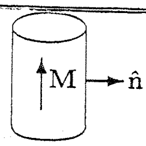
Øving 6 - Løsningsforslag

Problem 6.7

$\mathbf{J}_b = \nabla\times\mathbf{M} = 0$; $\mathbf{K}_b = \mathbf{M}\times\hat{\mathbf{n}} = M\hat{\boldsymbol{\phi}}$.



The field is that of a surface current $\mathbf{K}_b = M\hat{\boldsymbol{\phi}}$, but that's just a solenoid, so the field $\boxed{\text{outside is zero,}}$ and inside $B = \mu_0 K_b = \mu_0 M$. Moreover, it points upward (in the drawing), so $\boxed{\mathbf{B} = \mu_0\mathbf{M}.}$

Problem 6.26

At the interface, the perpendicular component of B is continuous (Eq. 6.26), and the parallel component of H is continuous (Eq. 6.25 with $K_f = 0$). So $B_1^{\perp} = B_2^{\perp}$, $\mathbf{H}_1^{\parallel} = \mathbf{H}_2^{\parallel}$. But $\mathbf{B} = \mu\mathbf{H}$ (Eq. 6.31), so $\frac{1}{\mu_1}\mathbf{B}_1^{\parallel} = \frac{1}{\mu_2}\mathbf{B}_2^{\parallel}$. Now $\tan\theta_1 = B_1^{\parallel}/B_1^{\perp}$, and $\tan\theta_2 = B_2^{\parallel}/B_2^{\perp}$, so

$$\frac{\tan\theta_2}{\tan\theta_1} = \frac{B_2^{\parallel}}{B_2^{\perp}}\frac{B_1^{\perp}}{B_1^{\parallel}} = \frac{B_2^{\parallel}}{B_1^{\parallel}} = \frac{\mu_2}{\mu_1}$$

(the same form, though for different reasons, as Eq. 4.68).

Problem 7.7

(a) $\mathcal{E} = -\frac{d\Phi}{dt} = -Bl\frac{dx}{dt} = -Blv$; $\mathcal{E} = IR \Rightarrow \boxed{I = \frac{Blv}{R}.}$ (Never mind the minus sign—it just tells you the *direction* of flow: $(\mathbf{v}\times\mathbf{B})$ is *upward*, in the bar, so *downward* through the resistor.)

(b) $F = IlB = \boxed{\frac{B^2l^2v}{R},}$ to the $\boxed{\text{left.}}$

(c) $F = ma = m\frac{dv}{dt} = -\frac{B^2l^2}{R}v \Rightarrow \frac{dv}{dt} = -(\frac{B^2l^2}{Rm})v \Rightarrow \boxed{v = v_0e^{-\frac{B^2l^2}{mR}t}.}$

(d) The energy goes into heat in the resistor. The power delivered to resistor is I^2R, so

$$\frac{dW}{dt} = I^2R = \frac{B^2l^2v^2}{R^2}R = \frac{B^2l^2}{R}v_0^2e^{-2\alpha t},\ \text{where } \alpha \equiv \frac{B^2l^2}{mR};\quad \frac{dW}{dt} = \alpha m v_0^2 e^{-2\alpha t}.$$

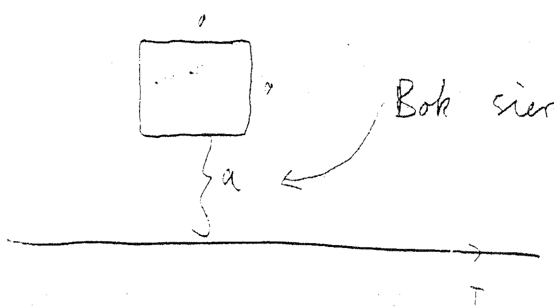
The total energy delivered to the resistor is $W = \alpha m v_0^2\int_0^{\infty} e^{-2\alpha t}dt = \alpha m v_0^2 \left.\frac{e^{-2\alpha t}}{-2\alpha}\right|_0^{\infty} = \alpha m v_0^2\frac{1}{2\alpha} = \frac{1}{2}mv_0^2.$ ✓

Problem 7.18

$$\Phi = \int \mathbf{B}\cdot d\mathbf{a};\ \mathbf{B} = \frac{\mu_0 I}{2\pi s}\hat{\boldsymbol{\phi}};\ \Phi = \frac{\mu_0 I a}{2\pi}\int_a^{2a}\frac{ds}{s} = \frac{\mu_0 I a\ln 2}{2\pi};\ \mathcal{E} = I_{\text{loop}}R = \frac{dQ}{dt}R = -\frac{d\Phi}{dt} = -\frac{\mu_0 a\ln 2}{2\pi}\frac{dI}{dt}.$$

$$dQ = -\frac{\mu_0 a\ln 2}{2\pi R}dI \Rightarrow \boxed{Q = \frac{I\mu_0 a\ln 2}{2\pi R}.}$$

The field of the wire, at the square loop, is *out of the page*, and *decreasing*, so the field of the induced current must point out of page, within the loop, and hence the induced current flows $\boxed{\text{counterclockwise.}}$



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