TFY 4240 Løsning Øving 8 2006

Problem 8.5

(a) $E_x = E_y = 0,\ E_z = -\sigma/\epsilon_0$. Therefore

$$T_{xy} = T_{xz} = T_{yz} = \cdots = 0; \quad T_{xx} = T_{yy} = -\frac{\epsilon_0}{2}E^2 = -\frac{\sigma^2}{2\epsilon_0}; \quad T_{zz} = \epsilon_0\left(E_z^2 - \frac{1}{2}E^2\right) = \frac{\epsilon_0}{2}E^2 = \frac{\sigma^2}{2\epsilon_0}.$$

$$\boxed{\overleftrightarrow{\mathbf{T}} = \frac{\sigma^2}{2\epsilon_0}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \end{pmatrix}.}$$

(b) $\mathbf{F} = \oint \overleftrightarrow{\mathbf{T}} \cdot d\mathbf{a}$ ($\mathbf{S} = 0$, since $\mathbf{B} = 0$); integrate over the xy plane: $d\mathbf{a} = -dx\,dy\,\hat{\mathbf{z}}$ (negative because *outward* with respect to a surface enclosing the upper plate). Therefore

$$F_z = \int T_{zz}\,da_z = -\frac{\sigma^2}{2\epsilon_0}A, \text{ and the force per unit area is } \mathbf{f} = \frac{\mathbf{F}}{A} = \boxed{-\frac{\sigma^2}{2\epsilon_0}\,\hat{\mathbf{z}}.}$$

(c) $-T_{zz} = \boxed{\sigma^2/2\epsilon_0}$ is the momentum in the z direction crossing a surface perpendicular to z, per unit area, per unit time (Eq. 8.31).

(d) The recoil force is the momentum delivered per unit time, so the force per unit area on the top plate is

$$\boxed{\mathbf{f} = -\frac{\sigma^2}{2\epsilon_0}\,\hat{\mathbf{z}}} \quad \text{(same as (b))}.$$

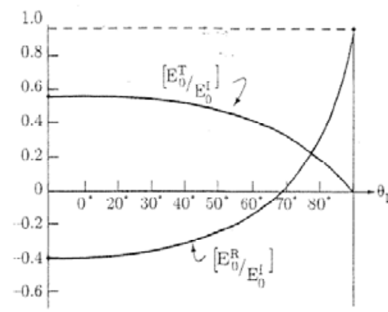
Problem 9.10

$P = \dfrac{I}{c} = \dfrac{1.3 \times 10^3}{3.0 \times 10^8} = \boxed{4.3 \times 10^{-6}\,\mathrm{N/m^2}.}$ For a perfect reflector the pressure is twice as great: $\boxed{8.6 \times 10^{-6}\,\mathrm{N/m^2}.}$ Atmospheric pressure is $1.03 \times 10^5\,\mathrm{N/m^2}$, so the pressure of light on a reflector is $(8.6 \times 10^{-6})/(1.03 \times 10^5) = \boxed{8.3 \times 10^{-11} \text{ atmospheres.}}$

Problem 9.17

Equation 9.106 $\Rightarrow \beta = 2.42$; Eq. 9.110 $\Rightarrow$

$\alpha = \dfrac{\sqrt{1-(\sin\theta/2.42)^2}}{\cos\theta}$.



(a) $\theta = 0 \Rightarrow \alpha = 1$. Eq. 9.109 $\Rightarrow \left(\dfrac{E_{0_R}}{E_{0_I}}\right) = \dfrac{\alpha-\beta}{\alpha+\beta} = \dfrac{1-2.42}{1+2.42} = -\dfrac{1.42}{3.42} = \boxed{-0.415;}$

$\left(\dfrac{E_{0_T}}{E_{0_I}}\right) = \dfrac{2}{\alpha+\beta} = \dfrac{2}{3.42} = \boxed{0.585.}$

(b) Equation 9.112 $\Rightarrow \theta_B = \tan^{-1}(2.42) = \boxed{67.5°.}$

(c) $E_{0_R} = E_{0_T} \Rightarrow \alpha - \beta = 2; \alpha = \beta + 2 = 4.42;$
$(4.42)^2\cos^2\theta = 1 - \sin^2\theta/(2.42)^2;$
$(4.42)^2(1-\sin^2\theta) = (4.42)^2 - (4.42)^2\sin^2\theta$
$= 1 - 0.171\sin^2\theta;\ 19.5 - 1 = (19.5 - 0.17)\sin^2\theta;$
$18.5 = 19.3\sin^2\theta;\ \sin^2\theta = 18.5/19.3 = 0.959;$
$\sin\theta = 0.979;\ \boxed{\theta = 78.3°.}$

Problem 9.18

(a) Equation 9.120 $\Rightarrow \tau = \epsilon/\sigma$. Now $\epsilon = \epsilon_0\epsilon_r$ (Eq. 4.34), $\epsilon_r \cong n^2$ (Eq. 9.70), and for glass the index of refraction is typically around 1.5, so $\epsilon \approx (1.5)^2 \times 8.85\times10^{-12} = 2\times10^{-11}\ \mathrm{C^2/N\,m^2}$, while $\sigma = 1/\rho \approx 10^{-12}\ \Omega\,\mathrm{m}$ (Table 7.1). Then $\tau = (2\times10^{-11})/10^{-12} = \boxed{20\ \mathrm{s.}}$ (But the resistivity of glass varies enormously from one type to another, so this answer could be off by a factor of 100 in either direction.)

(b) For silver, $\rho = 1.59\times10^{-8}$ (Table 7.1), and $\epsilon \approx \epsilon_0$, so $\omega\epsilon = 2\pi\times10^{10}\times8.85\times10^{-12} = 0.56$. Since $\sigma = 1/\rho = 6.25\times10^7 \gg \omega\epsilon$, the skin depth (Eq. 9.128) is

$$d = \frac{1}{\kappa} \cong \sqrt{\frac{2}{\omega\sigma\mu}} = \sqrt{\frac{2}{2\pi\times10^{10}\times6.25\times10^7\times4\pi\times10^{-7}}} = 6.4\times10^{-7}\ \mathrm{m} = 6.4\times10^{-4}\ \mathrm{mm}.$$

I'd plate silver to a depth of about $\boxed{0.001\ \mathrm{mm};}$ there's no point in making it any thicker, since the fields don't penetrate much beyond this anyway.

(c) For copper, Table 7.1 gives $\sigma = 1/(1.68\times10^{-8}) = 6\times10^7$, $\omega\epsilon_0 = (2\pi\times10^6)\times(8.85\times10^{-12}) = 6\times10^{-5}$. Since $\sigma \gg \omega\epsilon$, Eq. 9.126 $\Rightarrow k \approx \sqrt{\dfrac{\omega\sigma\mu}{2}}$, so (Eq. 9.129)

$$\lambda = 2\pi\sqrt{\frac{2}{\omega\sigma\mu_0}} = 2\pi\sqrt{\frac{2}{2\pi\times10^6\times6\times10^7\times4\pi\times10^{-7}}} = 4\times10^{-4}\ \mathrm{m} = \boxed{0.4\ \mathrm{mm.}}$$

¿From Eq. 9.129, the propagation speed is $v = \dfrac{\omega}{k} = \dfrac{\omega}{2\pi}\lambda = \lambda\nu = (4\times10^{-4})\times10^6 = \boxed{400\ \mathrm{m/s.}}$ In vacuum, $\lambda = \dfrac{c}{\nu} = \dfrac{3\times10^8}{10^6} = \boxed{300\ \mathrm{m};}$ $v = c = \boxed{3\times10^8\ \mathrm{m/s.}}$ (But really, in a good conductor the skin depth is so small, compared to the wavelength, that the notions of "wavelength" and "propagation speed" lose their meaning.)