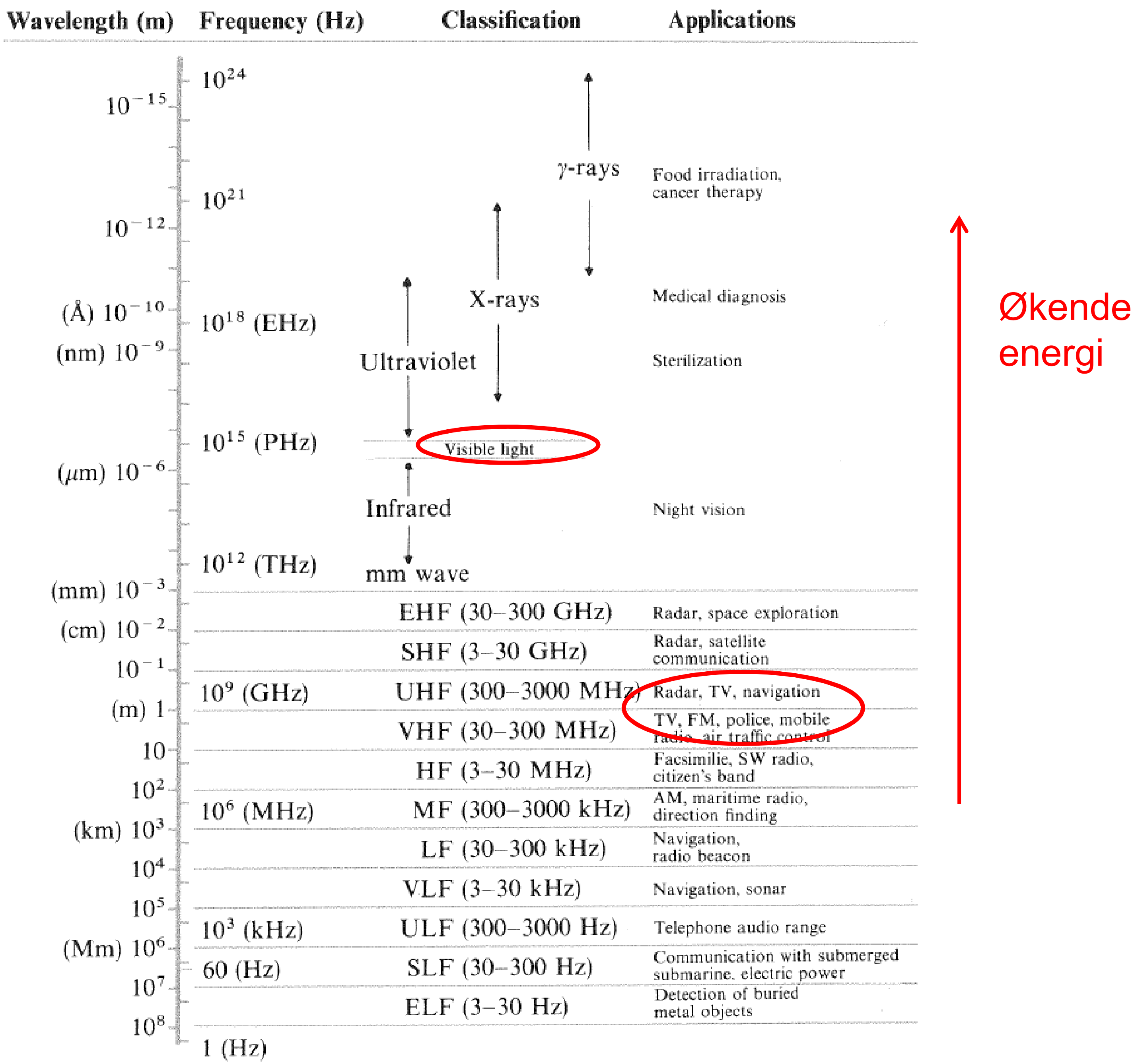
Kap. 32: Elektromagnetiske bølger

- 32.1 Oppsummering av Maxwells likninger.
  Emb-spekteret. Obs: Notat 4
- 32.2 Bølgelikninga:
  1. Matematisk utledning fra Maxwells diff.likninger
  2. "Visuell utledning" fra Maxwells integrallikninger
- 32.3 Harmoniske bølger

Spectrum of electromagnetic waves.



Maxwells likninger.

James Clerk Maxwell (1831-1879), skotsk fysiker.
(Aberdeen, London og Cambridge)
Blant de største vitenskapsmenn ved siden av Newton og Einstein.

$$\oint \vec{E} \cdot \mathrm{d}\vec{A} = \frac{Q}{\epsilon_0}$$

$$\oint \vec{B} \cdot \mathrm{d}\vec{A} = 0$$

$$\oint \vec{B} \cdot \mathrm{d}\vec{\ell} = \mu_0 I + \mu_0 \epsilon_0 \frac{\partial \Phi_E}{\partial t}$$

$$\oint \vec{E} \cdot \mathrm{d}\vec{\ell} = -\frac{\partial \Phi_B}{\partial t}$$

Integral-form:

$$\oint \vec{E} \cdot \mathrm{d}\vec{A} = \frac{Q}{\epsilon_0} \quad \text{(Gauss' lov for } \vec{E}\text{)}$$

Ladningsfritt

$$\oint \vec{B} \cdot \mathrm{d}\vec{A} = 0 \quad \text{(Gauss' lov for } \vec{B}\text{)}$$

Strømfritt

$$\oint \vec{B} \cdot \mathrm{d}\vec{\ell} = \mu_0 I + \mu_0 \epsilon_0 \frac{\partial \Phi_E}{\partial t} \quad \text{(Amperes lov)}$$

$$\oint \vec{E} \cdot \mathrm{d}\vec{\ell} = -\frac{\partial \Phi_B}{\partial t} \quad \text{(Faradays lov).}$$

Differen-sial-form:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

Ladningsfritt

$$\vec{\nabla} \cdot \vec{B} = 0$$

Strømfritt

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} .$$

Maxwells likninger

(ladningsfritt og strømfritt rom)

$$\vec{\nabla} \cdot \vec{E} = 0 \quad (1)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (2)$$

$$\vec{\nabla} \times \vec{B} = \mu\epsilon \frac{\partial \vec{E}}{\partial t} \quad (3)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}\,. \quad (4)$$

.. gir bølgelikningene

$$\frac{\partial^2 \vec{E}}{\partial^2 x} = \frac{1}{c^2} \cdot \frac{\partial^2 \vec{E}}{\partial^2 t} \qquad (5)$$

$$\frac{\partial^2 \vec{B}}{\partial^2 x} = \frac{1}{c^2} \cdot \frac{\partial^2 \vec{B}}{\partial^2 t} \qquad (6)$$

Bølgefart: $c = \sqrt{\frac{1}{\mu\varepsilon}}$

-- i vakuum:

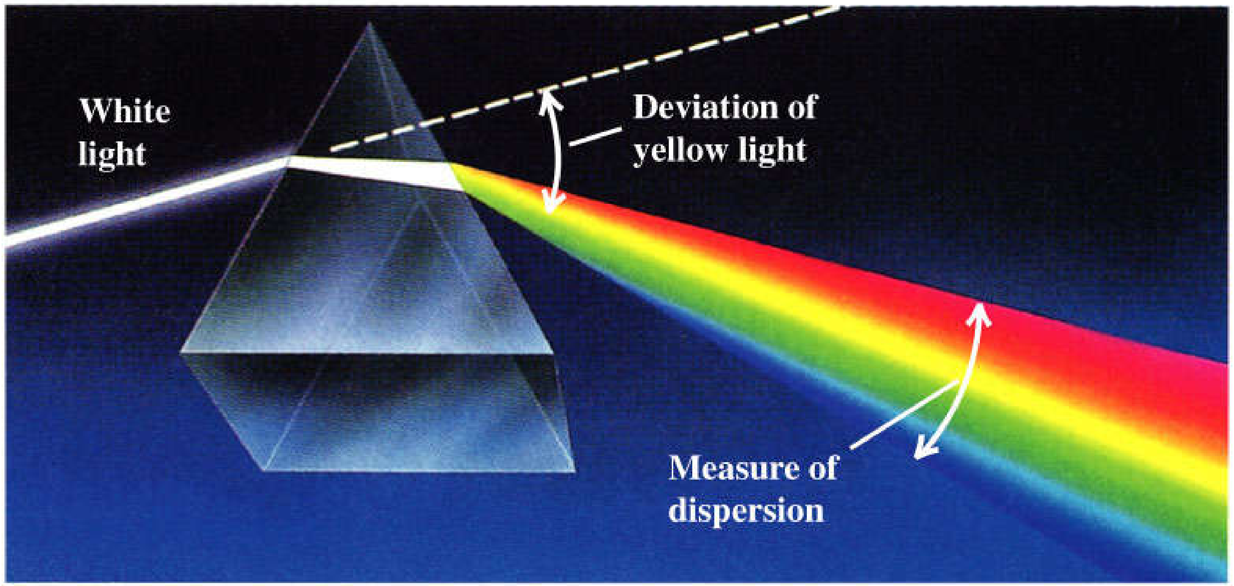
$$c_0 = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = 299\,792\,458 \text{ m/s} \approx 300 \text{ Mm/s}$$

-- i dielektrikum:

$$c = c_0 \sqrt{\frac{1}{\mu_0 \varepsilon_r \varepsilon_0}} = c_0 \sqrt{\frac{1}{\varepsilon_r}} < c_0$$

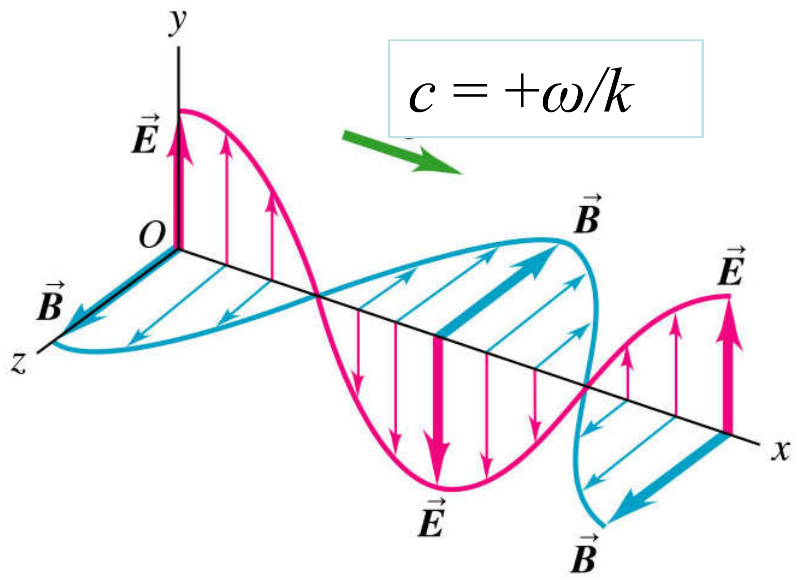
Relativ permittivitet ε_r , brytningsindeks $n = c_0/c$ og lysfart c/c_0 i vann ved 20 °C:

$f/10^{14}$Hz	λ / nm	Farge	ε_r	$n = \sqrt{\varepsilon_r}$	$c/c_0 = 1/n$
0	∞	(statisk ***E***)	83	-	-
4,24	707	Rød	1,7708	1,3307	0,7515
7,41	405	Fiolett	1,7924	1,3388	0,7469



E_y-bølge i positiv *x*-retning:

$$\boldsymbol{E}(x,t) = E_0\,\boldsymbol{j}\cos(kx - \omega t)$$

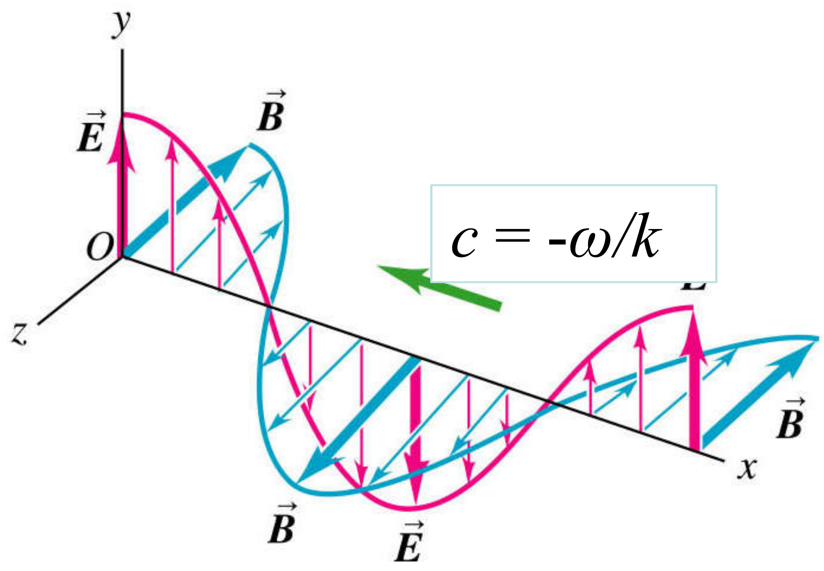


$\vec{E}$: *y*-component only
$\vec{B}$: *z*-component only

Alltid: ***E*** X ***B*** peker i forplantingsretningen

E_y-bølge i negativ *x*-retning:

$$\boldsymbol{E}(x,t) = E_0\,\boldsymbol{j}\cos(kx + \omega t)$$



$\vec{E}$: *y*-component only
$\vec{B}$: *z*-component only

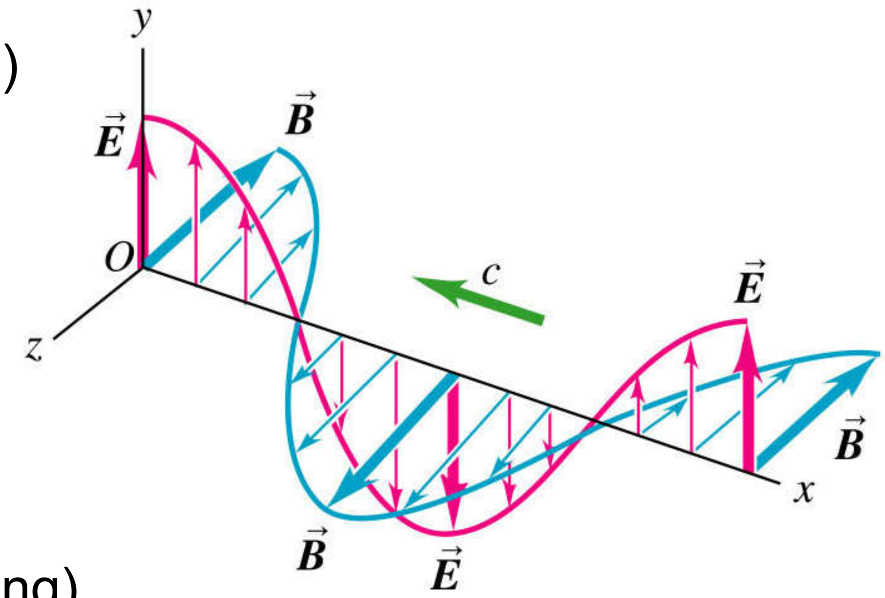
Ulike planpolariseringer (eks: bølger i $-x$-retning)

Planpolarisert vertikalt (***E*** i y-retning)

$$\boldsymbol{E} = E_y(x,t)\,\boldsymbol{j} = E_0\,\boldsymbol{j}\cos(kx + \omega t)$$

$$\boldsymbol{B} = B_z(x,t)\,\boldsymbol{k} = B_0\,\boldsymbol{k}\cos(kx + \omega t)$$

$$E_0 = -cB_0$$

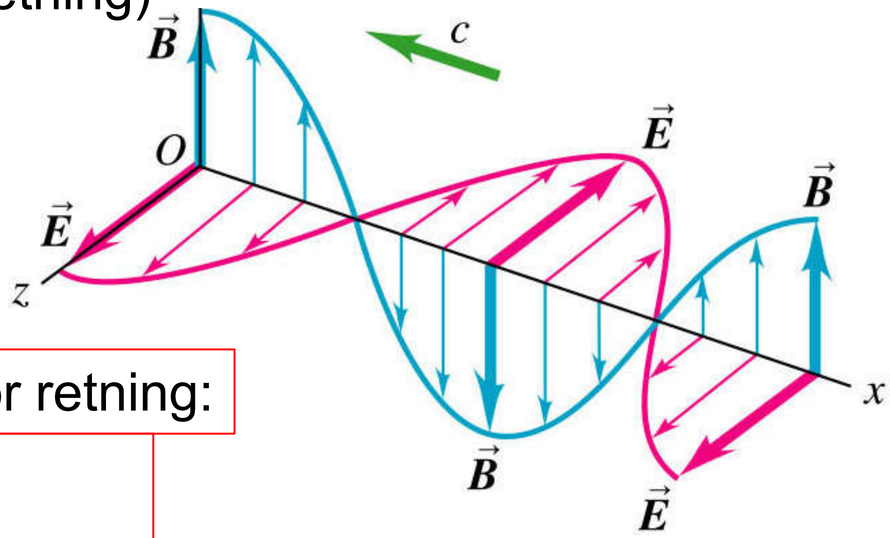


Planpolarisert horisontalt (***E*** i z-retning)

$$\boldsymbol{E} = E_z(x,t)\,\boldsymbol{k} = E_0\,\boldsymbol{k}\cos(kx + \omega t)$$

$$\boldsymbol{B} = B_y(x,t)\,\boldsymbol{j} = B_0\,\boldsymbol{j}\cos(kx + \omega t)$$

$$E_0 = cB_0$$



Bruk $|E_0| = c\,|B_0|$ og følgende for retning:

E X ***B***

peker i forplantingsretningen

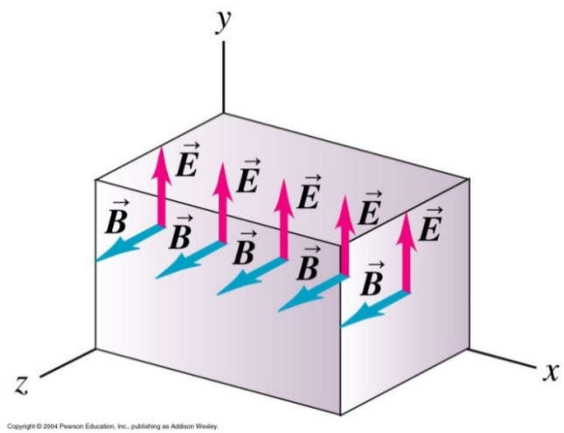
Utledning i Young & Freedman:

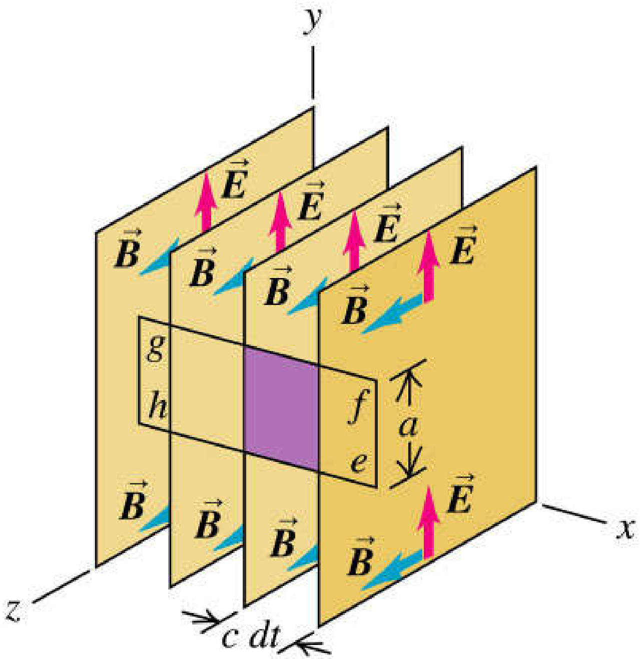
Faradays lov:

Integrasjonsveg *efghe*: $\oint \mathbf{E}\cdot d\mathbf{s} = -d\Phi_B/dt$

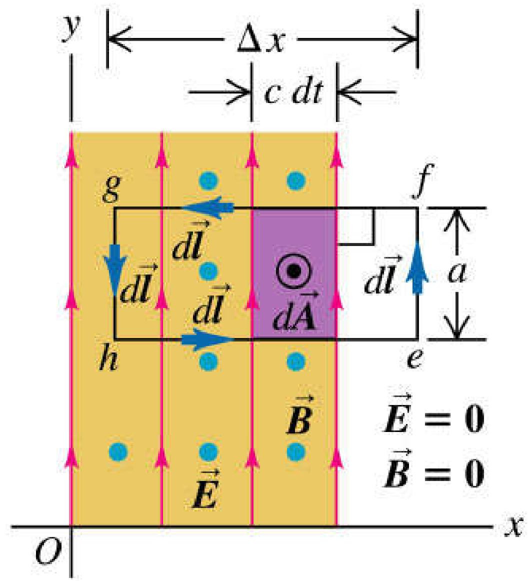
Endring i Φ_B pga. fiolett felt: $-E\cdot a = -B\cdot a\cdot (c\,dt)/dt$

$$E = Bc$$





(a) In time dt, the wave front moves a distance $c\,dt$ in the $+x$-direction



(b) Side view of situation in (a)

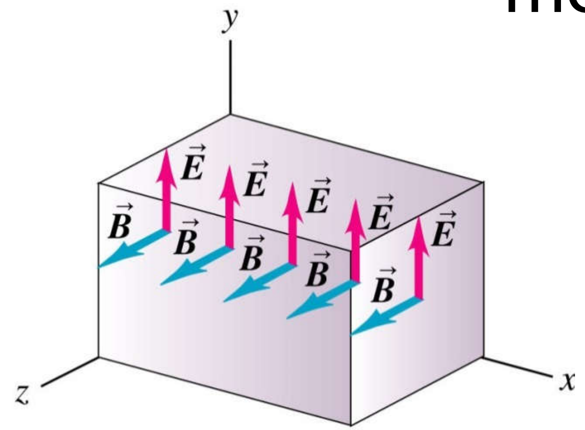
Utledning i Young & Freedman:

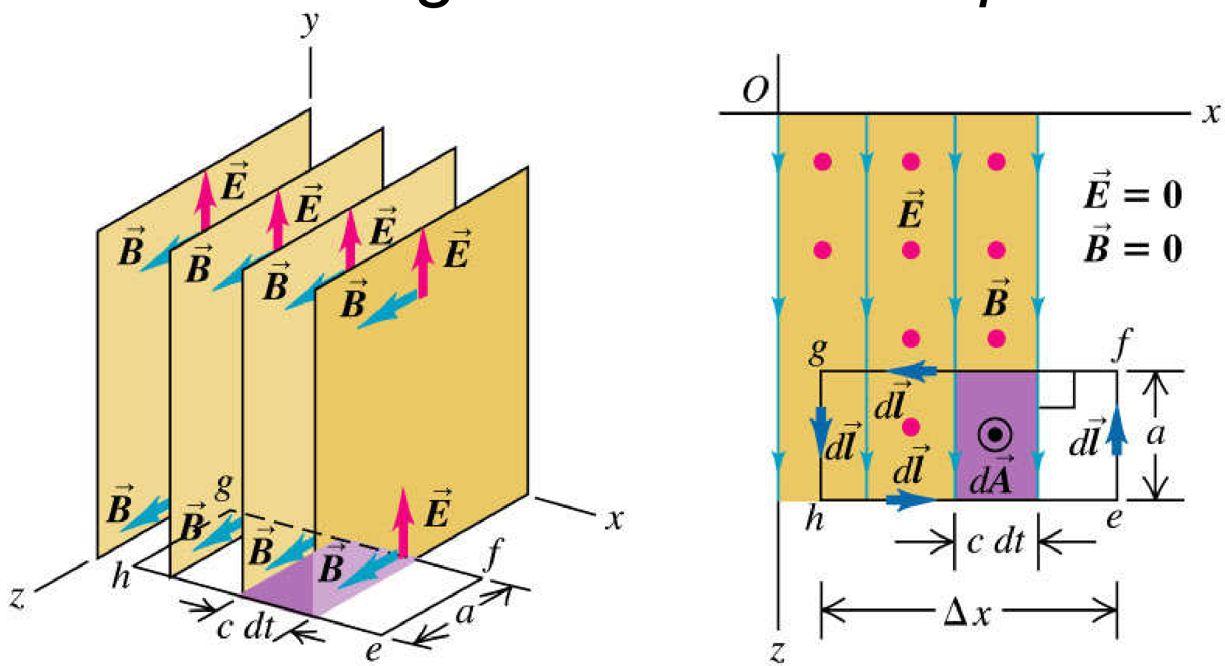
Amperes lov:

Integrasjonsveg *efghe*: $\int \mathbf{B}\cdot d\mathbf{s} = \mu\varepsilon \cdot d\Phi_E/dt$

Endring i Φ_E pga. fiolett felt: $B\cdot a = \mu\varepsilon \cdot E\cdot a\cdot (c\, dt)/dt$

med $E = Bc$ gir dette $c^2 = 1/\mu\varepsilon$





(a) In time dt, the wave front moves a distance $c\,dt$ in the $+x$-direction

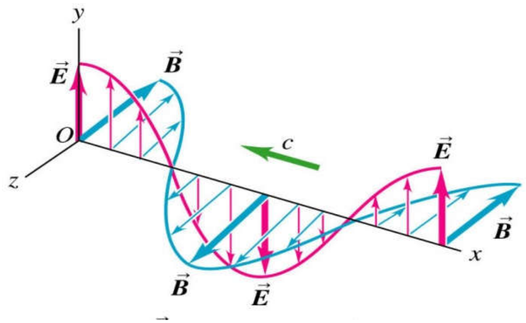
(b) Top view of situation in (a)

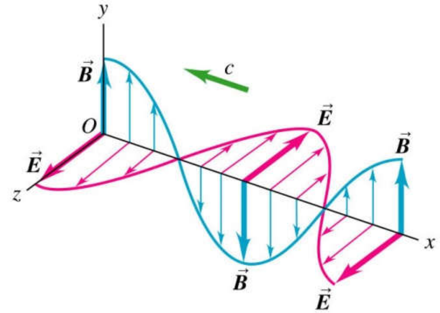
Ulike polariseringer

Planpolarisert:

vertikal $E_y(x,t)$ eller

horisontal $E_z(x,t)$





Sirkulærpolarisert:

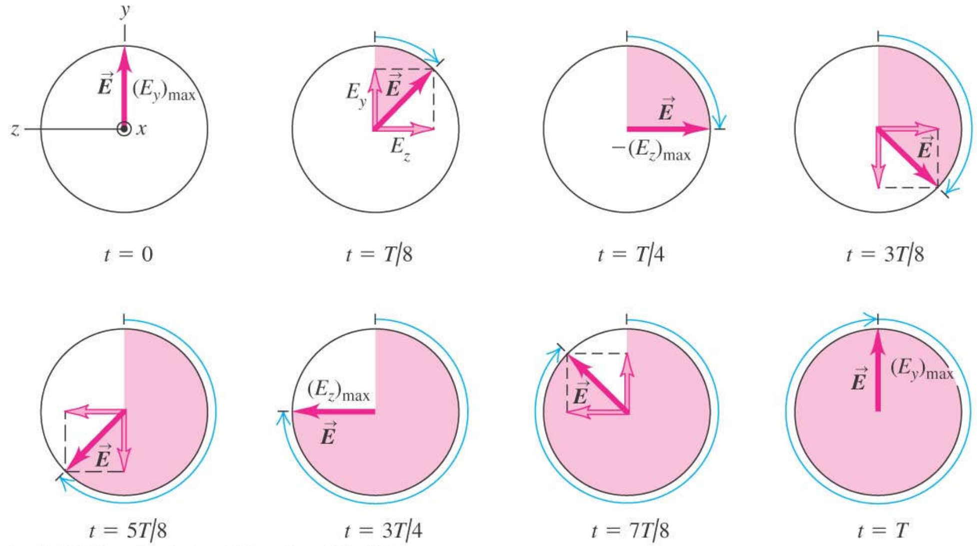
$\boldsymbol{E}$ roterer, dvs:

$|E_y(x,t)| = |E_z(x,t)|$

men 90^0 ute av fase.

$\boldsymbol{B}$ roterer,

retning 90^0 med $\boldsymbol{E}$.



Circular polarization: The $\vec{E}$ vector of the wave has constant magnitude and rotates in a circle.

Upolarisert:

Ingen ordning (like mye $E_y(x,t)$ som $E_z(x,t)$)

Kap. 32. Oppsummering, Elektromagnetiske bølger

- **Elektromagnetisk bølge, eks.:** $E_y(x,t) = E_0 \cos(kx \pm \omega t)$
  og $B_z(x,t) = B_0 \cos(kx \pm \omega t)$
- **Bølgellikning:** $\partial^2 E/\partial x^2 = 1/\mathrm{c}^2 \cdot \partial^2 E/\partial t^2$ og $\partial^2 B/\partial x^2 = 1/\mathrm{c}^2 \cdot \partial^2 B/\partial t^2$.
- Sammenheng $\boldsymbol{E}$ og $\boldsymbol{B}$: $E_0 = \pm c\, B_0$.
- Bølgefart vakuum: $\omega/k = f\lambda = c_0 = (\mu_0\, \varepsilon_0)^{-1/2} \approx 300$ Mm/s.
- Bølgefart annet medium: $c = (\mu\varepsilon)^{-1/2} = c_0 \cdot (\mu_\mathrm{r}\varepsilon_\mathrm{r})^{-1/2} < c_0$.
- Permittiviteten ε_r må beregnes for aktuell frekvens: $\varepsilon_\mathrm{r}(\omega)$.
- Frekvenser *f* fra 10^{54} Hz (radiobølger) til 10^{22} Hz (γ-bølger).
- Bølgelengder λ fra 10 km (radiobølger) til 10^{-14} m (γ-bølger).