ENGLISH.

TFY 4240
Øving 2006
Løsningsforslag

Oppgave 1

a) The energy stored in the field pr unit volume

$u = \frac{1}{2}\epsilon_0\epsilon_r E^2$

For 1 l we get

$u_\ell = \frac{1}{2}\epsilon_0\epsilon_r E^2 \cdot V$

$= \frac{1}{2} 8{,}85 \cdot 10^{-12} \cdot 1200\,(250 \cdot 10^6)^2\, 10^{-3}$ J

$= 0{,}332 \cdot 10^6$ J (per. liter)

For a car battery:

$W = V \cdot I \cdot t$

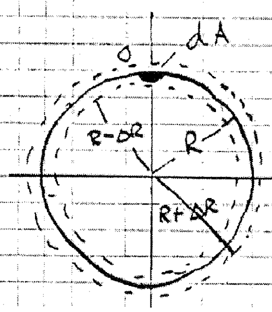
$= 12 \cdot 50 \cdot 3600 = 2{,}16 \cdot 10^6$ J

Per liter

$U_\ell = 2{,}16 \cdot 10^6/6$ J $= 0{,}36 \cdot 10^6$ J (per liter)

Oppgave 2

a)



The field at the point O is given as the average of the field inside and outside. See textbook p. 103

We use Gauss along the dotted surfaces just outside and inside the sphere

$$\oint \vec{E} \cdot \vec{u}_n \, dA = E \cdot A = \frac{Q_{INNE.}}{\epsilon_0}$$

$$E(R+\Delta R) = \frac{Q}{\epsilon_0 \cdot A} \approx \frac{\sigma}{\epsilon_0} \quad \text{når } \Delta R \to 0$$

$$E(R-\Delta R) = 0$$

$\Rightarrow$ The field in the surface E_0

$$E_0 = \frac{E(R+\Delta R) + E(R-\Delta R)}{2} = \frac{\sigma}{2\epsilon_0}$$

The force on dA (charge σdA)

$$dF = E_0 \, \sigma \, dA$$

$$dF = \frac{\sigma^2}{2\epsilon_0} dA$$

b) The force gives a pressure

$$\delta p_E = \frac{dF}{dA} = \frac{\sigma^2}{2\epsilon_0} = \frac{Q^2}{2\epsilon_0 (4\pi R^2)^2}$$

: This is set equal to the pressure from surface tension

$$2\alpha/R = \left(\frac{Q}{4\pi R^2}\right)^2 \cdot \frac{\lambda}{2\epsilon_0}$$

$$\Rightarrow \underline{Q = 8\pi R\sqrt{\alpha\epsilon_0 R}}$$

Tallverdier / Numerical

$$Q = 8\pi \cdot 5\cdot 10^{-6}\sqrt{73\cdot 10^{-3}\ 8.85\cdot 10^{-12}\cdot 5\cdot 10^{-6}}$$

$$\underline{\underline{Q = 2.25\cdot 10^{-13}\ C}}$$

Oppgave 3

Oppgave 1

For symmetry reasons, the field is radial

~~hjelp av~~ Gauss' lov: $\oiint_S \mathbf{D}\cdot d\mathbf{S} = \varepsilon E(r)2\pi r l = q = \lambda l$. ~~Dermed fås~~: Thus

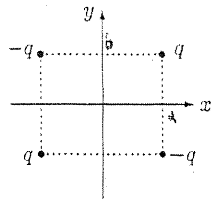
$E(r) = -\frac{d}{dr}V(r) = \frac{\lambda}{2\pi\varepsilon r}$, og direkte integrasjon gir: $V(r) = -\frac{\lambda}{2\pi\varepsilon}\ln r + A$. QED.

Ekvipotensialflatene er konsentriske sylindere. / Equipot. surfaces are concentric sylinders.

b) Se lærebok / See textbook

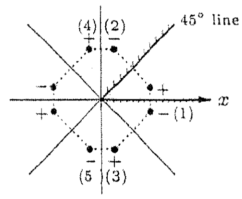
c)

The image configuration is as shown.



$$\boxed{\begin{aligned} V(x,y) = \frac{q}{4\pi\epsilon_0}\Bigg\{ & \frac{1}{\sqrt{(x-a)^2+(y-b)^2+z^2}} + \frac{1}{\sqrt{(x+a)^2+(y+b)^2+z^2}} \\ & - \frac{1}{\sqrt{(x+a)^2+(y-b)^2+z^2}} - \frac{1}{\sqrt{(x-a)^2+(y+b)^2+z^2}}\Bigg\}. \end{aligned}}$$

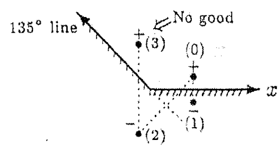
For this to work, $\boxed{\theta \text{ must be and integer divisor of } 180^\circ.}$ Thus 180°, 90°, 60°, 45°, etc., are OK, but no others. It works for 45°, say, with the charges as shown.



why it works for $\theta = 45°$

(Note the strategy: to make the x axis an equipotential ($V = 0$), you place the image charge (1) in the reflection point. To make the 45° line an equipotential, you place charge (2) at the image point. But that screws up the x axis, so you must now insert image (3) to balance (2). Moreover, to make the 45° line $V = 0$ you also need (4), to balance (1). But now, to restore the x axis to $V = 0$ you need (5) to balance (4), and so on.

The reason this doesn't work for *arbitrary* angles is that you are eventually forced to place an image charge *within the original region of interest*, and that's not allowed—*all* images must go *outside* the region, or you're no longer dealing with the same problem at all.)



why it *doesn't* work for $\theta = 135°$