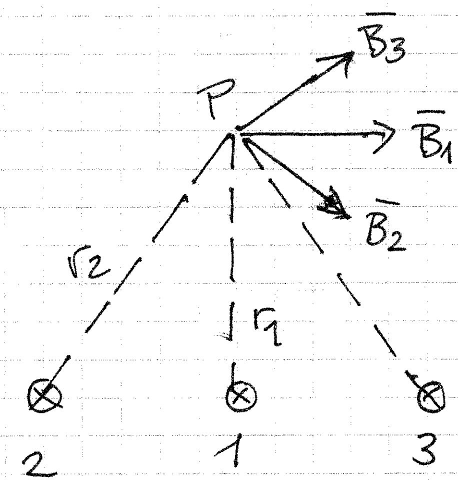
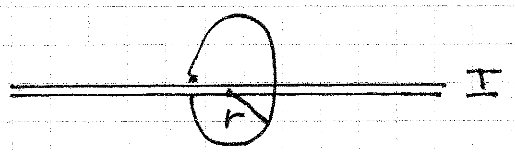
Oppgave 1

a)





Ampere's law

$$\oint \vec{B} d\vec{l} = \mu_0 I$$

$|\bar{B}|$ is constant and $\parallel d\bar{l}$ along the integr. path

$$B \cdot 2\pi r = \mu_0 I$$

$$B(r) = \frac{\mu_0 I}{2\pi r}$$

The total field is the vector sum of $\bar{B}_1$, $\bar{B}_2$ and $\bar{B}_3$

The vertical comp: $B_{vert} = 0$

The horisontal comp.

$$B_{HOR} = B_1 + B_2 \cos\theta + B_3 \cos\theta$$

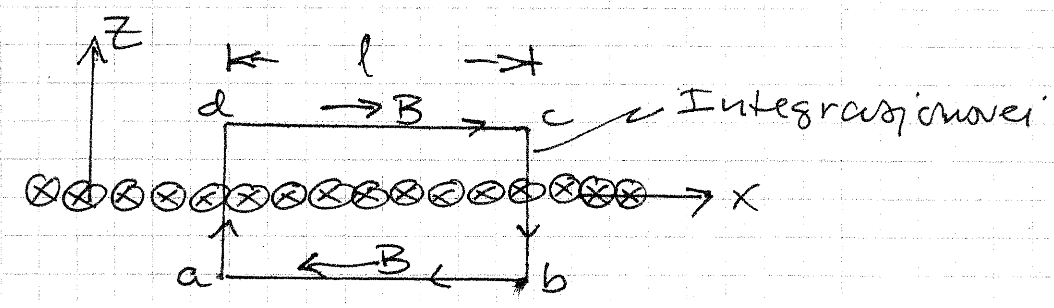
$$B_1 + 2B_2 \cos\theta$$

Med $B_1 = \frac{\mu_0 I}{2\pi r_1}$, $B_2 = \frac{\mu_0 I}{2\pi r_2} = \frac{\mu_0 I}{2\pi r_1/\cos\theta}$

Dette gir

$$\underline{B_{HOR} = \frac{\mu_0 I}{2\pi r_1}(1+2\cos^2\theta)}$$

b)



The B field will like in a) only have a horisontal component. Furthermore $\bar{B}$ is antisymmetric
Furthermore; $\bar{B}d\bar{l} = 0$ along ad and cb

$$\Rightarrow \quad \oint \vec{B}d\vec{l} = 2B\cdot l = \mu_0 I_{innenfor}$$

$$= \mu_0 n I \cdot l$$

Dette gir da:

$$\underline{B = \frac{\mu_0 n I}{2}}$$

The answer is independent of the distance from the sheet of current. Thus B is homogeneous.

c) We can use the principle of superposition. zero current is the same as a current I in positiv direction plus an equal current in the opposite direction. Thus we can calculate the field as a sum of the field from a complete current sheet + three wires carrying current in the opposite direction.

$$\vec{B} = \frac{1}{2}\mu_0 n I \,\hat{x} - \frac{\mu_0 I}{2\pi r_1}(1+2\cos^2\theta)\cdot\hat{x}$$

$$= \frac{1}{2}\mu_0 I\left(n - \frac{1}{\pi r_1}(1+2\cos^2\theta)\right)\hat{x}$$

Øving 5 - Løsningsforslag

Problem 5.10

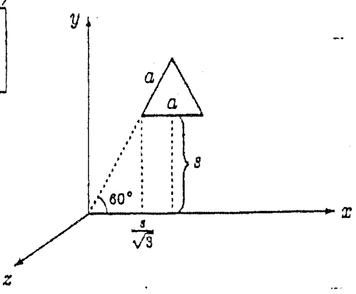
(a) The forces on the two sides cancel. At the bottom, $B = \frac{\mu_0 I}{2\pi s} \Rightarrow F = \left(\frac{\mu_0 I}{2\pi s}\right) Ia = \frac{\mu_0 I^2 a}{2\pi s}$ (up). At the top, $B = \frac{\mu_0 I}{2\pi(s+a)} \Rightarrow F = \frac{\mu_0 I^2 a}{2\pi(s+a)}$ (down). The net force is $\boxed{\frac{\mu_0 I^2 a^2}{2\pi s(s+a)}}$ (up). $\frac{a}{s}$

(b) The force on the bottom is the same as before, $\mu_0 I^2/2\pi$ (up). On the left side, $\mathbf{B} = \frac{\mu_0 I}{2\pi y}\,\hat{\mathbf{z}}$;

$d\mathbf{F} = I(d\mathbf{l} \times \mathbf{B}) = I(dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}) \times \left(\frac{\mu_0 I}{2\pi y}\,\hat{\mathbf{z}}\right) = \frac{\mu_0 I^2}{2\pi y}(-dx\,\hat{\mathbf{y}} + dy\,\hat{\mathbf{x}})$. But the x component cancels the corresponding term from the right side, and $F_y = -\frac{\mu_0 I^2}{2\pi}\int_{s/\sqrt{3}}^{(s/\sqrt{3}+a/2)} \frac{1}{y}\,dx$. Here $y = \sqrt{3}x$, so

$F_y = -\frac{\mu_0 I^2}{2\sqrt{3}\pi}\ln\left(\frac{s/\sqrt{3}+a/2}{s/\sqrt{3}}\right) = -\frac{\mu_0 I^2}{2\sqrt{3}\pi}\ln\left(1+\frac{\sqrt{3}a}{2s}\right)$. The force on the right side is the same, so the net force on the triangle is $\boxed{\frac{\mu_0 I^2}{2\pi}\left[1 - \frac{2}{\sqrt{3}}\ln\left(1+\frac{\sqrt{3}a}{2s}\right)\right]}$.

$\frac{a}{s}$



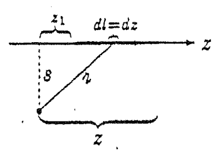
Problem 5.12

Magnetic attraction per unit length (Eqs. 5.37 and 5.13): $f_m = \frac{\mu_0}{2\pi}\frac{\lambda^2 v^2}{d}$.

Electric field of one wire (Eq. 2.9): $E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{s}$. Electric repulsion per unit length on the other wire: $f_e = \frac{1}{2\pi\epsilon_0}\frac{\lambda^2}{d}$. They balance when $\mu_0 v^2 = \frac{1}{\epsilon_0}$, or $\boxed{v = \frac{1}{\sqrt{\epsilon_0\mu_0}}.}$ Putting in the numbers,

$v = \frac{1}{\sqrt{(8.85\times10^{-12})(4\pi\times10^{-7})}} = \boxed{3.00\times10^8\ \mathrm{m/s}.}$ This is precisely the *speed of light(!)*, so in fact you could *never* get the wires going fast enough; the electric force always dominates.

Problem 5.22



$$\mathbf{A} = \frac{\mu_0}{4\pi}\int \frac{I\,\hat{\mathbf{z}}}{\mathscr{r}}\,dz = \frac{\mu_0 I}{4\pi}\,\hat{\mathbf{z}}\int_{z_1}^{z_2}\frac{dz}{\sqrt{z^2+s^2}}$$

$$= \frac{\mu_0 I}{4\pi}\,\hat{\mathbf{z}}\left[\ln\left(z+\sqrt{z^2+s^2}\right)\right]\Big|_{z_1}^{z_2} = \boxed{\frac{\mu_0 I}{4\pi}\ln\left[\frac{z_2+\sqrt{(z_2)^2+s^2}}{z_1+\sqrt{(z_1)^2+s^2}}\right]\hat{\mathbf{z}}}$$

$$\mathbf{B} = \nabla\times\mathbf{A} = -\frac{\partial A}{\partial s}\,\hat{\boldsymbol{\phi}} = -\frac{\mu_0 I}{4\pi}\left[\frac{1}{z_2+\sqrt{(z_2)^2+s^2}}\frac{s}{\sqrt{(z_2)^2+s^2}} - \frac{1}{z_1+\sqrt{(z_1)^2+s^2}}\frac{s}{\sqrt{(z_1)^2+s^2}}\right]\hat{\boldsymbol{\phi}}$$

$$= -\frac{\mu_0 I s}{4\pi}\left[\frac{z_2-\sqrt{(z_2)^2+s^2}}{(z_2)^2-[(z_2)^2+s^2]}\frac{1}{\sqrt{(z_2)^2+s^2}} - \frac{z_1-\sqrt{(z_1)^2+s^2}}{z_1^2-[(z_1)^2+s^2]}\frac{1}{\sqrt{(z_1)^2+s^2}}\right]\hat{\boldsymbol{\phi}}$$

$$= -\frac{\mu_0 I s}{4\pi}\left(-\frac{1}{s^2}\right)\left[\frac{z_2}{\sqrt{(z_2)^2+s^2}} - 1 - \frac{z_1}{\sqrt{(z_1)^2+s^2}} + 1\right]\hat{\boldsymbol{\phi}} = \frac{\mu_0 I}{4\pi s}\left[\frac{z_2}{\sqrt{(z_2)^2+s^2}} - \frac{z_1}{\sqrt{(z_1)^2+s^2}}\right]\hat{\boldsymbol{\phi}},$$

or, since $\sin\theta_1 = \frac{z_1}{\sqrt{(z_1)^2+s^2}}$ and $\sin\theta_2 = \frac{z_2}{\sqrt{(z_2)^2+s^2}}$,

$$= \boxed{\frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)\,\hat{\boldsymbol{\phi}}}\ \text{(as in Eq. 5.35).}$$