nabla}\times\vec{D} &= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ D_x & D_y & D_z \end{vmatrix}
\end{aligned}$$

Sylinderkoordinater (r,ϕ,z), med enhetsvektorer henholdsvis $\hat{\mathbf{r}}, \hat{\phi}$ og $\hat{\mathbf{k}}$:

$$\begin{aligned}
\vec{\nabla}V &= \hat{\mathbf{r}}\frac{\partial V}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial V}{\partial\phi} + \hat{\mathbf{k}}\frac{\partial V}{\partial z} \\
\vec{\nabla}\cdot\vec{D} &= \frac{1}{r}\frac{\partial}{\partial r}(rD_r) + \frac{1}{r}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z} \\
\vec{\nabla}^2 V &= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2}
\end{aligned}$$

Kulekoordinater (r,θ,ϕ), med enhetsvektorer henholdsvis $\hat{\mathbf{r}}, \hat{\theta}, \hat{\phi}$:

$$\begin{aligned}
\vec{\nabla}V &= \hat{\mathbf{r}}\frac{\partial V}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial V}{\partial\theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi} \\
\vec{\nabla}\cdot\vec{D} &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(D_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi} \\
\vec{\nabla}^2 V &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}
\end{aligned}$$

Divergensteoremet og Stokes' teorem for et tilfeldig vektorfelt $\vec{F}$:

$$\oiint \vec{F}\cdot\mathrm{d}\vec{A} = \iiint \vec{\nabla}\cdot\vec{F}\;\mathrm{d}\tau \qquad \oint \vec{F}\cdot\mathrm{d}\vec{s} = \iint \left(\vec{\nabla}\times\vec{F}\right)\cdot\mathrm{d}\vec{A}$$

Infinitesimale volumelement:

$$\begin{aligned}
\mathrm{d}\tau &= \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \\
\mathrm{d}\tau &= r^2\,\mathrm{d}r\,\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi \;\xrightarrow{\text{kulesymmetri}}\; 4\pi r^2\,\mathrm{d}r \\
\mathrm{d}\tau &= r\,\mathrm{d}r\,\mathrm{d}\phi\,\mathrm{d}z \;\xrightarrow{\text{syl.symmetri}}\; 2\pi r\,\mathrm{d}r\,\ell
\end{aligned}$$